Exercise 6.1: Trigonometric Identities

Prove the following trigonometric identities

Q1: (1-Cos^2 A) Cosec^2 A = 1

Ans: (1-Cos^2 A) Cosec^2 A = Sin^2 A Cosec^2 A

= (Sin A Cosec A)^2

= (Sin A x (1/Sin A))^2

= (1)^2 = 1

Q2: (1 + Cot^2 A) Sin^2 A = 1

Ans: We know, Cosec^2 A –Cot ^{2} A = 1

So,

(1 + Cot^2 A) Sin^2 A = Cosec^2 A Sin^2 A

= (Cosec A Sin A)^2

= ((1/Sin A) x Sin A)^2

= (1)^2 = 1

Q3: **tan^{2}θcos^{2}θ**$tan^2\theta\ cos^2\theta$**= 1−cos^{2}θ**$1 - cos^2\theta$

A3: We know ,

sin^{2}θ+cos^{2}θ=1$sin^2\theta + cos^2\theta = 1$

So,

$\tan^2\theta\cos^2\theta tan^2\theta\ cos^2\theta$ = $(\tan\theta\times\cos\theta)^2(tan\theta \times cos\theta)^2$

$=(\frac{\sin\theta}{\cos\theta}\times\cos\theta)^2 = (\frac{sin\theta}{cos\theta} \times cos\theta)^2$ $=(\sin\theta)^2 = (sin\theta)^2$ $=\sin^2\theta = sin^2\theta$ $1-\cos^2\theta 1 - cos^2\theta$

Q4: $\mathbf{cosec\theta\sqrt{1-cos^2\theta}=1}$$cosec\theta\sqrt{1 - cos^2\theta} = 1$

A4: We know ,

$\sin^2\theta+\cos^2\theta=1 sin^2\theta + cos^2\theta = 1$

So,

$cosec\theta\sqrt{1-\cos^2\theta}=cosec\theta\sqrt{\sin^2\theta} cosec\theta\sqrt{1 - cos^2\theta} = cosec\theta\sqrt{sin^2\theta}$

$= = cosec\theta\sin\theta = cosec\theta\ sin\theta$

$= = \frac{1}{\sin\theta}\sin\theta = \frac{1}{sin\theta}\ sin\theta$

= 1

Q5 : $\mathbf{(sec^2\theta-1)(cosec^2\theta-1)=1}$$(sec^2\theta - 1)(cosec^2\theta - 1) = 1$

A5: We know that,

$(\sec^2\theta-\tan^2\theta)=1 (sec^2\theta - tan^2\theta) = 1$ $(cosec^2\theta-\cot^2\theta)=1 (cosec^2\theta - cot^2\theta) = 1$

So,

$(\sec^2\theta-1)(cosec^2\theta-1)=\tan^2\theta\times\cot^2\theta (sec^2\theta - 1)(cosec^2\theta - 1) = tan^2\theta \times cot^2\theta$ $=(\tan\theta\times\cot\theta)^2 = (tan\theta \times cot\theta)^2$ $=(\tan\theta\times\frac{1}{\tan\theta})^2 = (tan\theta \times \frac{1}{tan\theta})^2$

$= 1^2 = 1$

Q6: $\mathbf{tan\theta+\frac{1}{tan\theta}=sec\theta cosec\theta}$$tan\theta + \frac{1}{tan\theta} = sec\theta\ cosec\theta$

A6: We know that,

$(\sec^2\theta-\tan^2\theta)=1 (sec^2\theta - tan^2\theta) = 1$

So,

tanθ+1tanθ=tan^2θ+1tanθ $tan\theta + \frac{1}{tan\theta} = \frac{tan^2\theta+1}{tan\theta}$ =sec^2θtanθ $= \frac{sec^2\theta}{tan\theta}$ =secθ secθtanθ $= sec\theta\frac{sec\theta}{tan\theta}$

=secθ 1cosθ sinθcosθ $= sec\theta\frac{\frac{1}{cos\theta}}{\frac{sin\theta}{cos\theta}}$ =secθ 1sinθ $= sec\theta\frac{1}{sin\theta}$ Undefined control sequence \thetacosec

Undefined control sequence \thetacosec

Q7: **cosθ1−sinθ=1+sinθcosθ** $\frac{cos\theta}{1-sin\theta} = \frac{1+sin\theta}{cos\theta}$

A7: We know ,

sin^2θ+cos^2θ=1 $sin^2\theta + cos^2\theta = 1$

So, Multiplying both numerator and denominator by (1+sinθ) $(1+sin\theta)$, we have

cosθ1−sinθ=cosθ(1+sinθ)(1−sinθ)(1+sinθ) $\frac{cos\theta}{1-sin\theta} = \frac{cos\theta(1+sin\theta)}{(1-sin\theta)(1+sin\theta)}$ =cosθ(1+sinθ)(1−sin^2θ) $= \frac{cos\theta(1+sin\theta)}{(1-sin^2\theta)}$

=cosθ(1+sinθ)cos^2θ $= \frac{cos\theta(1+sin\theta)}{cos^2\theta}$ =(1+sinθ)cosθ $= \frac{(1+sin\theta)}{cos\theta}$

Q8: **cosθ1+sinθ=1−sinθcosθ** $\frac{cos\theta}{1+sin\theta} = \frac{1-sin\theta}{cos\theta}$

A8: We know ,

sin^2θ+cos^2θ=1 $sin^2\theta + cos^2\theta = 1$

Multiplying both numerator and denominator by (1−sinθ) $(1-sin\theta)$, we have

cosθ1+sinθ=cosθ(1−sinθ)(1+sinθ)(1−sinθ) $\frac{cos\theta}{1+sin\theta} = \frac{cos\theta(1-sin\theta)}{(1+sin\theta)(1-sin\theta)}$ =cosθ(1−sinθ)(1−sin^2θ) $= \frac{cos\theta(1-sin\theta)}{(1-sin^2\theta)}$

=cosθ(1−sinθ)(cos^2θ) $= \frac{cos\theta(1-sin\theta)}{(cos^2\theta)}$ =(1−sinθ)cosθ $= \frac{(1-sin\theta)}{cos\theta}$ =(1−sinθ)cosθ $= \frac{(1-sin\theta)}{cos\theta}$

Q 9:cos^2A + 11+cot^2A $\frac{1}{1+cot^2A}$ **= 1**

A9: We know that,

$Sin^2A + cos^2A = 1$

$cosec^2A – cot^2A = 1$

So, cos^2A+11+cot^2A=cos^2A+1$cosec^2$A $cos^2A + \frac{1}{1+cot^2A} = cos^2A + \frac{1}{cosec^2A}$

=cos^2A+(1cosecA$)^2$ $= cos^2A + (\frac{1}{cosecA})^2$ =cos^2A+sinA^2 $= cos^2A + sinA^2$

= 1

Q10: **sinA^2+ 11+tan²A =1** $sinA^2+\frac{1}{1+tan^2A}=1$

A10: We know,

Sin^{2}A + cos^{2}A = 1

sec^{2}A – tan^{2}A = 1

So,

sinA^2+ 11+tan²A =sinA^2+ 1sec²A $sinA^2+\frac{1}{1+tan^2A}=sinA^2+\frac{1}{sec^2A}$ =sinA^2+(1secA)^2 $=sinA^2+(\frac{1}{secA})^2$ =sinA^2+cos^{2}A$=sinA^2+cos^2A$

= 1

Q11: **√1−cosθ1+cosθ =cosecθ−cotθ** $\sqrt{\frac{1-cos\theta}{1+cos\theta}}=cosec\theta-cot\theta$

A11: We know ,

sin^{2}θ+cos^{2}θ=1$sin^2\theta+cos^2\theta=1$

Multiplying both numerator and denominator by (1−cosθ)$(1-cos\theta)$, we have

√1−cosθ1+cosθ =√(1−cosθ)(1−cosθ)(1+cosθ)(1−cosθ) $\sqrt{\frac{1-cos\theta}{1+cos\theta}}=\sqrt{\frac{(1-cos\theta)(1-cos\theta)}{(1+cos\theta)(1-cos\theta)}}$ =√(1−cosθ)²1−cos²θ $=\sqrt{\frac{(1-cos\theta)^2}{1-cos^2\theta}}$ =√(1−cosθ)²sin²θ$=\sqrt{\frac{(1-cos\theta)^2}{sin^2\theta}}$ =(1−cosθ)sinθ$=\frac{(1-cos\theta)}{sin\theta}$ =1sinθ−cosθsinθ$=\frac{1}{sin\theta}-\frac{cos\theta}{sin\theta}$

\(=\frac{1}{sin\theta}-\frac{cos\theta}{sin\theta}[/latex\]

Q12: **[latex]\frac{1-cos\theta }{sin\theta}=\frac{sin\theta}{1+cos\theta}\)**

A12: We know ,

sin^{2}θ+cos^{2}θ=1$sin^2\theta+cos^2\theta=1$

Multiplying both numerator and denominator by (1+cosθ)$(1+cos\theta)$, we have

=(1−cos²θ)(1+cosθ)(sinθ)$=\frac{(1-cos^2\theta)}{(1+cos\theta)(sin\theta)}$ =(sin²θ)(1+cosθ)(sinθ)$=\frac{(sin^2\theta)}{(1+cos\theta)(sin\theta)}$ =(sinθ)(1+cosθ)$=\frac{(sin\theta)}{(1+cos\theta)}$

Q13. **sinθ1−cosθ** $\frac{sin\theta}{1-cos\theta}$ **= cosecθ**θ **+ cotθ**θ

Ans:

Given, L.H.S = sinθ1−cosθ $\frac{sin\theta}{1-cos\theta}$

Rationalize both nr and dr with 1+cos θθ

= sinθ1−cosθ $\frac{sin\theta}{1-cos\theta}$ * 1+cosθ1+cosθ $\frac{1+cos\theta}{1+cos\theta}$

We know that, (a-b)(a+b) = $a^2 - b^2$

=> sinθ(1+cosθ)1−cos²θ $\frac{sin\theta(1+cos\theta)}{1-cos^2\theta}$

Here, (1-cos² θθ) = sin² θθ

=> sinθ+(sinθ∗cosθ)sin²θ $\frac{sin\theta + (sin\theta * cos\theta)}{sin^2\theta}$

=> sinθsin²θ $\frac{sin\theta}{sin^2\theta}$ + sinθ∗cosθsin²θ $\frac{sin\theta * cos\theta}{sin^2\theta}$

=> 1sinθ $\frac{1}{sin\theta}$ + cosθsinθ $\frac{cos\theta}{sin\theta}$

=> cosec θθ + cot θθ

Hence, L.H.S = R.H.S

Q14. **1−sinθ1+sinθ** $\frac{1 - sin\theta}{1 + sin\theta}$ **= (secθ−tanθ)²**$(sec\theta \; - \; tan\theta)^2$

Ans:

Given, L.H.S = 1−sinθ1+sinθ $\frac{1 - sin\theta}{1 + sin\theta}$

Rationalize with nr and dr with 1 – sin θθ

=> 1−sinθ1+sinθ $\frac{1 - sin\theta}{1 + sin\theta}$ * 1−sinθ1−sinθ $\frac{1 - sin\theta}{1 - sin\theta}$

Here, (1-sin θθ)(1+sin θθ) = cos² θθ

=> (1−sinθ)²cos²θ $\frac{(1 - sin\theta)^2}{cos^2\theta}$

=> (1−sinθcosθ)²$(\frac{1 - sin\theta}{cos\theta})^2$

=> (1cosθ − sinθcosθ)²$(\frac{1}{cos\theta} \; - \; \frac{sin\theta}{cos\theta})^2$

=> (secθ−tanθ)^2 $(sec\theta\ -\ tan\theta)^2$

Hence, L.H.S = R.H.S

Q15. **(1+cot^{2}θ)tanθsec^{2}θ** $\frac{(1\ +\ cot^2\theta)tan\theta}{sec^2\theta}$ **= cot\theta**

Ans:

Given, L.H.S = (1+cot^{2}θ)tanθsec^{2}θ $\frac{(1\ +\ cot^2\theta)tan\theta}{sec^2\theta}$

Here, 1 + cot^2 θθ =cosec^2 θθ

=> cosec^{2}θ∗tanθsec^{2}θ $\frac{cosec^2\theta*tan\theta}{sec^2\theta}$

=> 1sin^{2}θ $\frac{1}{sin^2\theta}$ * cos^{2}θ1 $\frac{cos^2\theta}{1}$ * sinθcosθ $\frac{sin\theta}{cos\theta}$

=> cosθsinθ $\frac{cos\theta}{sin\theta}$

=> cot θθ

Hence, L.H.S = R.H.S

Q16. **tan^{2}θ−sin^{2}θ**$tan^2\theta\ -\ sin^2\theta$ **= tan^{2}θ∗sin^{2}θ**$tan^2\theta\ *\ sin^2\theta$

Ans:

Given, L.H.S = tan^{2}θ−sin^{2}θ$tan^2\theta\ -\ sin^2\theta$

Here, tan^2 θθ = sin^{2}θcos^{2}θ $\frac{sin^2\theta}{cos^2\theta}$

=> sin^{2}θcos^{2}θ $\frac{sin^2\theta}{cos^2\theta}$ – sin^{2}θ$sin^2\theta$

=> sin^{2}θ$sin^2\theta$[1cos^{2}θ $\frac{1}{cos^2\theta}$ – 1]

=> sin^{2}θ$sin^2\theta$[1−cos^{2}θcos^{2}θ $\frac{1\ -\ cos^2\theta}{cos^2\theta}$]

=> sin^{2}θcos^{2}θ $\frac{sin^2\theta}{cos^2\theta}$ * sin^{2}θ$sin^2\theta$

=> tan^{2}θ∗sin^{2}θ$tan^2\theta\ *\ sin^2\theta$

Hence, L.H.S = R.H.S

Q17. (cosec θθ + sin θθ)(cosec θθ – sin θθ) = cot^{2}θ+cos^{2}θ$cot^2\theta + cos^2\theta$

Ans:

Given, L.H.S = (cosec θθ + sin θθ)(cosec θθ – sin θθ)

Here, (a + b)(a – b) = $a^2 - b^2$

cosec^2 θθ can be written as 1 + cot^2 θθ and sin^2 θθ can be written as 1 – cos^2 θθ

=> 1 + cot^2 θθ – (1 – cos^2 θθ)

=> 1 + cot^2 θθ – 1 + cos^2 θθ

=> cot^2 θθ + cos^2 θθ

Hence, L.H.S = R.H.S

Q18. (secθ$sec\theta$ +cos θθ)(sec θθ – cos θθ) = tan^{2}θ+sin^{2}θ$tan^2\theta + sin^2\theta$

Ans:

Given, L.H.S = (sec θθ + cos θθ)(sec θθ – cos θθ)

Here, (a + b)(a – b) = $a^2 - b^2$

sec^{2}θθ can be written as 1 + tan^2 θθ and cos^2 θθ can be written as 1 – sin^2 θθ

=> 1 + tan^2 θθ – (1 – sin^2 θθ)

=> 1 + tan^2 θθ – 1 + sin^2 θθ

=> tan^2 θθ + sin^2 θθ

Hence, L.H.S = R.H.S

Q19. secA(1 – sinA)(secA + tanA) = 1

Ans:

Given, L.H.S = secA(1 – sinA)(secA + tanA)

Here, secA = 1cosA $\frac{1}{cos A}$ and tanA = sinAcosA $\frac{sin A}{cos A}$

=> 1cosA $\frac{1}{cos A}$ * (1 – sinA) * 1+sinAcosA $\frac{1+sin A}{cos A}$

=> \(\frac{cos^{2}A}{cos^{2}A}\)

=> 1

Hence, L.H.S = R.H.S

Q20. (cosecA – sinA)(secA – cosA)(tanA + cotA) = 1

Ans:

Given, L.H.S = (cosecA – sinA)(secA – cosA)(tanA + cotA)

Here, cosecA = 1sinA $\frac{1}{sinA}$, secA = 1cosA $\frac{1}{cosA}$, tanA = sinAcosA $\frac{sinA}{cosA}$, cotA = cosAsinA $\frac{cosA}{sinA}$

Substitute the above values in L.H.S

=> (1sinA $\frac{1}{sinA}$ – sinA)(1cosA $\frac{1}{cosA}$ – cosA)(sinAcosA $\frac{sinA}{cosA}$ + cosAsinA $\frac{cosA}{sinA}$)

=> (1–sin^{2}AsinA $\frac{1\ -sin^2A}{sinA}$) * (1–cos^{2}AcosA $\frac{1\ -cos^2A}{cosA}$) * (

sin^{2}A+cos^{2}AsinA∗cosA $\frac{sin^2A\ +\ cos^2A}{sinA\ *\ cosA}$

Here, (\(\frac{1\;-sin^{2}A}) = cos^{2}A, ([latex]\frac{1\;-cos^{2}A}) = sin^{2}A, sin^{2}A + cos^{2}A = 1

=> [latex]\frac{sin^{2}A\;*\;cos^{2}A\;*\;1}{sin^{2}A\;*\;cos^{2}A}\)

=> 1

Hence, L.H.S = R.H.S

Q21. (1 + tan^{2}θ$tan^2\theta$)(1 – sin θθ)(1 + sin θθ) = 1

Ans:

Given, L.H.S = (1 + tan^{2}θ$tan^2\theta$)(1 – sin\theta)(1 + sin\theta)

We know that,

Sin^2 θθ + cos^2 θθ = 1

And sec^2 θθ – tan^2 θθ = 1

So,

(1 + tan^{2}θ$tan^2\theta$)(1 – sin θθ)(1 + sin θθ) = (1 + tan^{2}θ$tan^2\theta$){(1 – sin θθ)(1 + sin θθ)}

= (1 + tan^{2}θ$tan^2\theta$)((1 – sin^{2}θ$sin^2\theta$)

= sec^{2}θ∗tan^{2}θ$sec^2\theta\ *\ tan^2\theta$

= (1cos²θ)∗cos^{2}θ$(\frac{1}{cos^2\theta})\ *\ cos^2\theta$

= 1

hence, L.H.S = R.H.S

Q22. (sin^{2}A∗cot^{2}A)+(cos^{2}A;∗tan^{2}A)$(sin^2 A\;*\;cot^2 A)\;+\;(cos^2 A;*\;tan^2 A)$ **= 1**

Ans:

Given, L.H.S = Undefined control sequence \A Undefined control sequence \A

Here, \((sin^{2}A\;+\;cos^{2}A) = 1

So,

[latex](sin^{2}\A\;*\;cot^{2}A)\;+\;(cos^{2}A\;*\;tan^{2}A)\) = sin^{2}A(cos^{2}Asin^{2}A $\frac{cos^2 A}{sin^2 A}$) + cos^{2}A(sin^{2}Acos^{2}A $\frac{sin^2 A}{cos^2 A}$

= cos^{2}A + sin^{2}A

= 1

Hence , L.H.S = R.H.S

Q23:

1. **cot θ**θ **– tan θ**θ **= 2cos^{2}θ−1sinθ∗cosθ** $\frac{2cos^2\theta - 1}{sin\theta * cos\theta}$

Ans:

Give, L.H.S = cot θθ – tan θθ

Here, Sin^2 θθ + cos^2 θθ = 1

So,

=> cot θθ – tan θθ = cosθsinθ $\frac{cos\theta}{sin\theta}$ – sinθcosθ $\frac{sin\theta}{cos\theta}$

= cos^{2}θ−sin^{2}θsinθ∗cosθ $\frac{cos^2\theta - sin^2\theta}{sin\theta * cos\theta}$

= cos^{2}θ−(1−cos^{2}θ)sinθ∗cosθ $\frac{cos^2\theta - (1 - cos^2\theta)}{sin\theta * cos\theta}$

= cos^{2}θ−1−cos^{2}θsinθ∗cosθ $\frac{cos^2\theta - 1 - cos^2\theta}{sin\theta * cos\theta}$

= (2cos^{2}θ−1sinθ∗cosθ)($\frac{2cos^2\theta - 1}{sin\theta * cos\theta}$)

Hence, L.H.S = R.H.S

1. **tanθ−cotθ=**$tan\theta\;-\;cot\theta\;=$ **(2sin²θ−1sinθ∗cosθ)**$(\frac{2sin^2\theta\;-\;1}{sin\theta*cos\theta})$

Sol:

Given, L.H.S = \(tan\theta\;-\;cot\theta

We know that,

Sin^2 [latex]\theta\) + cos^2 θθ = 1

tanθ−cotθ=[latex] sinθcosθ $tan\theta\;-\;cot\theta = [latex]\frac{sin\theta}{cos\theta}$ − cosθsinθ $\frac{cos\theta}{sin\theta}$

= sin²θ−cos²θsinθcosθ $\frac{sin^2\theta\;-\;cos^2\theta}{sin\theta cos\theta}$

= sin²θ−(1−sin²θ)sinθcosθ $\frac{sin^2\theta\;-\;(1\;-\;sin^2\theta)}{sin\theta cos\theta}$

= sin²θ−1+sin²θsinθcosθ $\frac{sin^2\theta\;-\;1\;+\;sin^2\theta}{sin\theta cos\theta}$

= (2sin²θ−1sinθ∗cosθ)$(\frac{2sin^2\theta\;-\;1}{sin\theta*cos\theta})$

Hence, L.H.S = R.H.S

Q24. **cos²θsinθ−cosecθ+sinθ**$\frac{cos^2\theta}{sin\theta}\;-\;cosec\theta\;+\;sin\theta$ **= 0**

Ans:

Given, L.H.S cos²θsinθ−cosecθ+sinθ$\frac{cos^2\theta}{sin\theta}\;-\;cosec\theta\;+\;sin\theta$

We know that,

Sin^2 θθ + cos^2 θθ = 1

So,

cos²θsinθ−cosecθ+sinθ$\frac{cos^2\theta}{sin\theta}\;-\;cosec\theta\;+\;sin\theta$ = (cos²θsinθ−cosecθ)+sinθ$\frac{cos^2\theta}{sin\theta}\;-\;cosec\theta)\;+\;sin\theta$

= (cos²θsinθ − 1sinθ)+sinθ$(\frac{cos^2\theta}{sin\theta}\;-\;\frac{1}{sin\theta})\;+\;sin\theta$

= (cos²θ−1sinθ)+sinθ$(\frac{cos^2\theta\;-\;1}{sin\theta})\;+\;sin\theta$

= (-sin²θsinθ)+sinθ$(\frac{-sin^2\theta}{sin\theta})\;+\;sin\theta$

= −sinθ+sinθ$-sin\theta\;+\;sin\theta$

= 0

Hence, L.H.S = R.H.S

Q 25 . 11+sinA $\frac{1}{1+sin\ A}$ + 11−sinA $\frac{1}{1-sin\ A}$ = 2 sec^2 A

Ans:

LHS = 11+sinA $\frac{1}{1+sin\ A}$ + 11−sinA $\frac{1}{1-sin\ A}$

(1−sinA)+(1+sinA)(1+sinA)(1−sinA) $\frac{(1-sinA)+(1+sinA)}{(1+sinA)(1-sinA)}$ 1-sinA+1+sinA1−sin^{2}A $\frac{1-sinA+1+sinA}{1-sin^2 A}$

⇒21−sin^{2}A⇒ $\frac{2}{1-sin^2\ A}$ [Since , (1 + sin A)(1 – sin A) = 1−sin^{2}A1 $- sin^2 A$]

⇒2cos^{2}A⇒ $\frac{2}{cos^2 A}$ [Since , 1-sin^2 A = cos2 A]

⇒2sec^{2}A⇒ $2sec^2 A$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

Q 26 . 1+sinθcosθ+cosθ1+sinθ =2secθ $\frac{1+sin\theta}{cos\theta} + \frac{cos\theta}{1+sin\theta} = 2sec\theta$

Ans:

LHS = 1+sinθcosθ+cosθ1+sinθ $\frac{1+sin\theta}{cos\theta} + \frac{cos\theta}{1+sin\theta}$

= (1+sinθ)^2+cos^{2}θcosθ(1+sinθ) $\frac{(1+sin\theta)^2+cos^2\theta}{cos\theta(1+sin\theta)}$

= 1+sin^{2}θ+2sinθ+cos^{2}θcosθ(1+sinθ) $\frac{1+sin^2\theta+2sin\theta+cos^2\theta}{cos\theta(1+sin\theta)}$

⇒2(1+sinθ)cosθ(1+sinθ)⇒ $\frac{2(1+sin\theta)}{cos\theta(1+sin\theta)}$ = 2 secθ$sec\theta$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

Q 27 . (1+sinθ)^2+(1−sinθ)^{2}2cos^{2}θ=1+sin^{2}θ1−sin^{2}θ $\frac{(1+sin\theta)^2+(1-sin\theta)^2}{2cos^2\theta} = \frac{1+sin^2\theta}{1-sin^2\theta}$

Ans:

We know that sin^{2}θ+cos^{2}θ=1$sin^2\theta + cos^2\theta = 1$

So,

LHS =

(1+sinθ)^2+(1−sinθ)^{2}2cos²θ = (1+2sinθ+sin^{2}θ)+

(1−2sinθ+sin^{2}θ)2cos²θ = 1+2sinθ+sin^{2}θ+1−2sinθ+sin^{2}θ2cos^{2}θ = 2+2sin^{2}θ2cos^{2}θ = 2(1+sin^{2}θ)2(1−sin^{2}θ) = (1+sin^{2}θ)(1−sin^{2}θ)

$$\frac{(1+sin\theta)^2+(1-sin\theta)^2}{2cos^2\theta}$$
$$=\frac{(1+2sin\theta+sin^2\theta)+(1-2sin\theta+sin^2\theta)}{2cos^2\theta}$$
$$=\frac{1+2sin\theta+sin^2\theta+1-2sin\theta+sin^2\theta}{2cos^2\theta}$$
$$=\frac{2+2sin^2\theta}{2cos^2\theta}$$
$$=\frac{2(1+sin^2\theta)}{2(1-sin^2\theta)}$$
$$=\frac{(1+sin^2\theta)}{(1-sin^2\theta)}$$

∴ Undefined control sequence \therefore LHS = RHS   Hence proved

Q 28 . 1+tan²θ1+cot²θ = [1−tanθcotθ]^2 −tan^{2}θ $\frac{1+tan^2\theta}{1+cot^2\theta}=\left[\frac{1-tan\theta}{cot\theta}\right]^2-tan^2\theta$

Ans :

LHS = 1+tan²θ1+cot²θ $\frac{1+tan^2\theta}{1+cot^2\theta}$

= sec²θcosec²θ $\frac{sec^2\theta}{cosec^2\theta}$ [Since , tan^{2}θ$tan^2\theta$ + 1 = sec^{2}θ$sec^2\theta$, 1 + cot^{2}θ $cot^2\theta$ = cosec^{2}θ$cosec^2\theta$]

= 1cos²θ·1 sin^{2}θ $\frac{1}{cos^2\theta\cdot 1}sin^2\theta$

= tan^{2}θ$tan^2\theta$

∴ Undefined control sequence \therefore LHS = RHS   Hence proved

Q 29 . **1+secθsecθ** $\frac{1+sec\theta}{sec\theta}$ **=** **sin²θ1−cosθ** $\frac{sin^2\theta}{1-cos\theta}$

Ans :

LHS = 1+secθsecθ $\frac{1+sec\theta}{sec\theta}$

= 1+1cosθ 1cosθ $\frac{1+\frac{1}{cos\theta}}{\frac{1}{cos\theta}}$

= cosθ+1cosθ ·cosθ $\frac{cos\theta+1}{cos\theta} \cdot cos\theta$

= 1+cosθ1 $+ cos\theta$

RHS = sin²θ1−cosθ $\frac{sin^2\theta}{1-cos\theta}$

= 1−cos²θ1−cosθ $\frac{1-cos^2\theta}{1-cos\theta}$

= (1−cosθ)(1+cosθ)1−cosθ $\frac{(1-cos\theta)(1+cos\theta)}{1-cos\theta}$

= 1+cosθ1 $+ cos\theta$

∴ Undefined control sequence \therefore LHS = RHS   Hence proved

Q 30 . **tanθ1−cotθ + cotθ1−tanθ** $\frac{tan\theta}{1-cot\theta} + \frac{cot\theta}{1-tan\theta}$ **= 1+tanθ+cotθ**$1 + tan\theta + cot\theta$

Ans:

LHS = tanθ1−1tanθ + cotθ1−tanθ $\frac{tan\theta}{1-\frac{1}{tan\theta}} + \frac{cot\theta}{1-tan\theta}$

= tan²θtanθ−1 + cotθ1−tanθ $\frac{tan^2\theta}{tan\theta-1} + \frac{cot\theta}{1-tan\theta}$

= 11−tanθ [1tanθ −tan²θ] $\frac{1}{1-tan\theta}\left[\frac{1}{tan\theta} - tan^2\theta\right]$

= 11−tanθ [1−tan³θtanθ] $\frac{1}{1-tan\theta}\left[\frac{1-tan^3\theta}{tan\theta}\right]$

= 11−tanθ (1−tanθ)(1+tanθ+tan²θ)tanθ $\frac{1}{1-tan\theta} \frac{(1-tan\theta)(1+tan\theta+tan^2\theta)}{tan\theta}$ [Since , a³−b³= (a−b)(a²+ab+b²)$a^3 - b^3 = (a-b)\left(a^2+ab+b^2\right)$]

= 1+tanθ+tan²θtanθ $\frac{1+tan\theta+tan^2\theta}{tan\theta}$

$= 1\tan\theta + \tan\theta\tan\theta + \tan^2\theta\tan\theta \frac{1}{tan\theta} + \frac{tan\theta}{tan\theta} + \frac{tan^2\theta}{tan\theta}$

$= 1+\tan\theta+\cot\theta 1 + tan\theta + cot\theta$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

Q 31 . $\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$ $sec^6\theta = tan^6\theta + 3tan^2\theta sec^2\theta + 1$

Ans :

We know that $\sec^2\theta-\tan^2\theta=1$ $sec^2\theta - tan^2\theta = 1$

Cubing both sides

$(\sec^2\theta-\tan^2\theta)^3=1$ $\left(sec^2\theta - tan^2\theta\right)^3 = 1$

$\sec^6\theta-\tan^6\theta-3\sec^2\theta\tan^2\theta(\sec^2\theta-\tan^2\theta)=1$ $sec^6\theta - tan^6\theta - 3sec^2\theta tan^2\theta\left(sec^2\theta - tan^2\theta\right) = 1$ [Since , $a^3-b^3=(a-b)(a^2+ab+b^2)$ $a^3 - b^3 = (a-b)\left(a^2+ab+b^2\right)$]

$\sec^6\theta-\tan^6\theta-3\sec^2\theta\tan^2\theta=1$ $sec^6\theta - tan^6\theta - 3sec^2\theta tan^2\theta = 1$ $\Rightarrow\sec^6\theta=\tan^6\theta+3\sec^2\theta\tan^2\theta+1$ $\Rightarrow sec^6\theta = tan^6\theta + 3sec^2\theta tan^2\theta + 1$

Hence proved.

Q 32 . $\mathrm{cosec}^6\theta=\cot^6\theta+3\cot^2\theta\,\mathrm{cosec}^2\theta+1$ $cosec^6\theta = cot^6\theta + 3cot^2\theta cosec^2\theta + 1$

Ans :

We know that $\mathrm{cosec}^2\theta-\cot^2\theta=1$ $cosec^2\theta - cot^2\theta = 1$

Cubing both sides

$(\mathrm{cosec}^2\theta-\cot^2\theta)^3=1$ $\left(cosec^2\theta - cot^2\theta\right)^3 = 1$

$\mathrm{cosec}^6\theta-\cot^6\theta-3\mathrm{cosec}^2\theta\cot^2\theta(\mathrm{cosec}^2\theta-\cot^2\theta)=1$ $cosec^6\theta - cot^6\theta - 3cosec^2\theta cot^2\theta\left(cosec^2\theta - cot^2\theta\right) = 1$ [Since , $a^3-b^3=(a-b)(a^2+ab+b^2)$ $a^3 - b^3 = (a-b)\left(a^2+ab+b^2\right)$]

$\mathrm{cosec}^6\theta-\cot^6\theta-3\mathrm{cosec}^2\theta\cot^2\theta=1$ $cosec^6\theta - cot^6\theta - 3cosec^2\theta cot^2\theta = 1$ $\Rightarrow\mathrm{cosec}^6\theta=\cot^6\theta+3\mathrm{cosec}^2\theta\cot^2\theta+1$ $\Rightarrow cosec^6\theta = cot^6\theta + 3cosec^2\theta cot^2\theta + 1$

Hence proved.

Q 33 . (1+tan²θ)cotθcosec²θ =tanθ $\frac{(1+tan^2\theta)cot\theta}{cosec^2\theta} = tan\theta$

Ans :

We know that sec²θ−tan²θ=1 $sec^2\theta - tan^2\theta = 1$

Therefore , sec²θ=1+tan²θ $sec^2\theta = 1 + tan^2\theta$

LHS = sec²θ·cotθcosec²θ $\frac{sec^2\theta \cdot cot\theta}{cosec^2\theta}$

= 1·sin²θcos²θ · cosθsinθ $\frac{1 \cdot sin^2\theta}{cos^2\theta} \cdot \frac{cos\theta}{sin\theta}$ [∵secθ= 1cosθ ,cosecθ= 1sinθ ,cotθ= cosθsinθ]

Undefined control sequence \because

⇒ sinθcosθ =tanθ⇒ $\frac{sin\theta}{cos\theta} = tan\theta$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

Q 34 . \(\frac{ 1+cosA}{sin ^{2} A}\) = 11−cosA $\frac{1}{1-cosA}$

Ans:

We know that sin²A+cos²A $sin^2A + cos^2A$ = 1

sin²A=1−cos²A $sin^2A = 1 - cos^2A$ ⇒sin²A=(1−cosA)(1+cosA)
⇒ $sin^2A = (1 - cosA)(1 + cosA)$ ⇒LHS= (1+cosA)(1−cosA)(1+cosA) ⇒ $LHS = \frac{(1+cosA)}{(1-cosA)(1+cosA)}$

= ⇒LHS= 1(1−cosA) ⇒ $LHS = \frac{1}{(1-cosA)}$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

Q 35 . secA−tanAsecA+tanA = cos²A(1+sinA)² $\frac{secA-tanA}{secA+tanA} = \frac{cos^2A}{(1+sinA)^2}$

Ans:

LHS = secA−tanAsecA+tanA $\frac{secA-tanA}{secA+tanA}$

Rationalizing the denominator by multiplying and dividing with sec A + tan A , we get

secA−tanAsecA+tanA × secA+tanAsecA+tanA $\frac{secA-tanA}{secA+tanA} \times \frac{secA+tanA}{secA+tanA}$

= sec²A−tan²A(secA+tanA)² $\frac{sec^2A-tan^2A}{(secA+tanA)^2}$

= 1(secA+tanA)^2 $\frac{1}{(secA+tanA)^2}$

= 1(sec^{2}A+tan^{2}A+2secAtanA) $\frac{1}{(sec^2A+tan^2A+2secAtanA)}$

= 1(1cos^{2}A + sin^{2}Acos^{2}A + 2sinAcosA) $\frac{1}{\left(\frac{1}{cos^2A}+\frac{sin^2A}{cos^2A}+\frac{2sinA}{cosA}\right)}$

⇒ cos^{2}A1+sin^{2}A+2sinA ⇒ $\frac{cos^2A}{1+sin^2A+2sinA}$

= cos^{2}A(1+sinA)^2 $\frac{cos^2A}{(1+sinA)^2}$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

Q 36 . 1+cosAsinA $\frac{1+cosA}{sinA}$ = sinA1−cosA $\frac{sinA}{1-cosA}$

Ans:

LHS = 1+cosAsinA $\frac{1+cosA}{sinA}$

Multiply both numerator and denominator with (1 – cos A) we get ,

(1+cosA)(1−cosA)sinA(1−cosA) $\frac{(1+cosA)(1-cosA)}{sinA(1-cosA)}$

= 1−cos^{2}AsinA(1−cosA) $\frac{1-cos^2A}{sinA(1-cosA)}$

= sin^{2}AsinA(1−cosA) $\frac{sin^2A}{sinA(1-cosA)}$

= sinA1−cosA $\frac{sinA}{1-cosA}$

∴ Undefined control sequence \therefore LHS = RHS Hence proved

37.

(i) √1+sinA1−sinA $\sqrt{\frac{1+sinA}{1-sinA}}$ = sec A + tan A

Ans:

To prove,

√1+sinA1−sinA $\sqrt{\frac{1+sinA}{1-sinA}}$ = sec A + tan A

Considering left hand side (LHS),

Rationalize the numerator and denominator with $\sqrt{1+sinA}\sqrt{1+sinA}$

$\sqrt{(1+sinA)(1+sinA)(1-sinA)(1+sinA}\sqrt{\frac{(1+sinA)(1+sinA)}{(1-sinA)(1+sinA}}$ = $\sqrt{(1+sinA)^2 1-sin^2A}\sqrt{\frac{(1+sinA)^2}{1-sin^2A}}$

= $\sqrt{(1+sinA)^2cos^2A}\sqrt{\frac{(1+sinA)^2}{cos^2A}}$

= $(1+sinA)cosA\ \frac{(1+sinA)}{cosA}$

= $1cosA + sinAcosA\ \frac{1}{cosA}+\frac{sinA}{cosA}$

= sec A + tan A

Therefore, LHS = RHS

Hence proved

(ii) $\sqrt{(1-cosA)(1+cosA)}+\sqrt{(1+cosA)(1-cosA)}\sqrt{\frac{(1-cosA)}{(1+cosA)}}+\sqrt{\frac{(1+cosA)}{(1-cosA)}}$ **= 2cosec A**

Ans:

To prove,

$\sqrt{(1-cosA)(1+cosA)}+\sqrt{(1+cosA)(1-cosA)}\sqrt{\frac{(1-cosA)}{(1+cosA)}}+\sqrt{\frac{(1+cosA)}{(1-cosA)}}$ = 2cosec A

Considering left hand side (LHS),

Rationalize the numerator and denominator.

= $\sqrt{(1-cosA)(1-cosA)(1+cosA)(1-cosA)}+\sqrt{(1+cosA)(1+cosA)(1-cosA)(1+cosA)}$

$\sqrt{\frac{(1-cosA)(1-cosA)}{(1+cosA)(1-cosA)}}+\sqrt{\frac{(1+cosA)(1+cosA)}{(1-cosA)(1+cosA)}}$

= $\sqrt{(1-cosA)^2(1-cos^2A)}+\sqrt{(1+cosA)^2(1-cos^2A)}\sqrt{\frac{(1-cosA)^2}{(1-cos^2A)}}+\sqrt{\frac{(1+cosA)^2}{(1-cos^2A)}}$

= $\sqrt{(1-cosA)^2(sin^2A)}+\sqrt{(1+cosA)^2(sin^2A)}\sqrt{\frac{(1-cosA)^2}{(sin^2A)}}+\sqrt{\frac{(1+cosA)^2}{(sin^2A)}}$

= $(1-cosA)(sinA)+(1+cosA)(sinA)\ \frac{(1-cosA)}{(sinA)}+\frac{(1+cosA)}{(sinA)}$

= $(1-cosA+1+cosA)(sinA)\ \frac{(1-cosA+1+cosA)}{(sinA)}$

= $(2)(sinA)\ \frac{(2)}{(sinA)}$

= 2cosec A

Therefore, LHS = RHS

Hence proved

38. **Prove that:**

(i) $\sqrt{(\sec\Theta-1)(\sec\Theta+1)} + \sqrt{(\sec\Theta+1)(\sec\Theta-1)}\ \sqrt{\frac{(sec\Theta-1)}{(sec\Theta+1)}} + \sqrt{\frac{(sec\Theta+1)}{(sec\Theta-1)}}$ **= 2cosec ΘΘ**

Ans:

To prove,

$= \sqrt{(\sec\Theta-1)(\sec\Theta+1)} + \sqrt{(\sec\Theta+1)(\sec\Theta-1)}\ \sqrt{\frac{(sec\Theta-1)}{(sec\Theta+1)}} + \sqrt{\frac{(sec\Theta+1)}{(sec\Theta-1)}}$ = 2cosec ΘΘ

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$= \sqrt{(\sec\Theta-1)(\sec\Theta-1)(\sec\Theta+1)(\sec\Theta-1)} + \sqrt{(\sec\Theta+1)(\sec\Theta+1)(\sec\Theta-1)(\sec\Theta+1)}$

$\sqrt{\frac{(sec\Theta-1)(sec\Theta-1)}{(sec\Theta+1)(sec\Theta-1)}} + \sqrt{\frac{(sec\Theta+1)(sec\Theta+1)}{(sec\Theta-1)(sec\Theta+1)}}$

$= \sqrt{(\sec\Theta-1)^2(\sec^2\Theta-1)} + \sqrt{(\sec\Theta+1)^2(\sec^2\Theta-1)}\ \sqrt{\frac{(sec\Theta-1)^2}{(sec^2\Theta-1)}} + \sqrt{\frac{(sec\Theta+1)^2}{(sec^2\Theta-1)}}$

$= \sqrt{(\sec\Theta-1)^2\tan^2\Theta} + \sqrt{(\sec\Theta+1)^2\tan^2\Theta}\ \sqrt{\frac{(sec\Theta-1)^2}{tan^2\Theta}} + \sqrt{\frac{(sec\Theta+1)^2}{tan^2\Theta}}$

$= (\sec\Theta-1)\tan\Theta + (\sec\Theta+1)\tan\Theta\ \frac{(sec\Theta-1)}{tan\Theta} + \frac{(sec\Theta+1)}{tan\Theta}$

$= (\sec\Theta-1+\sec\Theta+1)\tan\Theta\ \frac{(sec\Theta-1+sec\Theta+1)}{tan\Theta}$

$= (2\cos\Theta)\cos\Theta\sin\Theta\ \frac{(2cos\Theta)}{cos\Theta sin\Theta}$

$= 2\sin\Theta\ \frac{2}{sin\Theta}$

= 2cosec ΘΘ

Therefore, LHS = RHS

Hence proved

(ii) $\sqrt{(1+\sin\Theta)(1-\sin\Theta)} + \sqrt{(1-\sin\Theta)(1+\sin\Theta)}\ \sqrt{\frac{(1+sin\Theta)}{(1-sin\Theta)}} + \sqrt{\frac{(1-sin\Theta)}{(1+sin\Theta)}}$ **= 2sec ΘΘ**

Ans:

To prove,

= $\sqrt{(1+\sin\Theta)(1-\sin\Theta)}+\sqrt{(1-\sin\Theta)(1+\sin\Theta)}$ $\sqrt{\frac{(1+sin\Theta)}{(1-sin\Theta)}}+\sqrt{\frac{(1-sin\Theta)}{(1+sin\Theta)}}$ = 2sec ΘΘ

Considering left hand side (LHS),

Rationalize the numerator and denominator.

= $\sqrt{(1+\sin\Theta)(1+\sin\Theta)(1-\sin\Theta)(1+\sin\Theta)}+\sqrt{(1-\sin\Theta)(1-\sin\Theta)(1+\sin\Theta)(1-\sin\Theta)}$ $\sqrt{\frac{(1+sin\Theta)(1+sin\Theta)}{(1-sin\Theta)(1+sin\Theta)}}+\sqrt{\frac{(1-sin\Theta)(1-sin\Theta)}{(1+sin\Theta)(1-sin\Theta)}}$

= $\sqrt{(1+\sin\Theta)^2(1-\sin^2\Theta)}+\sqrt{(1-\sin\Theta)^2(1-\sin^2\Theta)}$ $\sqrt{\frac{(1+sin\Theta)^2}{(1-sin^2\Theta)}}+\sqrt{\frac{(1-sin\Theta)^2}{(1-sin^2\Theta)}}$

= $\sqrt{(1+\sin\Theta)^2(\cos^2\Theta)}+\sqrt{(1-\sin\Theta)^2(\cos^2\Theta)}$ $\sqrt{\frac{(1+sin\Theta)^2}{(cos^2\Theta)}}+\sqrt{\frac{(1-sin\Theta)^2}{(cos^2\Theta)}}$

= $(1+\sin\Theta)(\cos\Theta)+(1-\sin\Theta)(\cos\Theta)$ $\frac{(1+sin\Theta)}{(cos\Theta)}+\frac{(1-sin\Theta)}{(cos\Theta)}$

= $(1+\sin\Theta+1-\sin\Theta)(\cos\Theta)$ $\frac{(1+sin\Theta+1-sin\Theta)}{(cos\Theta)}$

= $(2)(\cos\Theta)$ $\frac{(2)}{(cos\Theta)}$

= 2secΘ$2sec\Theta$

Therefore, LHS = RHS

Hence proved

(iii) $\sqrt{(1+\cos\Theta)(1-\cos\Theta)}$ $\sqrt{\frac{(1+cos\Theta)}{(1-cos\Theta)}}$ **\sqrt{\frac{(1-cos \Theta)}{(1+cos \Theta)}} = 2cosecΘ**$2cosec\Theta$

Ans:

To prove,

$\sqrt{(1-\cos\Theta)(1+\cos\Theta)}+\sqrt{(1+\cos\Theta)(1-\cos\Theta)}$ $\sqrt{\frac{(1-cos\Theta)}{(1+cos\Theta)}}+\sqrt{\frac{(1+cos\Theta)}{(1-cos\Theta)}}$ = 2cosec \Theta

Considering left hand side (LHS),

Rationalize the numerator and denominator.

= $\sqrt{(1-\cos\Theta)(1-\cos\Theta)(1+\cos\Theta)(1-\cos\Theta)}+\sqrt{(1+\cos\Theta)(1+\cos\Theta)(1-\cos\Theta)(1+\cos\Theta)}$

$\sqrt{\frac{(1-cos\Theta)(1-cos\Theta)}{(1+cos\Theta)(1-cos\Theta)}}+\sqrt{\frac{(1+cos\Theta)(1+cos\Theta)}{(1-cos\Theta)(1+cos\Theta)}}$

= $\sqrt{(1-\cos\Theta)^2(1-\cos^2\Theta)}+\sqrt{(1+\cos\Theta)^2(1-\cos^2\Theta)}$ $\sqrt{\frac{(1-cos\Theta)^2}{(1-cos^2\Theta)}}+\sqrt{\frac{(1+cos\Theta)^2}{(1-cos^2\Theta)}}$

= √(1−cosΘ)²(sin²Θ) + √(1+cosΘ)²(sin²Θ) $\sqrt{\frac{(1-cos\Theta)^2}{(sin^2\Theta)}} + \sqrt{\frac{(1+cos\Theta)^2}{(sin^2\Theta)}}$

= (1−cosΘ)(sinΘ) + (1+cosΘ)(sinΘ) $\frac{(1-cos\Theta)}{(sin\Theta)} + \frac{(1+cos\Theta)}{(sin\Theta)}$

= (1−cosΘ+1+cosΘ)(sinΘ) $\frac{(1-cos\Theta+1+cos\Theta)}{(sin\Theta)}$

= (2)(sinΘ) $\frac{(2)}{(sin\Theta)}$

= 2cosec \Theta

Therefore, LHS = RHS

Hence proved

(iv) secΘ−1secΘ+1 $\frac{sec\Theta-1}{sec\Theta+1}$ **= (sinΘ1+cosΘ)²**$(\frac{sin\Theta}{1+cos\Theta})^2$

Ans:

To prove,

secΘ−1secΘ+1 $\frac{sec\Theta-1}{sec\Theta+1}$ = (sinΘ1+cosΘ)²$(\frac{sin\Theta}{1+cos\Theta})^2$

Considering left hand side (LHS),

= secΘ−1secΘ+1 $\frac{sec\Theta-1}{sec\Theta+1}$

= 1−cosΘ1+cosΘ $\frac{1-cos\Theta}{1+cos\Theta}$

Multiply and divide with (1+cosΘΘ)

= (1−cosΘ)(1+cosΘ)(1+cosΘ)(1+cosΘ) $\frac{(1-cos\Theta)(1+cos\Theta)}{(1+cos\Theta)(1+cos\Theta)}$

= (1−cos²Θ)(1+cosΘ)² $\frac{(1-cos^2\Theta)}{(1+cos\Theta)^2}$

= (sin²Θ)(1+cosΘ)² $\frac{(sin^2\Theta)}{(1+cos\Theta)^2}$

= (sinΘ1+cosΘ)²$(\frac{sin\Theta}{1+cos\Theta})^2$

Therefore, LHS = RHS

Hence proved

39. **(sec A – tan A)² = 1−sinA1+sinA** $\frac{1-sinA}{1+sinA}$

Ans:

To prove,

(sec A – tan A)^2 = 1−sinA1+sinA $\frac{1-sinA}{1+sinA}$

Considering left hand side (LHS),

= (sec A – tan A)^2

= [1cosA − sinAcosA]^2 $[\frac{1}{cosA} - \frac{sinA}{cosA}]^2$

= (1−sinA)^{2}cos^{2}A $\frac{(1-sinA)^2}{cos^2A}$

= (1−sinA)^{2}1−sin^{2}A $\frac{(1-sinA)^2}{1-sin^2A}$

= (1−sinA)^2(1+sinA)(1−sinA) $\frac{(1-sinA)^2}{(1+sinA)(1-sinA)}$

= (1−sinA)(1+sinA) $\frac{(1-sinA)}{(1+sinA)}$

Therefore, LHS = RHS

Hence proved

40. **1−cosA1+cosA $\frac{1-cosA}{1+cosA}$ = (cot A – cosec A)^2**

Ans:

To prove,

1−cosA1+cosA $\frac{1-cosA}{1+cosA}$ = (cot A – cosecA)

Considering left hand side (LHS),

Rationalize the numerator and denominator with (1 – cos A)

= (1−cosA)(1−cosA)(1+cosA)(1−cosA) $\frac{(1-cosA)(1-cosA)}{(1+cosA)(1-cosA)}$

= (1−cosA)^2(1−cos^{2}A) $\frac{(1-cosA)^2}{(1-cos^2A)}$

= (1−cosA)^2(sin^{2}A) $\frac{(1-cosA)^2}{(sin^2A)}$

= (1sinA − cosAsinA)^2 $(\frac{1}{sinA} - \frac{cosA}{sinA})^2$

= (cosec A – cot A)^2

= (cot A – cosec)^2

Therefore, LHS = RHS

Hence proved

41. **1secA–1 + 1secA+1 =2cosecAcotA** $\frac{1}{secA-1} + \frac{1}{secA+1} = 2cosecAcotA$

Ans:

To prove,

1secA–1 + 1secA+1 =2cosecAcotA $\frac{1}{secA-1} + \frac{1}{secA+1} = 2cosecAcotA$

Considering left hand side (LHS),

= secA+1+secA−1(secA+1)(secA−1) $\frac{secA+1+secA-1}{(secA+1)(secA-1)}$

= 2secA(sec²A−1) $\frac{2secA}{(sec^2A-1)}$

= 2secA(tan²A) $\frac{2secA}{(tan^2A)}$

= 2cos²A(cosAsin²A) $\frac{2cos^2A}{(cosAsin^2A)}$

= 2cosA(sin²A) $\frac{2cosA}{(sin^2A)}$

= 2cosA(sinA)(sinA)) $\frac{2cosA}{(sinA)(sinA))}$

= 2cosec A cot A

Therefore, LHS = RHS

Hence proved

42. **cosA1−tanA + sinA1−cotA** $\frac{cosA}{1-tanA} + \frac{sinA}{1-cotA}$ **= sin A + cos A**

Ans:

To prove,

cosA1−tanA + sinA1−cotA $\frac{cosA}{1-tanA} + \frac{sinA}{1-cotA}$ = sin A + cos A

Considering left hand side (LHS),

= cosA1−tanA + sinA1−cotA $\frac{cosA}{1-tanA} + \frac{sinA}{1-cotA}$

= cosA1−sinAcosA + sinA1−cosAsinA $\frac{cosA}{1-\frac{sinA}{cosA}} + \frac{sinA}{1-\frac{cosA}{sinA}}$

= cos^2AcosA−sinA − sin^2AcosA−sinA $\frac{cos^2 A}{cosA-sinA} - \frac{sin^2 A}{cosA-sinA}$

= cos^2A−sin^2AcosA−sinA $\frac{cos^2 A-sin^2 A}{cosA-sinA}$

= (cosA+sinA)(cosA−sinA)cosA−sinA $\frac{(cosA+sinA)(cosA-sinA)}{cosA-sinA}$

= cos A + sin A

Therefore, LHS = RHS

Hence proved

43. **(cosecA)(cosecA−1) + (cosecA)(cosecA+1)** $\frac{(cosecA)}{(cosecA-1)} + \frac{(cosecA)}{(cosecA+1)}$ **= 2sec^2 A**

Ans:

To prove,

(cosecA)(cosecA−1) + (cosecA)(cosecA+1) $\frac{(cosecA)}{(cosecA-1)} + \frac{(cosecA)}{(cosecA+1)}$ = 2sec^2 A

Considering left hand side (LHS),

= (cosecA)(cosecA+1+cosecA−1)(cosec^{2}A−1)) $\frac{(cosecA)(cosecA+1+cosecA-1)}{(cosec^2 A-1)}$)

= (2cosec^{2}A)cot^{2}A $\frac{(2cosec^2 A)}{cot^2 A}$

= (2sin^{2}A)sin^{2}A.cos^{2}A $\frac{(2sin^2 A)}{sin^2 A.cos^2 A}$

= 2cos^{2}A $\frac{2}{cos^2 A}$

= 2sec^{2}A$2sec^2 A$

Therefore, LHS = RHS

Hence proved

44. tan^{2}A1+tan^{2}A + cot^{2}A1+cot^{2}A $\frac{tan^2 A}{1+tan^2 A} + \frac{cot^2 A}{1+cot^2 A}$ **= 1**

Ans:

To prove,

tan²A1+tan²A + cot²A1+cot²A $\frac{tan^2A}{1+tan^2A} + \frac{cot^2A}{1+cot^2A}$ = 1

Considering left hand side (LHS),

= sin²Acos²A cos²A+sin²Acos²A + cos²Asin²A cos²A+sin²Asin²A $\frac{\frac{sin^2A}{cos^2A}}{\frac{cos^2A+sin^2A}{cos^2A}} + \frac{\frac{cos^2A}{sin^2A}}{\frac{cos^2A+sin^2A}{sin^2A}}$

= sin²Acos²A+sin²A + cos²Acos²A+sin²A $\frac{sin^2A}{cos^2A+sin^2A} + \frac{cos^2A}{cos^2A+sin^2A}$

= sin²A+cos²Acos²A+sin²A $\frac{sin^2A+cos^2A}{cos^2A+sin^2A}$

= 1

Therefore, LHS = RHS

Hence proved

45. **cotA−cosAcotA+cosA** $\frac{cotA-cosA}{cotA+cosA}$ **= cosecA−1cosecA+1** $\frac{cosecA-1}{cosecA+1}$

Ans:

To prove,

cotA−cosAcotA+cosA $\frac{cotA-cosA}{cotA+cosA}$ = cosecA−1cosecA+1 $\frac{cosecA-1}{cosecA+1}$

Considering left hand side (LHS),

= cosAsinA−cosAcosAsinA+cosA $\frac{\frac{cosA}{sinA}-cosA}{\frac{cosA}{sinA}+cosA}$

= cosAcosecA−cosAcosAcosecA+cosA $\frac{cosAcosecA-cosA}{cosAcosecA+cosA}$

= cosA(cosecA−1)cosA(cosecA+1) $\frac{cosA(cosecA-1)}{cosA(cosecA+1)}$

= (cosecA−1)(cosecA+1) $\frac{(cosecA-1)}{(cosecA+1)}$

Therefore, LHS = RHS

Hence proved

46. **1+cosΘ−sin²ΘsinΘ(1+cosΘ)** $\frac{1+cos\Theta-sin^2\Theta}{sin\Theta(1+cos\Theta)}$ **= cot ΘΘ**

Ans:

To prove,

1+cosΘ−sin^{2}ΘsinΘ(1+cosΘ) $\frac{1+cos\Theta-sin^2\Theta}{sin\Theta(1+cos\Theta)}$ = cot ΘΘ

Considering left hand side (LHS),

= 1+cosΘ−(1−cos^{2}Θ)sinΘ(1+cosΘ) $\frac{1+cos\Theta-(1-cos^2\Theta)}{sin\Theta(1+cos\Theta)}$

= 1+cosΘ−1+cos^{2}ΘsinΘ(1+cosΘ) $\frac{1+cos\Theta-1+cos^2\Theta}{sin\Theta(1+cos\Theta)}$

= cosΘ+cos^{2}ΘsinΘ(1+cosΘ) $\frac{cos\Theta+cos^2\Theta}{sin\Theta(1+cos\Theta)}$

= cosΘ(1+cosΘ)sinΘ(1+cosΘ) $\frac{cos\Theta(1+cos\Theta)}{sin\Theta(1+cos\Theta)}$

= (cosΘ)(sinΘ) $\frac{(cos\Theta)}{(sin\Theta)}$

= cotΘ$cot\Theta$

Therefore, LHS = RHS

Hence, proved.

(i) 1+cosΘ+sinΘ1+cosΘ−sinΘ $\frac{1+cos\Theta+sin\Theta}{1+cos\Theta-sin\Theta}$ **= 1+sinΘcosΘ** $\frac{1+sin\Theta}{cos\Theta}$

Ans:

To prove,

1+cosΘ+sinΘ1+cosΘ−sinΘ $\frac{1+cos\Theta+sin\Theta}{1+cos\Theta-sin\Theta}$ = 1+sinΘcosΘ $\frac{1+sin\Theta}{cos\Theta}$

Dividing the numerator and denominator with cosΘ$cos\Theta$

Considering LHS, we get,

= 1+cosΘ+sinΘcosΘ 1+cosΘ−sinΘcosΘ $\frac{\frac{1+cos\Theta+sin\Theta}{cos\Theta}}{\frac{1+cos\Theta-sin\Theta}{cos\Theta}}$

= secΘ+1+tanΘsecΘ+1−tanΘ $\frac{sec\Theta+1+tan\Theta}{sec\Theta+1-tan\Theta}$

= 1+secΘ+tanΘ1+secΘ−tanΘ $\frac{1+sec\Theta+tan\Theta}{1+sec\Theta-tan\Theta}$

[As we know,

$(sec^2\Theta)-(tan^2\Theta)=1(sec\Theta+tan\Theta)(sec\Theta-tan\Theta)=1(sec\Theta+tan\Theta)=1(sec\Theta-tan\Theta)$

$$(sec^2\Theta)-(tan^2\Theta)=1 \quad]$$
$$(sec\Theta+tan\Theta)(sec\Theta-tan\Theta)=1$$
$$(sec\Theta+tan\Theta)=\frac{1}{(sec\Theta-tan\Theta)}$$

= $1(sec\Theta-tan\Theta)+11+sec\Theta-tan\Theta$ $\frac{\frac{1}{(sec\Theta-tan\Theta)}+1}{1+sec\Theta-tan\Theta}$

= $1+sec\Theta-tan\Theta1+sec\Theta-tan\Theta \times 1sec\Theta-tan\Theta$ $\frac{1+sec\Theta-tan\Theta}{1+sec\Theta-tan\Theta}\times\frac{1}{sec\Theta-tan\Theta}$

= $sec\Theta+tan\Theta sec\Theta+tan\Theta$

= $1+sin\Theta cos\Theta$ $\frac{1+sin\Theta}{cos\Theta}$

Therefore, LHS = RHS

Hence proved

(ii) **$sin\Theta-cos\Theta+1sin\Theta+cos\Theta-1$** $\frac{sin\Theta-cos\Theta+1}{sin\Theta+cos\Theta-1}$ **= $1sec\Theta-tan\Theta$** $\frac{1}{sec\Theta-tan\Theta}$

Ans:

To prove,

$sin\Theta-cos\Theta+1sin\Theta+cos\Theta-1$ $\frac{sin\Theta-cos\Theta+1}{sin\Theta+cos\Theta-1}$ = $1sec\Theta-tan\Theta$ $\frac{1}{sec\Theta-tan\Theta}$

Considering LHS, we get,

$sin\Theta-cos\Theta+1sin\Theta+cos\Theta-1$ $\frac{sin\Theta-cos\Theta+1}{sin\Theta+cos\Theta-1}$

Dividing the numerator and denominator with $cos\Theta cos\Theta$, we get,

= $tan\Theta+sec\Theta-1tan\Theta-sec\Theta+1$ $\frac{tan\Theta+sec\Theta-1}{tan\Theta-sec\Theta+1}$

[As we know, $(sec\Theta+tan\Theta)=1(sec\Theta-tan\Theta)$ $(sec\Theta+tan\Theta)=\frac{1}{(sec\Theta-tan\Theta)}$]

= $1(sec\Theta-tan\Theta)-1tan\Theta-sec\Theta+1$ $\frac{\frac{1}{(sec\Theta-tan\Theta)}-1}{tan\Theta-sec\Theta+1}$

= $tan\Theta-sec\Theta+1tan\Theta-sec\Theta+1 \times 1(sec\Theta-tan\Theta)$ $\frac{tan\Theta-sec\Theta+1}{tan\Theta-sec\Theta+1}\times\frac{1}{(sec\Theta-tan\Theta)}$

= $1(sec\Theta-tan\Theta)$ $\frac{1}{(sec\Theta-tan\Theta)}$

Therefore, LHS = RHS

Hence proved

(iii) cosΘ−sinΘ+1cosΘ+sinΘ−1 $\frac{cos\Theta - sin\Theta + 1}{cos\Theta + sin\Theta - 1}$ **= cosecΘ+cotΘ**$cosec\Theta + cot\Theta$

Ans:

To prove,

cosΘ−sinΘ+1cosΘ+sinΘ−1 $\frac{cos\Theta - sin\Theta + 1}{cos\Theta + sin\Theta - 1}$ = cosecΘ+cotΘ$cosec\Theta + cot\Theta$

Considering LHS, we get,

Dividing the numerator and denominator with sinΘ$sin\Theta$, we get,

= cosΘ−sinΘ+1sinΘ cosΘ+sinΘ−1sinΘ $\frac{\frac{cos\Theta - sin\Theta + 1}{sin\Theta}}{\frac{cos\Theta + sin\Theta - 1}{sin\Theta}}$

= cotΘ+cosecΘ−1cotΘ−cosecΘ+1 $\frac{cot\Theta + cosec\Theta - 1}{cot\Theta - cosec\Theta + 1}$

[As we know,

(cosec^{2}Θ)−(cot^{2}Θ)=1(cosecΘ+cotΘ)

$(cosec^2\Theta) - (cot^2\Theta) = 1$]

$(cosec\Theta + cot\Theta)(cosec\Theta - cot\Theta) = 1$

(cosecΘ−cotΘ)=1(cosecΘ+cotΘ)=1(cosecΘ−cotΘ) $(cosec\Theta + cot\Theta) = \frac{1}{(cosec\Theta - cot\Theta)}$

= 1(cosecΘ−cotΘ)−1cotΘ−cosecΘ+1 $\frac{\frac{1}{(cosec\Theta - cot\Theta)} - 1}{cot\Theta - cosec\Theta + 1}$

= cotΘ−cosecΘ+1cotΘ−cosecΘ+1 × 1(cosecΘ−cotΘ) $\frac{cot\Theta - cosec\Theta + 1}{cot\Theta - cosec\Theta + 1} \times \frac{1}{(cosec\Theta - cot\Theta)}$

= 1(cosecΘ−cotΘ) $\frac{1}{(cosec\Theta - cot\Theta)}$

= cosecΘ+cotΘ$cosec\Theta + cot\Theta$

Therefore, LHS = RHS

Hence proved

(iv) (sinΘ+cosΘ)(tanΘ+cotΘ)$(sin\Theta + cos\Theta)(tan\Theta + cot\Theta)$ **= secΘ+cosecΘ**$sec\Theta + cosec\Theta$

Ans:

To prove,

(sinΘ+cosΘ)(tanΘ+cotΘ)$(sin\Theta + cos\Theta)(tan\Theta + cot\Theta)$ = cosecΘ+cosecΘ$cosec\Theta + cosec\Theta$

Considering LHS, we get,

= (sinΘ+cosΘ)(sinΘcosΘ + cosΘsinΘ)$(sin\Theta + cos\Theta)(\frac{sin\Theta}{cos\Theta} + \frac{cos\Theta}{sin\Theta})$

= (sin^{2}ΘcosΘ +cosΘ+sinΘ+ cos^{2}ΘsinΘ ($\frac{sin^2\Theta}{cos\Theta}+cos\Theta+sin\Theta+\frac{cos^2\Theta}{sin\Theta}$

= sinΘ(tanΘ+1)+cosΘ(1tanΘ +1) $sin\Theta(tan\Theta+1)+cos\Theta(\frac{1}{tan\Theta}+1)$

= sinΘ(tanΘ+1)+ cosΘtanΘ (tanΘ+1) $sin\Theta(tan\Theta+1)+\frac{cos\Theta}{tan\Theta}(tan\Theta+1)$

= (sinΘ+ cosΘtanΘ)(tanΘ+1) $(sin\Theta+\frac{cos\Theta}{tan\Theta})(tan\Theta+1)$

= (sin^{2}Θ+cos^{2}ΘsinΘ)(tanΘ+1) $(\frac{sin^2\Theta+cos^2\Theta}{sin\Theta})(tan\Theta+1)$

= (1sinΘ)(tanΘ+1) $(\frac{1}{sin\Theta})(tan\Theta+1)$

= Undefined control sequence \Thetasin Undefined control sequence \Thetasin

= secΘ+cosecΘ $sec\Theta+cosec\Theta$

Therefore, LHS = RHS

Hence proved

50. **\frac{tanA}{1+secA}-\frac{tanA}{1-secA}= 2 cosec A**

Ans:

To prove,

\frac{tanA}{1+secA}-\frac{tanA}{1-secA}= 2 cosec A

Considering LHS, we get,

= sinAcosA cosA+1cosA − sinAcosA cosA−1cosA $\frac{\frac{sinA}{cosA}}{\frac{cosA+1}{cosA}}-\frac{\frac{sinA}{cosA}}{\frac{cosA-1}{cosA}}$

= sinAcosA+1 − sinAcosA−1 $\frac{sinA}{cosA+1}-\frac{sinA}{cosA-1}$

= sinA(1cosA+1 − 1cosA−1) $sinA(\frac{1}{cosA+1}-\frac{1}{cosA-1})$

= sinA(cosA−1-cosA−1cos^{2}A−1) $sinA(\frac{cosA-1-cosA-1}{cos^2A-1})$

= sinA(cosA−1-cosA−1cos^{2}A−1) $sinA(\frac{cosA-1-cosA-1}{cos^2A-1})$

= sinA(−2−sin^{2}A) $sinA(\frac{-2}{-sin^2A})$

= 2sinA) $\frac{2}{sinA}$)

= 2 cosec A

Therefore, LHS = RHS

Hence proved

Q51: **1+cot²Θ1+cosecΘ=cosecΘ** $1+\frac{cot^2\Theta}{1+cosec\Theta}=cosec\Theta$

Ans:

1+cosec²Θ−11+cosecΘ[∵cot²Θ=cosec²Θ−1] Undefined control sequence \because 1+(cosecΘ−1)(cosecΘ+1)1+cosecΘ $1+\frac{(cosec\Theta-1)(cosec\Theta+1)}{1+cosec\Theta}$ =1+cosecΘ−1[∵(a+b)(a−b)=a²−b²)]

Undefined control sequence \because =cosecΘ= $cosec\Theta$

Therefore, LHS = RHS

Hence, proved.

Q52: **cosΘcosecΘ+1+cosΘcosecΘ−1=2tanΘ** $\frac{cos\Theta}{cosec\Theta+1}+\frac{cos\Theta}{cosec\Theta-1}=2tan\Theta$

Ans:

cosΘ1sinΘ+1+cosΘ1sinΘ−1 $\frac{cos\Theta}{\frac{1}{sin\Theta}+1}+\frac{cos\Theta}{\frac{1}{sin\Theta}-1}$ cosΘ1+sinΘsinΘ+cosΘ1−sinΘsinΘ $\frac{cos\Theta}{\frac{1+sin\Theta}{sin\Theta}}+\frac{cos\Theta}{\frac{1-sin\Theta}{sin\Theta}}$ (cosΘ)(sinΘ)1+sinΘ+(cosΘ)(sinΘ)1−sinΘ $\frac{(cos\Theta)(sin\Theta)}{1+sin\Theta}+\frac{(cos\Theta)(sin\Theta)}{1-sin\Theta}$ (1−sinΘ)(sinΘcosΘ)+(sinΘcosΘ)(1+sinΘ)(1−sinΘ) $\frac{(1-sin\Theta)(sin\Theta cos\Theta)+(sin\Theta cos\Theta)}{(1+sin\Theta)(1-sin\Theta)}$ sinΘcosΘ−sinΘcosΘ+sinΘcosΘ+sin²Θcos²Θ1−sin²Θ $\frac{sin\Theta cos\Theta-sin\Theta cos\Theta+sin\Theta cos\Theta+sin^2\Theta cos^2\Theta}{1-sin^2\Theta}$ =sinΘcosΘcos²Θ= $\frac{sin\Theta cos\Theta}{cos^2\Theta}$ =2sinΘcosΘ= $\frac{2sin\Theta}{cos\Theta}$ =2tanΘ= $2tan\Theta$

Therefore, LHS = RHS

Hence, proved

Q53) (1+tan²A)+(1+1tan²A)=1sin²A−sin⁴A $(1+tan^2A)+(1+\frac{1}{tan^2A})=\frac{1}{sin^2A-sin^4A}$

Ans:

LHS = (1+sin²Acos²A)+(1+cos²Asin²A) $(1+\frac{sin^2A}{cos^2A})+(1+\frac{cos^2A}{sin^2A})$

=> cos²A+sin²Acos²A+sin²A+cos²Asin²A $\frac{cos^2A+sin^2A}{cos^2A}+\frac{sin^2A+cos^2A}{sin^2A}$

=> 1cos²A+1sin²A[∵sin²A+cos²A=1] Undefined control sequence \because

=> sin²A+cos²Asin²Acos²A=1sin²A(1−sin²A)[∵cos²A=1−sin²A] Undefined control sequence \because

=> $1sin^2A-sin^4A$ $\frac{1}{sin^2A-sin^4A}$

Therefore, LHS = RHS.

Hence Proved.

Q54) $sin^2Acos^2B - cos^2Asin^2B = sin^2A - sin^2B$

Ans:

LHS = $sin^2Acos^2B - cos^2Asin^2B$

= $sin^2A(1-sin^2B)-(1-sin^2A)(sin^2A)[\because cos^2A=1-sin^2A]$ Undefined control sequence \because

= $sin^2A-sin^2Asin^2B-sin^2B+sin^2Asin^2B$ $sin^2A - sin^2Asin^2B - sin^2B + sin^2Asin^2B$

= sin^2A-sin^2B $sin^2A - sin^2B$

= RHS

Hence Proved.

Q55: (i) cotA+tanBcotB+tanA =cotAtanB $\frac{cotA+tanB}{cotB+tanA} = cotAtanB$

Ans:

LHS = cotA+tanBcotB+tanA $\frac{cotA+tanB}{cotB+tanA}$

= cosAsinA+sinBcosB cosBsinB+sinAcosA $\frac{\frac{cosA}{sinA}+\frac{sinB}{cosB}}{\frac{cosB}{sinB}+\frac{sinA}{cosA}}$

= cosAcosB+sinAsinBsinAcosB cosAcosB+sinAsinBcosAsinB $\frac{\frac{cosAcosB+sinAsinB}{sinAcosB}}{\frac{cosAcosB+sinAsinB}{cosAsinB}}$

= cosAcosB+sinAsinBsinAcosB × cosAsinBcosAcosB+sinAsinB $\frac{cosAcosB+sinAsinB}{sinAcosB} \times \frac{cosAsinB}{cosAcosB+sinAsinB}$

= cosAsinBsinAcosB $\frac{cosAsinB}{sinAcosB}$

= cotAtanB

= RHS

Hence Proved.

(ii) tanA+tanBcotA+cotB =tanAtanB $\frac{tanA+tanB}{cotA+cotB} = tanAtanB$

Ans:

LHS = tanA+tanBcotA+cotB $\frac{tanA+tanB}{cotA+cotB}$

= sinAcosA+sinBcosB cosAsinA+cosBsinB $\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{\frac{cosA}{sinA}+\frac{cosB}{sinB}}$

= sinAcosB+cosAsinBcosAcosB cosAsinB+cosBsinAsinAsinB $\frac{\frac{sinAcosB+cosAsinB}{cosAcosB}}{\frac{cosAsinB+cosBsinA}{sinAsinB}}$

= sinAcosB+cosAsinBcosAcosB × sinAsinBcosAsinB+cosBsinA $\frac{sinAcosB+cosAsinB}{cosAcosB} \times \frac{sinAsinB}{cosAsinB+cosBsinA}$

= sinAsinBcosAcosB $\frac{sinAsinB}{cosAcosB}$

= tanAtanB

= RHS

Hence Proved.

Q56) cot²Acosec²B−cot²Bcosec²A=cot²A−cot²B
$cot^2 Acosec^2 B - cot^2 Bcosec^2 A = cot^2 A - cot^2 B$

Ans:

LHS = cot²Acosec²B−cot²Bcosec²A$cot^2 Acosec^2 B - cot^2 Bcosec^2 A$

= cot²A(1+cot²B)−cot²B(1+cot²A)[∵cosec²θ=1+cot²θ] Undefined control sequence \because

= cot²A+cot²Acot²B−cot²B−cot²Bcot²A$cot^2 A + cot^2 Acot^2 B - cot^2 B - cot^2 Bcot^2 A$

= cot²A – cot²B

= RHS

Hence Proved.

Q57) tan²Asec²B−sec²Atan²B=tan²A−tan²B$tan^2 Asec^2 B - sec^2 Atan^2 B = tan^2 A - tan^2 B$

Ans:

LHS = tan²Asec²B−sec²Atan²B$tan^2 Asec^2 B - sec^2 Atan^2 B$

= tan²A(1+tan²B)−sec²A(tan²A)$tan^2 A(1 + tan^2 B) - sec^2 A(tan^2 A)$

= tan²A+tan²Atan²B−tan²B(1+tan²A)[∵sec²A=1+tan²A] Undefined control sequence \because

$= tan^2A + tan^2A tan^2B - tan^2B - tan^2A tan^2B$ $tan^2A + tan^2Atan^2B - tan^2B - tan^2Atan^2B$

$= tan^2A - tan^2B$ $tan^2A - tan^2B$

= RHS

Hence Proved.

Q58) If x = $a sec\theta + b tan\theta$ $asec\theta + btan\theta$ and y = $a tan\theta + b sec\theta$ $a\ tan\theta + b\ sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

Ans:

LHS = $x^2 - y^2$

$= (asec\theta + btan\theta)^2 - (atan\theta + bsec\theta)^2$ $(asec\theta + btan\theta)^2 - (atan\theta + bsec\theta)^2$

$= a^2sec^2\theta + b^2tan^2\theta + 2absec\theta tan\theta - a^2tan^2\theta - b^2sec^2\theta - 2absec\theta tan\theta$
$a^2sec^2\theta + b^2tan^2\theta + 2absec\theta tan\theta - a^2tan^2\theta - b^2sec^2\theta - 2absec\theta tan\theta$

$= a^2sec^2\theta + b^2tan^2\theta - a^2tan^2\theta - b^2sec^2\theta$ $a^2sec^2\theta + b^2tan^2\theta - a^2tan^2\theta - b^2sec^2\theta$

$= a^2sec^2\theta - b^2sec^2\theta + b^2tan^2\theta - a^2tan^2\theta$ $a^2sec^2\theta - b^2sec^2\theta + b^2tan^2\theta - a^2tan^2\theta$

$= sec^2\theta(a^2 - b^2) + tan^2\theta(b^2 - a^2)$ $sec^2\theta(a^2 - b^2) + tan^2\theta(b^2 - a^2)$

$= sec^2\theta(a^2 - b^2) - tan^2\theta(a^2 - b^2)$ $sec^2\theta(a^2 - b^2) - tan^2\theta(a^2 - b^2)$

$= (sec^2\theta - tan^2\theta)(a^2 - b^2)$ $(sec^2\theta - tan^2\theta)(a^2 - b^2)$

$= a^2 - b^2$

= RHS

Hence Proved.

Q59) If $3sin\theta + 5cos\theta = 5$ $3sin\theta + 5cos\theta = 5$, prove that $5sin\theta - 3cos\theta = \pm 3$ $5sin\theta - 3cos\theta = \pm 3$.

Ans:

Given $3sin\theta + 5cos\theta = 5$ $3sin\theta + 5cos\theta = 5$

$3sin\theta = 5 - 5cos\theta$ $3sin\theta = 5 - 5cos\theta$ $3sin\theta = 5(1 - cos\theta)$ $3sin\theta = 5(1 - cos\theta)$ $3sin\theta = 5(1-cos\theta)(1-cos\theta)1+cos\theta$ $3sin\theta = \frac{5(1-cos\theta)(1-cos\theta)}{1+cos\theta}$ $3sin\theta = 5(1-cos^2\theta)1+cos\theta$ $3sin\theta = \frac{5(1-cos^2\theta)}{1+cos\theta}$ $3sin\theta = 5sin^2\theta 1+cos\theta$ $3sin\theta = \frac{5sin^2\theta}{1+cos\theta}$ $3 + 3cos\theta = 5sin\theta$ $3 + 3cos\theta = 5sin\theta$ $3 = 5sin\theta - 3cos\theta$ $3 = 5sin\theta - 3cos\theta$

= RHS

Hence Proved.

Q60) If cosecθ+cotθ$cosec\theta + cot\theta$=m and cosecθ−cotθ$cosec\theta - cot\theta$=n, prove that mn = 1.

Ans:

LHS = mn

= (cosecθ+cotθ)(cosecθ−cotθ)$(cosec\theta + cot\theta)(cosec\theta - cot\theta)$

= $cosec^2\theta - cot^2\theta$ $cosec^2\theta - cot^2\theta$

= 1

= RHS

Hence Proved.

Q 62 . If $T_n = sin^n\theta + cos_n\theta$ $T_n = sin^n\theta + cos_n\theta$, prove that $T_3 - T_5 T_1 = T_5 - T_7 T_3$ $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$.

Ans:

LHS = $(sin^3\theta + cos^3\theta) - (sin^5\theta + cos^5\theta) sin\theta + cos\theta$ $\frac{(sin^3\theta + cos^3\theta) - (sin^5\theta + cos^5\theta)}{sin\theta + cos\theta}$

= $sin^3\theta(1 - sin^2\theta) + cos^3\theta(1 - cos^2\theta) sin\theta + cos\theta$ $\frac{sin^3\theta(1 - sin^2\theta) + cos^3\theta(1 - cos^2\theta)}{sin\theta + cos\theta}$

= $sin^3\theta \times cos^2\theta + cos^3\theta \times sin^2\theta sin\theta + cos\theta$ $\frac{sin^3\theta \times cos^2\theta + cos^3\theta \times sin^2\theta}{sin\theta + cos\theta}$

= $sin^2\theta cos^2\theta(sin\theta + cos\theta) sin\theta + cos\theta$ $\frac{sin^2\theta cos^2\theta(sin\theta + cos\theta)}{sin\theta + cos\theta}$

= $sin^2\theta cos^2\theta$ $sin^2\theta cos^2\theta$

RHS = Missing close brace Missing close brace

= Missing close brace Missing close brace

= $sin^5\theta \times cos^2\theta + cos^5\theta \times sin^2\theta sin^3\theta + cos^3\theta$ $\frac{sin^5\theta \times cos^2\theta + cos^5\theta \times sin^2\theta}{sin^3\theta + cos^3\theta}$

= $sin^2\theta cos^2\theta(sin^3\theta + cos^3\theta) sin\theta + cos\theta$ $\frac{sin^2\theta cos^2\theta(sin^3\theta + cos^3\theta)}{sin\theta + cos\theta}$

= $sin^2\theta cos^2\theta$ $sin^2\theta cos^2\theta$

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 63 . $(\tan\theta+\frac{1}{\cos\theta})^2+(\tan\theta-\frac{1}{\cos\theta})^2$ $\left(tan\theta+\frac{1}{cos\theta}\right)^2+\left(tan\theta-\frac{1}{cos\theta}\right)^2$ **=** $2(\frac{1+\sin^2\theta}{1-\sin^2\theta})$ $2\left(\frac{1+sin^2\theta}{1-sin^2\theta}\right)$

Ans:

$(\tan\theta+\sec\theta)^2+(\tan\theta-\sec\theta)^2$ $(tan\theta+sec\theta)^2+(tan\theta-sec\theta)^2$

$= \tan^2\theta+\sec^2\theta+2\tan\theta\sec\theta+\tan^2\theta+\sec^2\theta-2\tan\theta\sec\theta$
$tan^2\theta+sec^2\theta+2tan\theta sec\theta+tan^2\theta+sec^2\theta-2tan\theta sec\theta$

$= 2\tan^2\theta+2\sec^2\theta$ $2tan^2\theta+2sec^2\theta$

$= 2[\tan^2\theta+\sec^2\theta]$ $2\left[tan^2\theta+sec^2\theta\right]$

$= 2[\frac{\sin^2\theta}{\cos^2\theta}+\frac{1}{\cos^2\theta}]$ $2\left[\frac{sin^2\theta}{cos^2\theta}+\frac{1}{cos^2\theta}\right]$

$= 2(\frac{1+\sin^2\theta}{\cos^2\theta})$ $2\left(\frac{1+sin^2\theta}{cos^2\theta}\right)$

$= 2(\frac{1+\sin^2\theta}{1-\sin^2\theta})$ $2\left(\frac{1+sin^2\theta}{1-sin^2\theta}\right)$

= RHS

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 64 . $(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\csc^2\theta-\sin^2})\sin^2\theta\cos^2\theta$ $\left(\frac{1}{sec^2\theta-cos^2\theta}+\frac{1}{cosec^2\theta-sin^2}\right)sin^2\theta cos^2\theta$ **=** $\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$ $\frac{1-sin^2\theta cos^2\theta}{2+sin^2\theta cos^2\theta}$

Ans:

$[\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta}+\frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}]\sin^2\theta\cos^2$ $\left[\frac{1}{\frac{1}{cos^2\theta}-cos^2\theta}+\frac{1}{\frac{1}{sin^2\theta}-sin^2\theta}\right]sin^2\theta cos^2$

$= [\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}}+\frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}]\sin^2\theta\cos^2\theta$ $\left[\frac{1}{\frac{1-cos^4\theta}{cos^2\theta}}+\frac{1}{\frac{1-sin^4\theta}{sin^2\theta}}\right]sin^2\theta cos^2\theta$

$= [\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}]\sin^2\theta\cos^2\theta$ $\left[\frac{cos^2\theta}{1-cos^4\theta}+\frac{sin^2\theta}{1-sin^4\theta}\right]sin^2\theta cos^2\theta$

$= [\frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta-\cos^4\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta-\sin^4\theta}]\sin^2\theta\cos^2\theta$
$\left[\frac{cos^2\theta}{cos^2\theta+sin^2\theta-cos^4\theta}+\frac{sin^2\theta}{cos^2\theta+sin^2\theta-sin^4\theta}\right]sin^2\theta cos^2\theta$

$= [\frac{\cos^2\theta}{\cos^2\theta(1-\cos^2\theta)+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta(1-\sin^2\theta)}]\sin^2\theta\cos^2\theta$
$\left[\frac{cos^2\theta}{cos^2\theta(1-cos^2\theta)+sin^2\theta}+\frac{sin^2\theta}{cos^2\theta+sin^2\theta(1-sin^2\theta)}\right]sin^2\theta cos^2\theta$

$= [\cos^2\theta\cos^2\theta\sin^2\theta+\sin^2\theta + \sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta]\sin^2\theta\cos^2\theta \left[\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta\cos^2\theta}\right]\sin^2\theta\cos^2\theta$

$= [\cos^2\theta\sin^2\theta(\cos^2\theta+1) + \sin^2\theta\cos^2\theta(\sin^2\theta+1)]\sin^2\theta\cos^2\theta \left[\frac{\cos^2\theta}{\sin^2\theta(\cos^2\theta+1)}+\frac{\sin^2\theta}{\cos^2\theta(\sin^2\theta+1)}\right]\sin^2\theta\cos^2\theta$

$= \cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)\sin^2\theta\cos^2\theta(\cos^2\theta+1)(\sin^2\theta+1)\sin^2\theta\cos^2\theta \frac{\cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)}{\sin^2\theta\cos^2\theta(\cos^2\theta+1)(\sin^2\theta+1)}\sin^2\theta\cos^2\theta$

$= \cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)(\cos^2\theta+1)(\sin^2\theta+1) \frac{\cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)}{(\cos^2\theta+1)(\sin^2\theta+1)}$

$= \cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta 1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta \frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$

$= 1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)1+1+\cos^2\theta\sin^2\theta \frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{1+1+\cos^2\theta\sin^2\theta}$

$= 1-\sin^2\theta\cos^2\theta 2+\sin^2\theta\cos^2\theta \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 65 . (i) . $\left[1+\sin\theta-\cos\theta 1+\sin\theta+\cos\theta\right]^2 \left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2$ **=** $1-\cos\theta 1+\cos\theta \frac{1-\cos\theta}{1+\cos\theta}$

Ans:

$= \left(1+\sin\theta-\cos\theta 1+\sin\theta+\cos\theta \times 1+\sin\theta-\cos\theta 1+\sin\theta-\cos\theta\right)^2 \left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\times\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2$

$= \left[(1+\sin\theta-\cos\theta)^2(1+\sin\theta)^2-\cos^2\theta\right]^2 \left[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2-\cos^2\theta}\right]^2$

$= \left[(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)-2\cos\theta 1-\cos^2\theta+\sin^2\theta+2\sin\theta\right]^2$

$\left[\frac{(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2$

$= \left[1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta\sin^2\theta+\sin^2\theta+2\sin\theta\right]^2 \left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2$

$= \left[2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta 2\sin^2\theta+2\sin\theta\right]^2 \left[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2\sin^2\theta+2\sin\theta}\right]^2$

$= \left[2(1+\sin\theta)-2\cos\theta(\sin\theta+1)2\sin\theta(\sin\theta+1)\right]^2 \left[\frac{2(1+\sin\theta)-2\cos\theta(\sin\theta+1)}{2\sin\theta(\sin\theta+1)}\right]^2$

$= \left[(1+\sin\theta)(2-2\cos\theta)2\sin\theta(\sin\theta+1)\right]^2 \left[\frac{(1+\sin\theta)(2-2\cos\theta)}{2\sin\theta(\sin\theta+1)}\right]^2$

$= \left[(2-2\cos\theta)2\sin\theta\right]^2 \left[\frac{(2-2\cos\theta)}{2\sin\theta}\right]^2$

= [(1−cosθ)sinθ]^2 $\left[\frac{(1-cos\theta)}{sin\theta}\right]^2$

= (1−cosθ)^{2}1−cos^{2}θ $\frac{(1-cos\theta)^2}{1-cos^2\theta}$

= (1−cosθ)×(1−cosθ)(1−cosθ)(1+cosθ) $\frac{(1-cos\theta)\times(1-cos\theta)}{(1-cos\theta)(1+cos\theta)}$

= 1−cosθ1+cosθ $\frac{1-cos\theta}{1+cos\theta}$

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 65 (ii) . 1+secθ−tanθ1+secθ+tanθ $\frac{1+sec\theta-tan\theta}{1+sec\theta+tan\theta}$ = 1−sinθcosθ $\frac{1-sin\theta}{cos\theta}$

Ans:

= LHS = 1+secθ−tanθ1+secθ+tanθ $\frac{1+sec\theta-tan\theta}{1+sec\theta+tan\theta}$

= (sec^{2}θ−tan^{2}θ)+(secθ−tanθ)1+secθ+tanθ $\frac{(sec^2\theta-tan^2\theta)+(sec\theta-tan\theta)}{1+sec\theta+tan\theta}$ [since , sec^{2}θ−tan^{2}θ=1 $sec^2\theta - tan^2\theta = 1$]

= (secθ−tanθ)(secθ+tanθ)+(secθ−tanθ)1+secθ+tanθ $\frac{(sec\theta-tan\theta)(sec\theta+tan\theta)+(sec\theta-tan\theta)}{1+sec\theta+tan\theta}$

= (secθ−tanθ)(1+secθ+tanθ)1+secθ+tanθ $\frac{(sec\theta-tan\theta)(1+sec\theta+tan\theta)}{1+sec\theta+tan\theta}$

= (secθ−tanθ)$(sec\theta - tan\theta)$

= 1cosθ − sinθcosθ $\frac{1}{cos\theta} - \frac{sin\theta}{cos\theta}$

= 1−sinθcosθ $\frac{1-sin\theta}{cos\theta}$

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 66 . (sec A + tan A – 1)(sec A – tan A + 1) = 2 tan A

Ans:

= (secA+tanA−{sec^{2}A−tan^{2}A})[secA−tanA+(sec^{2}A−tan^{2}A)]
$\left(secA + tanA - \left\{sec^2 A - tan^2 A\right\}\right) \left[secA - tanA + \left(sec^2 A - tan^2 A\right)\right]$

=

(secA+tanA−(secA+tanA)(secA−tanA))[secA−tanA+(secA−tanA)(secA+tanA)]
$(secA + tanA - (secA + tanA)(secA - tanA))\,[secA - tanA + (secA - tanA)(secA + tanA)]$

= (secA+tanA)(1−(secA−tanA))(secA−tanA)(1+(secA+tanA))
$(secA+tanA)\,(1-(secA-tanA))\,(secA-tanA)\,(1+(secA+tanA))$

= (sec^2A−tan^2A)(1−secA+tanA)(1+secA+tanA)
$(sec^2A-tan^2A)\,(1-secA+tanA)\,(1+secA+tanA)$

= (1− 1cosA + sinAcosA)(1+ 1cosA + sinAcosA) $\left(1-\frac{1}{cosA}+\frac{sinA}{cosA}\right)\left(1+\frac{1}{cosA}+\frac{sinA}{cosA}\right)$

= (cosA–1+sinAcosA)(cosA+1+sinAcosA) $\left(\frac{cosA-1+sinA}{cosA}\right)\left(\frac{cosA+1+sinA}{cosA}\right)$

= ((cosA+sinA)^2−1cos^{2}A) $\left(\frac{(cosA+sinA)^2-1}{cos^2A}\right)$

= (cos^{2}A+sin^{2}A+2sinAcosB−1cos^{2}A) $\left(\frac{cos^2A+sin^2A+2sinAcosB-1}{cos^2A}\right)$

= (1+2sinAcosB−1cos^{2}A) $\left(\frac{1+2sinAcosB-1}{cos^2A}\right)$

= (2sinAcosBcos^{2}A) $\left(\frac{2sinAcosB}{cos^2A}\right)$

= 2 tan A

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 67 . (1 + cot A – cosec A)(1 + tan A + sec A) = 2

Ans:

LHS = (1 + cot A – cosec A)(1 + tan A + sec A)

= (1+ cosAsinA − 1sinA)(1+ sinAcosA + 1cosA) $\left(1+\frac{cosA}{sinA}-\frac{1}{sinA}\right)\left(1+\frac{sinA}{cosA}+\frac{1}{cosA}\right)$

= (sinA+cosA−1sinA)(cosA+sinA+1cosA) $\left(\frac{sinA+cosA-1}{sinA}\right)\left(\frac{cosA+sinA+1}{cosA}\right)$

= ((sinA−cosA)^2−1sinAcosA) $\left(\frac{(sinA-cosA)^2-1}{sinAcosA}\right)$

= sin^{2}A+2sinAcosA+cos^{2}A−1sinAcosA $\frac{sin^2A+2sinAcosA+cos^2A-1}{sinAcosA}$

= (1+2sinAcosA−1sinAcosA) $\left(\frac{1+2sinAcosA-1}{sinAcosA}\right)$

= 2

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 68 . (cosecθ−secθ)(cotθ−tanθ)$(cosec\theta-sec\theta)\ (cot\theta-tan\theta)$ = (cosecθ+secθ) (secθcosecθ–2)$(cosec\theta+sec\theta)\ (sec\theta cosec\theta-2)$

Ans:

LHS = (cosecθ−secθ)(cotθ−tanθ)$(cosec\theta-sec\theta)\ (cot\theta-tan\theta)$

[1sinθ − 1cosθ][cosθsinθ − sinθcosθ] $\left[\frac{1}{sin\theta}-\frac{1}{cos\theta}\right]\left[\frac{cos\theta}{sin\theta}-\frac{sin\theta}{cos\theta}\right]$ [cosθ−sinθsinθcosθ][cos²θ−sin²θsinθcosθ]

$\left[\frac{cos\theta-sin\theta}{sin\theta cos\theta}\right]\left[\frac{cos^2\theta-sin^2\theta}{sin\theta cos\theta}\right]$ [(cosθ−sinθ)²(cosθ+sinθ)sin²θcos²θ] $\left[\frac{(cos\theta-sin\theta)^2(cos\theta+sin\theta)}{sin^2\theta cos^2\theta}\right]$

RHS = (cosecθ+secθ)(secθcosecθ–2)$(cosec\theta+sec\theta)\ (sec\theta cosec\theta-2)$

[1sinθ + 1cosθ][1cosθ − 1sinθ −2] $\left[\frac{1}{sin\theta}+\frac{1}{cos\theta}\right]\left[\frac{1}{cos\theta}-\frac{1}{sin\theta}-2\right]$

= [sinθ+cosθsinθcosθ][1−2sinθcosθsinθcosθ] $\left[\frac{sin\theta+cos\theta}{sin\theta cos\theta}\right]\left[\frac{1-2sin\theta cos\theta}{sin\theta cos\theta}\right]$

= [sinθ+cosθsinθcosθ][cos²θ+sin²θ−2sinθcosθsinθcosθ] $\left[\frac{sin\theta+cos\theta}{sin\theta cos\theta}\right]\left[\frac{cos^2\theta+sin^2\theta-2sin\theta cos\theta}{sin\theta cos\theta}\right]$

= [(cosθ−sinθ)²(cosθ+sinθ)sin²θcos²θ] $\left[\frac{(cos\theta-sin\theta)^2(cos\theta+sin\theta)}{sin^2\theta cos^2\theta}\right]$ [∵cos²θ+sin²θ=1]

Undefined control sequence \because

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q 70 . cosAcosecA−sinAsecAcosA+sinA $\frac{cosAcosecA-sinAsecA}{cosA+sinA}$ = cosec A – sec A

Ans:

LHS = cosAcosecA−sinAsecAcosA+sinA $\frac{cosAcosecA-sinAsecA}{cosA+sinA}$

= cosA×1sinA−sinA×1cosAcosA+sinA $\frac{cosA\times\frac{1}{sinA}-sinA\times\frac{1}{cosA}}{cosA+sinA}$

= cosAsinA − sinAcosAcosA+sinA $\frac{\frac{cosA}{sinA}-\frac{sinA}{cosA}}{cosA+sinA}$

= cos²A−sin²AcosAsinAcosA+sinA $\frac{\frac{cos^2A-sin^2A}{cosAsinA}}{cosA+sinA}$

= cos²A−sin²AcosAsinA × 1cosA+sinA $\frac{cos^2A-sin^2A}{cosAsinA}\times\frac{1}{cosA+sinA}$

= (cosA−sinA)(cosA+sinA)cosAsinA×(cosA+sinA) $\frac{(cosA-sinA)(cosA+sinA)}{cosAsinA\times(cosA+sinA)}$

= (cosA−sinA)cosAsinA $\frac{(cosA-sinA)}{cosAsinA}$

= cosAcosAsinA − sinAcosAsinA $\frac{cosA}{cosAsinA}-\frac{sinA}{cosAsinA}$

= 1sinA − 1cosA $\frac{1}{sinA} - \frac{1}{cosA}$

= cosecA−secA $cosecA - secA$

= RHS

∴ Undefined control sequence \therefore LHS = RHS   Hence proved .

Q 71 . sinAsecA+tanA−1 + cosAcosecA+cotA−1 $\frac{sinA}{secA+tanA-1} + \frac{cosA}{cosecA+cotA-1}$ **= 1**

Ans:

LHS : sinAsecA+tanA−1 + cosAcosecA+cotA−1 $\frac{sinA}{secA+tanA-1} + \frac{cosA}{cosecA+cotA-1}$

= sinA1cosA+sinAcosA−1 + cosA1sinA+cosAsinA−1 $\frac{sinA}{\frac{1}{cosA}+\frac{sinA}{cosA}-1} + \frac{cosA}{\frac{1}{sinA}+\frac{cosA}{sinA}-1}$

= sinA1+sinA−cosAcosA + cosA1+cosA−sinAsinA $\frac{sinA}{\frac{1+sinA-cosA}{cosA}} + \frac{cosA}{\frac{1+cosA-sinA}{sinA}}$

= sinAcosA1+sinA−cosA + cosAsinA1+cosA−sinA $\frac{sinAcosA}{1+sinA-cosA} + \frac{cosAsinA}{1+cosA-sinA}$

= (sinAcosA)[11+sinA−cosA + 11+cosA−sinA] $(sinAcosA)\left[\frac{1}{1+sinA-cosA} + \frac{1}{1+cosA-sinA}\right]$

= (sinAcosA)[2cosA−sinA+sinA+sinAcosA−sin²A−cosA−cos²A+cosAsinA]

$(sinAcosA)\left[\frac{2}{cosA-sinA+sinA+sinAcosA-sin^2A-cosA-cos^2A+cosAsinA}\right]$

= (sinAcosA)[21−sin²A−cos²A+2sinAcosA] $(sinAcosA)\left[\frac{2}{1-sin^2A-cos^2A+2sinAcosA}\right]$

= (sinAcosA)[21−(sin²A−cos²A)+2sinAcosA] $(sinAcosA)\left[\frac{2}{1-(sin^2A-cos^2A)+2sinAcosA}\right]$

= (sinAcosA)[21−1+2sinAcosA] $(sinAcosA)\left[\frac{2}{1-1+2sinAcosA}\right]$

= (sinAcosA)× 22sinAcosA $(sinAcosA) \times \frac{2}{2sinAcosA}$

= 1

= RHS

∴ Undefined control sequence \therefore LHS = RHS   Hence proved .

Q 72 . tanA(1+tan²A)² + cotA(1+cot²A)² $\frac{tanA}{(1+tan^2A)^2} + \frac{cotA}{(1+cot^2A)^2}$ **= sin A cos A**

Ans:

tanA(sec²A)²+cotA(cosec²A)² $\frac{tanA}{(sec^2A)^2} + \frac{cotA}{(cosec^2A)^2}$ [1 +tan² A = sec² A , 1 +cot² A = cosec² A]

= sinAcosAsec⁴A+cosAsinAcosec⁴A $\frac{\frac{sinA}{cosA}}{sec^4A} + \frac{\frac{cosA}{sinA}}{cosec^4A}$

= sinAcosA 1cos⁴A + cosAsinA 1sin⁴A $\frac{\frac{sinA}{cosA}}{\frac{1}{cos^4A}} + \frac{\frac{cosA}{sinA}}{\frac{1}{sin^4A}}$

= sinAcosA × cos⁴A1 + cosAsinA × sin⁴A1 $\frac{sinA}{cosA} \times \frac{cos^4A}{1} + \frac{cosA}{sinA} \times \frac{sin^4A}{1}$

= sinA×cos³A+cosA×sin³A$sinA \times cos^3A + cosA \times sin^3A$

= sinAcosA(cos²A+sin²A)$sinAcosA\left(cos^2A + sin^2A\right)$

= sinAcosA$sinAcosA$

∴ Undefined control sequence \therefore LHS = RHS Hence proved .

Q73. sec⁴A(1−sin⁴A)−2tan²A=1$sec^4A(1\;-\;sin^4A)\;-\;2tan^2A = 1$

Ans:

Given, L.H.S = \(sec^{4}A(1\;-\;sin^{4}A)\;-\;2tan^{2}A

= [latex]sec^{4}A\;-\;sec^{4}A\;\times \;sin^{4}A\;-\;2tan^{4}A\)

= sec⁴A−(1cos⁴A ×sin⁴A)−2tan⁴A$sec^4A\;-\;(\frac{1}{cos^4A}\;\times\;sin^4A)\;-\;2tan^4A$

= sec⁴A−tan⁴A−2tan⁴A$sec^4A\;-\;tan^4A\;-\;2tan^4A$

= (sec²A)²−tan⁴A−2tan⁴A$(sec^2A)^2\;-\;tan^4A\;-\;2tan^4A$

= (1+tan²A)²−tan⁴A−2tan⁴A$(1\;+\;tan^2A)^2\;-\;tan^4A\;-\;2tan^4A$

= 1+tan⁴A+2tan²A−tan⁴A−2tan⁴A$1 + tan^4A + 2tan^2A -\;tan^4A\;-\;2tan^4A$

= 1

Hence, L.H.S = R.H.S

Q74. **cot²A(secA−1)1+sinA** $\frac{cot^2A(secA\;-\;1)}{1\;+\;sinA}$ **= sec²A[1−sinA1+sinA]**$sec^2A[\frac{1\;-\;sinA}{1\;+\;sinA}]$

Ans:

Given, L.H.S = cot^{2}A(secA−1)1+sinA $\frac{cot^2A(secA\ -\ 1)}{1\ +\ sinA}$

Here, sin^{2}A+cos^{2}A$sin^2A\ +\ cos^2A$ = 1

= cos^{2}Asin^{2}A(1cosA−1)1+sinA $\frac{\frac{cos^2A}{sin^2A}(\frac{1}{cosA}-1)}{1+sinA}$

= cos^{2}Asin^{2}A(1−cosAcosA)1+sinA $\frac{\frac{cos^2A}{sin^2A}(\frac{1-cosA}{cosA})}{1+sinA}$

= cosA×cosA(1−cos^{2}A)(1−cosAcosA)1+sinA $\frac{\frac{cosA\times cosA}{(1-cos^2A)}(\frac{1-cosA}{cosA})}{1+sinA}$

= (cosA)(1+cosA) 11+sinA $\frac{(cosA)}{(1+cosA)}\ \frac{1}{1+sinA}$

Solving,

RHS => sec^{2}a[1-sinA1+secA]$sec^2a[\frac{1-sinA}{1+secA}]$

= 1cos^{2}A[1−sinA1+secA]$\frac{1}{cos^2A}[\frac{1-sinA}{1+secA}]$

= 1cos^{2}A[1−sinA1+secA]$\frac{1}{cos^2A}[\frac{1-sinA}{1+secA}]$

= 1cos^{2}A[1−sinAcosA+1]cosA$\frac{1}{cos^2A}[\frac{1-sinA}{cosA+1}]cosA$

= (1−sinA)(cosA+1)(cosA) $\frac{(1-sinA)}{(cosA+1)(cosA)}$

Multiplying Nr. And Dr. with (1+SinA)

= (1−sinA)(cosA+1)(cosA) × 1+sinA1+sinA $\frac{(1-sinA)}{(cosA+1)(cosA)}\ \times\ \frac{1+sinA}{1+sinA}$

= (1^2−sin^{2}A)(cosA+1)(cosA)(1+sinA) $\frac{(1^2-sin^2A)}{(cosA+1)(cosA)(1+sinA)}$

= cos^{2}A(cosA+1)(cosA)(1+sinA) $\frac{cos^2A}{(cosA+1)(cosA)(1+sinA)}$

= cosA(cosA+1)(1+sinA) $\frac{cosA}{(cosA+1)(1+sinA)}$

Hence, LHS= RHS

Q75. (1+cotA+tanA)(sinA−cosA)$(1\ +\ cotA\ +tanA)(sinA\ -\ cosA)$ **= secAcosec²A** $\frac{secA}{cosec^2A}$ **− cosecAsec²A** $\frac{cosecA}{sec^2A}$ **= sinAtanA – cotAcosA**

Ans:

Given, L.H.S = (1+cotA+tanA)(sinA−cosA)$(1\ +\ cotA\ +tanA)(sinA\ -\ cosA)$

=> sinA – cosA + cotAsinA – cotAcosA + sinAtanA – tanAcosA

=> sinA – cosA + cosAsinA ×sinA $\frac{cosA}{sinA} \times sinA$ – cotAcosA + sinAtanA – sinAcosA ×cosA $\frac{sinA}{cosA} \times cosA$

=> sinA – cosA + cosA – cotAcosA + sinAtanA – sinA

=> sinAtanA – cosAcotA

=> secAcosec^{2}A $\frac{secA}{cosec^2A}$ – cosecAsec^{2}A $\frac{cosecA}{sec^2A}$

Here, secA = 1cosA $\frac{1}{cosA}$ and cosecA = 1sinA $\frac{1}{sinA}$

=> sin^{2}AcosA $\frac{sin^2A}{cosA}$ – cos^{2}AsinA $\frac{cos^2A}{sinA}$

=> sin^{2}A−cos^{2}AcosAsinA $\frac{sin^2A\;-\;cos^2A}{cosAsinA}$

=> (sinA× sinAcosA)$(sinA \times \frac{sinA}{cosA})$ – (cosA× cosAcotA)$(cosA \times \frac{cosA}{cotA})$

=> sinAtanA – cosAcotA

Hence, L.H.S = R.H.S

Q76. If xa cosθ+ yb sinθ $\frac{x}{a}cos\theta\;+\;\frac{y}{b}sin\theta$ = 1 and xa cosθ− yb sinθ $\frac{x}{a}cos\theta\;-\;\frac{y}{b}sin\theta$ = 1, prove that x²a² + y²b² $\frac{x^2}{a^2}\;+\;\frac{y^2}{b^2}$ = 2

Ans:

Given,

=> (xa cosθ+ yb sinθ $\frac{x}{a}cos\theta\;+\;\frac{y}{b}sin\theta)^2$ + (xa cosθ− yb sinθ $\frac{x}{a}cos\theta\;-\;\frac{y}{b}sin\theta)^2$ = $1^2 + 1^2$

=> x²a² cos^{2}θ+ y²b² sin^{2}θ+ 2xyab cosθsinθ+ x²a² sin^{2}θ+ y²b² − 2xyab sinθcosθ
$\frac{x^2}{a^2}cos^2\theta\;+\;\frac{y^2}{b^2}sin^2\theta\;+\;\frac{2xy}{ab}cos\theta sin\theta\;+\;\frac{x^2}{a^2}sin^2\theta\;+\;\frac{y^2}{b^2}\;-\;\frac{2xy}{ab}sin\theta cos\theta$ = 1 + 1

=> x²a² cos^{2}θ+ y²b² sin^{2}θ+ x²a² sin^{2}θ+ y²b² sin^{2}θ $\frac{x^2}{a^2}cos^2\theta\;+\;\frac{y^2}{b^2}sin^2\theta\;+\;\frac{x^2}{a^2}sin^2\theta\;+\;\frac{y^2}{b^2}sin^2\theta$ = 2

=> cos^{2}θ[x²a² + y²b²]$cos^2\theta[\frac{x^2}{a^2}\;+\;\frac{y^2}{b^2}]$ + sin^{2}θ[x²a² + y²b²]$sin^2\theta[\frac{x^2}{a^2}\;+\;\frac{y^2}{b^2}]$ = 2

=> (cos^{2}θ+sin^{2}θ)[x²a² + y²b²]$(cos^2\theta\;+\;sin^2\theta)[\frac{x^2}{a^2}\;+\;\frac{y^2}{b^2}]$ = 2

Here cos^{2}A +sin^{2}A = 1

=> (1) [\frac{x^{2}}{a^{2}}\;+\;\frac{y^{2}}{b^{2}}][/latex] = 2

=> [\frac{x^{2}}{a^{2}}\;+\;\frac{y^{2}}{b^{2}}][/latex] = 2

Q77. If cosecθ−sinθ=a^3 $cosec\theta - sin\theta = a^3$, secθ−cosθ=b^3 $sec\theta - cos\theta = b^3$, prove that a^{2}b^2(a^2+b^2) $a^2b^2(a^2 + b^2)$ = 1

Ans:

Given, cosecθ−sinθ=a^3 $cosec\theta - sin\theta = a^3$

Here, cosecθ=1sinθ $cosec\theta = \frac{1}{sin\theta}$

=> 1sinθ $\frac{1}{sin\theta}$ – sinθ $sin\theta$ = a^3 a^3

=> 1-sin^{2}θsinθ $\frac{1 - sin^2\theta}{sin\theta}$ = a^3 a^3

Here cos^{2}A +sin^{2}A = 1

=> cos^{2}θsinθ $\frac{cos^2\theta}{sin\theta}$ = a^3 a^3

=> cos^{23}θsin^{13}θ $\frac{cos^{\frac{2}{3}}\theta}{sin^{\frac{1}{3}}\theta}$ = a

Squaring on both sides

=> a^2 = cos^{43}θsin^{23}θ $\frac{cos^{\frac{4}{3}}\theta}{sin^{\frac{2}{3}}\theta}$

secθ−cosθ=b^3 $sec\theta - cos\theta = b^3$

=> 1cosθ $\frac{1}{cos\theta}$ – cosθ $cos\theta$ = b^3 b^3

=> 1-cos^{2}θcosθ $\frac{1 - cos^2\theta}{cos\theta}$ = b^3 b^3

=> sin^{2}θcosθ $\frac{sin^2\theta}{cos\theta}$ = b^3 b^3

=> sin^{23}θcos^{13}θ $\frac{sin^{\frac{2}{3}}\theta}{cos^{\frac{1}{3}}\theta}$ = b

Squaring on both sides

=> b^2 = sin^{43}θcos^{23}θ $\frac{sin^{\frac{4}{3}}\theta}{cos^{\frac{2}{3}}\theta}$

Now, a^{2}b^2(a^2+b^2) $a^2b^2(a^2 + b^2)$

=> cos^{43}θsin^{23}θ $\frac{cos^{\frac{4}{3}}\theta}{sin^{\frac{2}{3}}\theta}$ ×$\times$ sin^{43}θcos^{23}θ $\frac{sin^{\frac{4}{3}}\theta}{cos^{\frac{2}{3}}\theta}$(cos^{43}θsin^{23}θ $\frac{cos^{\frac{4}{3}}\theta}{sin^{\frac{2}{3}}\theta}$ + sin^{43}θcos^{23}θ $\frac{sin^{\frac{4}{3}}\theta}{cos^{\frac{2}{3}}\theta}$)

=> cos^{23}θ $cos^{\frac{2}{3}}\theta$ sin^{23}θ $sin^{\frac{2}{3}}\theta$(Missing close brace Missing close brace sin^{23}θ $sin^{\frac{2}{3}}\theta$}[/latex]

= 1

Hence, L.H.S = R.H.S

Q78. If acos^3 θθ + 3acosθθ sin^{2}θ$sin^2\theta$ = m, asin^{3}θ$sin^3\theta$ + 3acos^{2}θ$cos^2\theta$ sinθ$sin\theta$ = n, prove that (m+n)^{23}$(m + n)^{\frac{2}{3}}$ + (m−n)^{23}$(m - n)^{\frac{2}{3}}$ = 2(a)^{23}$(a)^{\frac{2}{3}}$

Ans:

Given, (m+n)^{23}$(m + n)^{\frac{2}{3}}$ + (m−n)^{23}$(m - n)^{\frac{2}{3}}$

Substitute the values of m and n in the above equation

=> (\((acos^3 [latex]\theta\) + 3acosθθ sin^{2}θ$sin^2\theta$\;+\; asin^{3}θ$sin^3\theta$ + 3acos^{2}θ$cos^2\theta$ sinθ$sin\theta$)^{\frac{2}{3}}[/latex] + (\((acos^3 [latex]\theta\) + 3acosθθ sin^{2}θ$sin^2\theta$\;-\; asin^{3}θ$sin^3\theta$ – 3acos^{2}θ$cos^2\theta$ sinθ $sin\theta$)^{\frac{2}{3}}[/latex]

=> (a)^{23}$(a)^{\frac{2}{3}}$ ((\((cos^3 [latex]\theta\) + 3cosθθ sin^{2}θ$sin^2\theta$\;+\; sin^{3}θ$sin^3\theta$ + 3cos^{2}θ$cos^2\theta$ sinθ $sin\theta$)^{\frac{2}{3}}[/latex] + (a)^{23}$(a)^{\frac{2}{3}}$ ((\((cos^3 [latex]\theta\) + 3cosθθ sin^{2}θ$sin^2\theta$\;-\; sin^{3}θ$sin^3\theta$ – 3cos^{2}θ$cos^2\theta$ sinθ$sin\theta$)^{\frac{2}{3}}[/latex]

=> (a)^{23}$(a)^{\frac{2}{3}}$ ((cosθ+sinθ)^3)^{23}$((cos\theta + sin\theta)^3)^{\frac{2}{3}}$ + (a)^{23}$(a)^{\frac{2}{3}}$ ((cosθ−sinθ)^3)^{23} $((cos\theta - sin\theta)^3)^{\frac{2}{3}}$

=> (a)^{23}$(a)^{\frac{2}{3}}$ [(cosθ+sinθ)^2$(cos\theta + sin\theta)^2$] + (a)^{23}$(a)^{\frac{2}{3}}$ [(cosθ−sinθ)^2$(cos\theta - sin\theta)^2$]

=> (a)^{23}$(a)^{\frac{2}{3}}$ ((cos^{2}θ+sin^{2}θ+2sinθcosθ)$(cos^2\theta + sin^2\theta + 2sin\theta cos\theta)$) + (a)^{23}$(a)^{\frac{2}{3}}$ ((cos^{2}θ+sin^{2}θ−2sinθcosθ)$(cos^2\theta + sin^2\theta - 2sin\theta cos\theta)$)

=> (a)^{23}$(a)^{\frac{2}{3}}$ [1 + 2sinθcosθ$2sin\theta cos\theta$] + (a)^{23}$(a)^{\frac{2}{3}}$ [1 – 2sinθcosθ$2sin\theta cos\theta$]

=> (a)^{23}$(a)^{\frac{2}{3}}$ [1 + 2sinθcosθ$2sin\theta cos\theta$] + 1 – 2sinθcosθ$2sin\theta cos\theta$]

=> (a)^{23}$(a)^{\frac{2}{3}}$ (1 + 1)

=> 2(a)^{23}$(a)^{\frac{2}{3}}$

Hence, L.H.S = R.H.S

Q79) Ifx=acos^{3}Θ,y=bsin^{3}Θ,provethat(xa)^{23}+(yb)^{23}=1

$If\ x = acos^3\Theta,\ y = bsin^3\Theta,\ prove\ that\ (\frac{x}{a})^{\frac{2}{3}} + (\frac{y}{b})^{\frac{2}{3}} = 1$

Ans:

x=acos^{3}Θ:y=bsin^{3}Θ$x = acos^3\Theta : \ y = bsin^3\Theta$ xa=cos^{3}Θ:yb=sin^{3}Θ$\frac{x}{a} = cos^3\Theta : \ \frac{y}{b} = sin^3\Theta$

L.H.S = [xa]^23+[yb]^23 $[\frac{x}{a}]^{\frac{2}{3}}+[\frac{y}{b}]^{\frac{2}{3}}$

=[cos³Θ]^23+[sin³Θ]^23 $=[cos^3\Theta]^{\frac{2}{3}}+[sin^3\Theta]^{\frac{2}{3}}$ =cos²Θ+sin²Θ(∵cos²Θ+sin²Θ=1)

Undefined control sequence \because

=1

Hence proved.

Q80) IfacosΘ+bsinΘ=mandasinΘ−bcosΘ=n,provethata²+b²=m²+n²
$If\ acos\Theta+bsin\Theta=m\ and\ asin\Theta-bcos\Theta=n,\ prove\ that\ a^2+b^2=m^2+n^2$

Ans:

R.H.S=m²+n² $R.H.S=m^2+n^2$

=(acosΘ+bsinΘ)²+

(asinΘ−bcosΘ)²=a²cos²Θ+b²sin²Θ+2absinΘcosΘ+a²sin²Θ+b²cos²Θ−2absinΘcosΘ=a²cos²Θ+b²(

Undefined control sequence \because

=a²(sin²Θ+cos²Θ)+b²(sin²Θ+cos²Θ)=a²+b²[∵sin²Θ+cos²Θ=1]

= m² + n²

Q81: IfcosA+cos²A=1,provethatsin²A+sin⁴A=1
$If\ cosA+cos^2A=1,\ prove\ that\ sin^2A+sin^4A=1$

Ans:

Given- cos A + cos^2 A = 1

We have to prove sin^2 A + sin^4 A = 1

Now, cos A + cos^2 A = 1

$\cos A = 1 - \cos^2 A$

$\cos A = \sin^2 A$

$\sin^2 A = \cos A$

Therefore, we have $\sin^2 A + \sin^4 A = \cos A + (\cos A)^2 = \cos A + \cos^2 A = 1$

Hence proved.

Q82:

If $\cos\Theta + \cos^2\Theta = 1$, prove that $\sin^{12}\Theta + 3\sin^{10}\Theta + 3\sin^8\Theta + \sin^6\Theta + 2\sin^4\Theta + 2\sin^2\Theta - 2 = 1$
$If\ cos\Theta + cos^2\Theta = 1,\ prove\ that\ sin^{12}\Theta + 3sin^{10}\Theta + 3sin^8\Theta + sin^6\Theta + 2sin^4\Theta + 2sin^2\Theta - 2 = 1$

Ans:

$\cos\Theta + \cos^2\Theta = 1$ $cos\Theta + cos^2\Theta = 1$ $\cos\Theta = 1 - \cos^2\Theta$ $cos\Theta = 1 - cos^2\Theta$

$\cos\Theta = \sin^2\Theta$ $cos\Theta = sin^2\Theta$......(i)

Now, $\sin^{12}\Theta + 3\sin^{10}\Theta + 3\sin^8\Theta + \sin^6\Theta + 2\sin^4\Theta + 2\sin^2\Theta - 2$
$Now,\ sin^{12}\Theta + 3sin^{10}\Theta + 3sin^8\Theta + sin^6\Theta + 2sin^4\Theta + 2sin^2\Theta - 2$ =
$(\sin^4\Theta)^3 + 3\sin^4\Theta.\sin^2\Theta(\sin^4\Theta + \sin^2\Theta) + (\sin^2\Theta)^3 + 2(\sin^2\Theta)^2 + 2\sin^2\Theta - 2$
$= (sin^4\Theta)^3 + 3sin^4\Theta.\ sin^2\Theta(sin^4\Theta + sin^2\Theta) + (sin^2\Theta)^3 + 2(sin^2\Theta)^2 + 2sin^2\Theta - 2$
Using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ and also from (i) $\cos\Theta = \sin^2\Theta$
$Using\ (a+b)^3 = a^3 + b^3 + 3ab(a+b)\ and\ also\ from\ (i)\ cos\Theta = sin^2\Theta$
$(\sin^4\Theta + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $(sin^4\Theta + sin^2\Theta)^3 + 2cos^2\Theta + 2cos\Theta - 2$
$((\sin^2\Theta)^2 + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $((sin^2\Theta)^2 + sin^2\Theta)^3 + 2cos^2\Theta + 2cos\Theta - 2$
$(\cos^2\Theta + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $(cos^2\Theta + sin^2\Theta)^3 + 2cos^2\Theta + 2cos\Theta - 2$
$1 + 2\cos^2\Theta + 2\sin^2\Theta - 2 [\because \sin^2\Theta + \cos^2\Theta = 1]$ Undefined control sequence \because $1 + 2(\cos^2\Theta + \sin^2\Theta) - 2$
$1 + 2(cos^2\Theta + sin^2\Theta) - 2$ $1 + 2(1) - 2$ $1 + 2(1) - 2$ $= 1$ $= 1$

L.H.S = R.H.S

Hence proved.

Q83: Given that: $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$
$(1 + cos\alpha)(1 + cos\beta)(1 + cos\gamma) = (1 - cos\alpha)(1 - cos\beta)(1 - cos\gamma)$. **Show that one of the values of each member of this equality is $\sin\alpha \sin\beta \sin\gamma$** $sin\alpha\ sin\beta\ sin\gamma$.

Ans:

We know that $1 + \cos\Theta = 1 + \cos^2\frac{\Theta}{2} - \sin^2\frac{\Theta}{2} = 2\cos^2\frac{\Theta}{2}$ $1 + cos\Theta = 1 + cos^2\frac{\Theta}{2} - sin^2\frac{\Theta}{2} = 2cos^2\frac{\Theta}{2}$

⇒2cos^{2}α2.2cos^{2}β2.2cos^{2}γ2…..(i)⇒ $2cos^2\frac{\alpha}{2}.2cos^2\frac{\beta}{2}.2cos^2\frac{\gamma}{2}.....(i)$

Multiply(i)withsinαsinβsinγanddivideitwithsameweget
Multiply (i) with sinα sinβ sinγ and divide it with same we get

8cos^{2}α2.cos^{2}β2.cos^{2}γ2sinα.sinβ.sinγ×sinα.sinβ.sinγ $\frac{8cos^2\frac{\alpha}{2}.cos^2\frac{\beta}{2}.cos^2\frac{\gamma}{2}}{sin\alpha.sin\beta.sin\gamma}\times sin\alpha.sin\beta.sin\gamma$

⇒2cos^{2}α2.cos^{2}β2.cos^{2}γ2×sinα.sinβ.sinγsinα2.sinβ2.sinγ2 $\Rightarrow \frac{2cos^2\frac{\alpha}{2}.\ cos^2\frac{\beta}{2}.\ cos^2\frac{\gamma}{2}\times sin\alpha.\ sin\beta.\ sin\gamma}{sin\frac{\alpha}{2}.\ sin\frac{\beta}{2}.\ sin\frac{\gamma}{2}}$

sinα.sinβ.sinγ×cotα2.cotβ2.cotγ2 $sin\alpha.\ sin\beta.\ sin\gamma\times cot\frac{\alpha}{2}.\ cot\frac{\beta}{2}.\ cot\frac{\gamma}{2}$ RHS(1−cosα)(1−cosβ)(1−cosγ) $RHS\ (1-cos\alpha)(1-cos\beta)(1-cos\gamma)$

We know that 1−cosΘ=1−cos^{2}Θ2+sin^{2}Θ2=2sin^{2}Θ2 $1-cos\Theta=1-cos^2\frac{\Theta}{2}+sin^2\frac{\Theta}{2}=2sin^2\frac{\Theta}{2}$

⇒2.sin^{2}α2 2.sin^{2}β2 2.sin^{2}γ2 $\Rightarrow 2.sin^2\frac{\alpha}{2}\ 2.sin^2\frac{\beta}{2}\ 2.sin^2\frac{\gamma}{2}$

Multiply(i)withsinαsinβsinγanddivideitwithsameweget
Multiply (i) with sinα sinβ sinγ and divide it with same we get

8sin^{2}α2.sin^{2}β2.sin^{2}γ2sinα.sinβ.sinγ×sinα.sinβ.sinγ $\frac{8sin^2\frac{\alpha}{2}.sin^2\frac{\beta}{2}.sin^2\frac{\gamma}{2}}{sin\alpha.sin\beta.sin\gamma}\times sin\alpha.sin\beta.sin\gamma$

⇒8sin^{2}α2.sin^{2}β2.sin^{2}γ2×sinα.sinβ.sinγ2sinα2cosα2.2sinβ2cosβ2.2sinγ2cosγ2 $\Rightarrow \frac{8sin^2\frac{\alpha}{2}.\ sin^2\frac{\beta}{2}.\ sin^2\frac{\gamma}{2}\times sin\alpha.\ sin\beta.\ sin\gamma}{2sin\frac{\alpha}{2}cos\frac{\alpha}{2}.\ 2sin\frac{\beta}{2}cos\frac{\beta}{2}.\ 2sin\frac{\gamma}{2}cos\frac{\gamma}{2}}$

⇒sinα.sinβ.sinγ×tanα2.tanβ2.tanγ2 $\Rightarrow sin\alpha.\ sin\beta.\ sin\gamma\times tan\frac{\alpha}{2}.\ tan\frac{\beta}{2}.\ tan\frac{\gamma}{2}$

Hence sinαsinβsinγ $sin\alpha\ sin\beta\ sin\gamma$ is the member of equality.

Q84: If sinΘ+cosΘ=x,provethatsin^{6}Θ+cos^{6}Θ=4−3(x^2−1)^{2}4

$sin\Theta+cos\Theta=x,\ prove\ that\ sin^6\Theta+cos^6\Theta=\frac{4-3(x^2-1)^2}{4}.$

Ans:

sinΘ+cosΘ=x $sin\Theta+cos\Theta=x$

Squaring on both sides

(sinΘ+cosΘ)^2=x^2 $(sin\Theta+cos\Theta)^2=x^2$ ⇒sinΘ^2+cosΘ^2+2sinΘcosΘ=x^2
$\Rightarrow sin\Theta^2+cos\Theta^2+2sin\Theta cos\Theta=x^2$ ∴sinΘcosΘ=x2−12……(i) Undefined control sequence \therefore

Weknowthatsin^{2}Θ+cos^{2}Θ=1 $We\ know\ that\ sin^2\Theta+cos^2\Theta=1$

Cubing on both sides

(sin^{2}Θ+cos^{2}Θ)^3=1^3 $(sin^2\Theta+cos^2\Theta)^3=1^3$ sin^{6}Θ+cos^{6}Θ+3sin^{2}Θcos^{2}Θ(sin^{2}Θ+cos^{2}Θ)=1
$sin^6\Theta+cos^6\Theta+3sin^2\Theta cos^2\Theta(sin^2\Theta+cos^2\Theta)=1$ ⇒sin^{6}Θ+cos^{6}Θ=1−3sin^{2}Θcos^{2}Θ
$\Rightarrow sin^6\Theta+cos^6\Theta=1-3sin^2\Theta cos^2\Theta$ ⇒sin^{6}Θ+cos^{6}Θ=1−3(x^2−1)^{2}4

$\Rightarrow sin^6\Theta+cos^6\Theta=1-\frac{3(x^2-1)^2}{4}$ ∴sin^{6}Θ+cos^{6}Θ=4−3(x^2−1)^{2}4 Undefined control sequence \therefore

Q85. If x = asecθcosϕ$asec\theta cos\phi$, y =bsecθsinϕ$bsec\theta sin\phi$ and z= ctanϕ$c\ tan\phi$, show that $x^2a^2+y^2b^2-z^2c^2$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ = 1

Ans:

Given, x = asecθcosϕ$asec\theta cos\phi$

y =bsecθsinϕ$bsec\theta sin\phi$

z= ctanϕ$ctan\phi$

squaring x,y,z on the sides

$x^2 x^2$ = $a^2sec^2\theta cos^2\phi$ $a^2sec^2\theta cos^2\phi$

x^2a^2 $\frac{x^2}{a^2}$ = $sec^2\theta cos^2\phi$ $sec^2\theta cos^2\phi$ — 1

$y^2 y^2$ = $b^2sec^2\theta sin^2\phi$ $b^2sec^2\theta sin^2\phi$

y^2b^2 $\frac{y^2}{b^2}$ = $sec^2\theta sin^2\phi$ $sec^2\theta sin^2\phi$ — 2

$z^2 z^2$ = $c^2tan^2\phi$ $c^2tan^2\phi$

z^2c^2 $\frac{z^2}{c^2}$ = $tan^2\phi$ $tan^2\phi$ —— 3

Substitute eq 1,2,3 in $x^2a^2+y^2b^2-z^2c^2$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

=> $x^2a^2+y^2b^2-z^2c^2$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

=> $sec^2\theta cos^2\phi$ $sec^2\theta cos^2\phi$ + $sec^2\theta sin^2\phi$ $sec^2\theta sin^2\phi$ – $tan^2\phi$ $tan^2\phi$

=> $sec^2\theta(cos^2\phi+sin^2\phi)$ $sec^2\theta(cos^2\phi + sin^2\phi)$ – $tan^2\phi$ $tan^2\phi$

We know that, $cos^2\phi+sin^2\phi$ $cos^2\phi + sin^2\phi$ = 1

=> $sec^2\theta$ $sec^2\theta(1)$ – $tan^2\phi$ $tan^2\phi$

And, $sec^2\theta-tan^2\theta$ $sec^2\theta - tan^2\theta$ = 1

=> 1

Hence, L.H.S= R.H.S

Q86. If sinθ+2cosθ$sin\theta + 2cos\theta$ prove that 2sinθ−cosθ$2sin\theta - cos\theta$ = 2

Ans:

Given, $\sin\theta+2\cos\theta$ $sin\theta + 2cos\theta$ =1

Squaring on both sides

=> $(\sin\theta+2\cos\theta)^2$ $(sin\theta + 2cos\theta)^2$ = 1^2

=> $\sin^2\theta+4\cos^2\theta+4\sin\theta\cos\theta$ $sin^2\theta + 4cos^2\theta + 4sin\theta cos\theta$ = 1

=> $4\cos^2\theta+4\sin\theta\cos\theta$ $4cos^2\theta + 4sin\theta cos\theta$ = 1 – $\sin^2\theta$ $sin^2\theta$

Here, 1 – $\sin^2\theta$ $sin^2\theta$ = $\cos^2\theta$ $cos^2\theta$

=> $4\cos^2\theta+4\sin\theta\cos\theta$ $4cos^2\theta + 4sin\theta cos\theta$ – $\cos^2\theta$ $cos^2\theta$ = 0

=> $3\cos^2\theta+4\sin\theta\cos\theta$ $3cos^2\theta + 4sin\theta cos\theta$ =0 — 1

We have, $2\sin\theta-\cos\theta$ $2sin\theta - cos\theta$ = 2

Squaring L.H.S

$(2\sin\theta-\cos\theta)^2$ $(2sin\theta - cos\theta)^2$ = $4\sin^2\theta+\cos^2\theta-4\sin\theta\cos\theta$ $4sin^2\theta + cos^2\theta - 4sin\theta cos\theta$

Here, $4\sin\theta\cos\theta$ $4sin\theta cos\theta$ = $3\cos^2\theta$ $3cos^2\theta$

= $4\sin^2\theta+\cos^2\theta+3\cos^2\theta$ $4sin^2\theta + cos^2\theta + 3cos^2\theta$

= $4\sin^2\theta+4\cos^2\theta$ $4sin^2\theta + 4cos^2\theta$

= $4(\sin^2\theta+\cos^2\theta)$ $4(sin^2\theta + cos^2\theta)$

= 4(1)

= 4

$(2\sin\theta-\cos\theta)^2$ $(2sin\theta - cos\theta)^2$ = 4

=> $2\sin\theta-\cos\theta$ $2sin\theta - cos\theta$ = 2

Hence proved

Exercise 6.2: Trigonometric Identities

Q1) If cosθ=45 $cos\theta = \frac{4}{5}$, find all other trigonometric ratios of angle ΘΘ.

Solution:

We have:

sinΘ=√1−cos²Θ=√1−(45)² $sin\Theta = \sqrt{1 - cos^2\Theta} = \sqrt{1 - (\frac{4}{5})^2}$

= √1−1625 $\sqrt{1 - \frac{16}{25}}$

= √25−1625 $\sqrt{\frac{25-16}{25}}$

= √925=35 $\sqrt{\frac{9}{25}} = \frac{3}{5}$

Therefore, sinΘ=35 $sin\Theta = \frac{3}{5}$

tanΘ=sinΘcosΘ=3545=34 secΘ=1cosΘ=145=54

$$tan\Theta = \frac{sin\Theta}{cos\Theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4} sec\Theta = \frac{1}{cos\Theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

i.e. cosecΘ=1secΘ=135=53cotΘ=1tanΘ=134=43

$$cosec\Theta = \frac{1}{sec\Theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3} cot\Theta = \frac{1}{tan\Theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Q2) If sinΘ=1√2 $sin\Theta = \frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle ΘΘ.

Solution:

We have,

cosΘ=√1−sin²Θ=√1−(1√2)² $cos\Theta = \sqrt{1 - sin^2\Theta} = \sqrt{1 - (\frac{1}{\sqrt{2}})^2}$

= √1−12 $\sqrt{1 - \frac{1}{2}}$

= √2−12 $\sqrt{\frac{2-1}{2}}$

= cosΘ=1√2 $cos\Theta = \frac{1}{\sqrt{2}}$

= tanΘ=sinΘcosΘ=1√21√2=1 $tan\Theta = \frac{sin\Theta}{cos\Theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

= cosecΘ=1sinΘ=11√2=√2 $cosec\Theta = \frac{1}{sin\Theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

= secΘ=1cosΘ=11√2=√2 $sec\Theta = \frac{1}{cos\Theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

= cotΘ=1tanΘ=11=1 $cot\Theta = \frac{1}{tan\Theta} = \frac{1}{1} = 1$

Q3) If tanΘ=1√2 $tan\Theta = \frac{1}{\sqrt{2}}$, find the value of cosec²Θ−sec²Θcosec²Θ+cot²Θ

$\frac{cosec^2\Theta - sec^2\Theta}{cosec^2\Theta + cot^2\Theta}$.

Solution:

We know that secΘ=√1+tan²Θ $sec\Theta = \sqrt{1 + tan^2\Theta}$

=√1+(1√2)² $\sqrt{1+(\frac{1}{\sqrt{2}})^2}$

= √1+12=√32 $\sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$

= cotΘ=1tanΘ=11√2=√2 $cot\Theta = \frac{1}{tan\Theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

= cosecΘ=√1+cot²Θ=√1+2=√3 $cosec\Theta = \sqrt{1+cot^2\Theta} = \sqrt{1+2} = \sqrt{3}$

Substituting it in equation (1) we get

= (√3)²−(√32)²(√3)²+(√2)²=3−323+2=325=310 $\frac{(\sqrt{3})^2-(\sqrt{\frac{3}{2}})^2}{(\sqrt{3})^2+(\sqrt{2})^2} = \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5} = \frac{3}{10}$

Q4) If tanΘ=34 $tan\Theta = \frac{3}{4}$, find the value of 1−cosΘ1+cosΘ $\frac{1-cos\Theta}{1+cos\Theta}$

Solution:

We know that

secΘ=√1+tan²Θ $sec\Theta = \sqrt{1+tan^2\Theta}$

= √1+(34)² $\sqrt{1+(\frac{3}{4})^2}$

= √1+916 $\sqrt{1+\frac{9}{16}}$

=√16+916 $\sqrt{\frac{16+9}{16}}$

= √2516 $\sqrt{\frac{25}{16}}$

= secΘ=54 $sec\Theta = \frac{5}{4}$

= secΘ=1cosΘ=154=45=cosΘ $sec\Theta = \frac{1}{cos\Theta} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = cos\Theta$

Therefore, Weget 1−451+45=1595=19 $We\ get\ \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}$

Q5) If tanΘ=125 $tan\Theta = \frac{12}{5}$, find the value of 1+sinΘ1−sinΘ $\frac{1+sin\Theta}{1-sin\Theta}$.

Solution:

cotΘ= 1tanΘ = 1 125 = 512 $cot\Theta = \frac{1}{tan\Theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$

= cosecΘ=√1+cot²Θ=√1+[512]²=√144+25144 =√169144 = 1312

$cosec\Theta = \sqrt{1+cot^2\Theta} = \sqrt{1+[\frac{5}{12}]^2} = \sqrt{\frac{144+25}{144}} = \sqrt{\frac{169}{144}} = \frac{13}{12}$

= sinΘ= 1cosecΘ = 1 1312 = 1213 $sin\Theta = \frac{1}{cosec\Theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$

i.e. Weget 1+1213 1−1213 = 13+1218 13−1218 = 251 =25 $We\ get\ \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}} = \frac{25}{1} = 25.$

Q6) If cotΘ=1√3 $cot\Theta = \frac{1}{\sqrt{3}}$, find the value of 1−cos²Θ2−sin²Θ $\frac{1-cos^2\Theta}{2-sin^2\Theta}$.

Solution:

cosecΘ=√1+cot²Θ=√1+ 13 =√43 $cosec\Theta = \sqrt{1+cot^2\Theta} = \sqrt{1+\frac{1}{3}} = \sqrt{\frac{4}{3}}$

= cosecΘ= 2√3 $cosec\Theta = \frac{2}{\sqrt{3}}$

= sinΘ= 1cosecΘ = 1 2√3 = √32 $sin\Theta = \frac{1}{cosec\Theta} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$

= and 1cotΘ = sinΘcosΘ =cosΘ=sinΘ×cotΘ= √32 × 1√3 = 12

$and\ \frac{1}{cot\Theta} = \frac{sin\Theta}{cos\Theta} = cos\Theta = sin\Theta \times cot\Theta = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$

Therefore, on substituting we get

= 1−(12)²2−(√32)² = 1−142−34 = 34 54 = 35 $\frac{1-(\frac{1}{2})^2}{2-(\frac{\sqrt{3}}{2})^2} = \frac{1-\frac{1}{4}}{2-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}.$

–

Q7) If cosecA=√2$cosecA = \sqrt{2}$, find the value of 2sin²A+3cot²A4(tan²A−cos²A)

$\frac{2sin^2A+3cot^2A}{4(tan^2A-cos^2A)}$.

Solution:

We know that cotA=√cosec²A−1$cotA = \sqrt{cosec^2A - 1}$

= √(2)²−1=√2−1$\sqrt{(2)^2 - 1} = \sqrt{2-1}$ =1.

= tanA=1cotA=11=1$tanA = \frac{1}{cotA} = \frac{1}{1} = 1$

= sinA=1cosecA=1√2$sinA = \frac{1}{cosecA} = \frac{1}{\sqrt{2}}$

= sinA=1√2$sinA = \frac{1}{\sqrt{2}}$

cosA=√1−sin²A=√1−(1√2)²=√1√2=1√2

$$cosA = \sqrt{1 - sin^2A} = \sqrt{1 - (\frac{1}{\sqrt{2}})^2} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

On substituting we get:

= 2[1√2]²+3[1]²4[[1]−[1√2]²]=2×12+34[1−12] $\frac{2[\frac{1}{\sqrt{2}}]^2+3[1]^2}{4[[1]-[\frac{1}{\sqrt{2}}]^2]} = \frac{2\times\frac{1}{2}+3}{4[1-\frac{1}{2}]}$

⇒1+34.12=42=2⇒ $\frac{1+3}{4.\frac{1}{2}} = \frac{4}{2} = 2$

Q8) If cotΘ=√3$cot\Theta = \sqrt{3}$, find the value of cosec²Θ+cot²Θcosec²Θ−sec²Θ

$\frac{cosec^2\Theta+cot^2\Theta}{cosec^2\Theta-sec^2\Theta}$.

Solution:

cosecΘ=√1+cot²Θ=√1+(√3)²=√1+3=2

$$cosec\Theta = \sqrt{1 + cot^2\Theta} = \sqrt{1 + (\sqrt{3})^2} = \sqrt{1+3} = 2$$

sinΘ= 1cosecΘ = 12 cotΘ= cosΘsinΘ cosΘ=cotΘ.sinΘ

$sin\Theta = \frac{1}{cosec\Theta} = \frac{1}{2} cot\Theta = \frac{cos\Theta}{sin\Theta}$ $\cos\Theta = cot\Theta.\, sin\Theta$ ⇒cosΘ=√32

$\Rightarrow cos\Theta = \frac{\sqrt{3}}{2}$

= secΘ= 1cosΘ = 2√3 $sec\Theta = \frac{1}{cos\Theta} = \frac{2}{\sqrt{3}}$

On substituting we get:

(2)2+(√3)2(2)2−(2√3)2 = 4+3 12−43 = 7 83 $\frac{(2)^2+(\sqrt{3})^2}{(2)^2-(\frac{2}{\sqrt{3}})^2} = \frac{4+3}{\frac{12-4}{3}} = \frac{7}{\frac{8}{3}}$

= 218 $\frac{21}{8}$

Q9) If 3cosΘ=1$3cos\Theta = 1$, find the value of 6sin²Θ+tan²Θ4cosΘ $\frac{6sin^2\Theta+tan^2\Theta}{4cos\Theta}$.

Solution:

cosΘ= 13 ,sinΘ=√1−cos²Θ$cos\Theta = \frac{1}{3},$ $sin\Theta = \sqrt{1 - cos^2\Theta}$

= √1− 19 =√89 = 2√23 $\sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

tanΘ= sinΘcosΘ = 2√23. 13 =2√2$tan\Theta = \frac{sin\Theta}{cos\Theta} = \frac{2\sqrt{2}}{3.\frac{1}{3}} = 2\sqrt{2}$

On substituting we get

6[2√23]2+(2√2)24. 13 = 163+843 = 16+243 43 $\frac{6[\frac{2\sqrt{2}}{3}]^2+(2\sqrt{2})^2}{4.\frac{1}{3}} = \frac{\frac{16}{3}+8}{\frac{4}{3}} = \frac{\frac{16+24}{3}}{\frac{4}{3}}$

= 404 =10$\frac{40}{4} = 10$

Q10) If √3tanΘ=sinΘ$\sqrt{3}tan\Theta = sin\Theta$, find the value of sin²Θ−cos²Θ $sin^2\Theta - cos^2\Theta$.

Solution:

—

√3 sinΘcosΘ =sinΘ$\sqrt{3}\frac{sin\Theta}{cos\Theta} = sin\Theta$

= cosΘ=√33⇒1√3$cos\Theta = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\sqrt{3}}$

= sinΘ=√1−cos²Θ=√1−(1√3)²$sin\Theta = \sqrt{1 - cos^2\Theta} = \sqrt{1 - (\frac{1}{\sqrt{3}})^2}$

= sin²Θ−cos²Θ=(√23)²−(1√3)²$sin^2\Theta - cos^2\Theta = (\sqrt{\frac{2}{3}})^2 - (\frac{1}{\sqrt{3}})^2$

= 23−13=13$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

Q11) If cosecΘ=1312$cosec\Theta = \frac{13}{12}$, find the value of 2sinΘ−3cosΘ4sinΘ−9cosΘ
$\frac{2sin\Theta - 3cos\Theta}{4sin\Theta - 9cos\Theta}$.

Solution:

sinΘ=1cosecΘ=11312=1213$sin\Theta = \frac{1}{cosec\Theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$

= cosΘ=√1−sin²Θ=√1−[1213]²=√1−144169
$cos\Theta = \sqrt{1 - sin^2\Theta} = \sqrt{1 - [\frac{12}{13}]^2} = \sqrt{1 - \frac{144}{169}}$

= √25169=513$\sqrt{\frac{25}{169}} = \frac{5}{13}$

⇒2.1213−3.5134.1213−9.513=24−151348−1513=93=3$\Rightarrow \frac{2.\frac{12}{13} - 3.\frac{5}{13}}{4.\frac{12}{13} - 9.\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-15}{13}} = \frac{9}{3} = 3$

Q12) If sinΘ+cosΘ=√2cos(90°−Θ)$sin\Theta + cos\Theta = \sqrt{2}cos(90° - \Theta)$, find cotΘ
$cot\Theta$.

Solution:

= sinΘ+cosΘ=√2sinΘ[cos(90−Θ)=sinΘ]
$sin\Theta + cos\Theta = \sqrt{2}sin\Theta \quad [cos(90 - \Theta) = sin\Theta]$

$\Rightarrow cos\Theta = \sqrt{2}sin\Theta - sin\Theta$

⇒cosΘ=√2sinΘ−sinΘ⇒cosΘ=sinΘ(√2−1)$\Rightarrow cos\Theta = sin\Theta(\sqrt{2} - 1)$

Divide both sides with sinΘ$sin\Theta$ we get

= cosΘsinΘ = sinΘsinΘ (√2−1)$\frac{cos\Theta}{sin\Theta} = \frac{sin\Theta}{sin\Theta}(\sqrt{2} - 1)$

= cotΘ=√2−1$cot\Theta = \sqrt{2} - 1.$

Q-13. If 2sin^{2}Θ–cos^{2}Θ=2$2sin^2\Theta - cos^2\Theta = 2$, then find the value of ΘΘ.

Solution.

2sin^{2}Θ–cos^{2}Θ=2$2sin^2\Theta - cos^2\Theta = 2$

⇒2sin^{2}Θ−(1−sin^{2}Θ)=2$\Rightarrow 2sin^2\Theta - \left(1 - sin^2\Theta\right) = 2$ ⇒2sin^{2}Θ–1+sin^{2}Θ=2 $\Rightarrow 2sin^2\Theta - 1 + sin^2\Theta = 2$ ⇒3sin^{2}Θ=3$\Rightarrow 3sin^2\Theta = 3$ ⇒sin^{2}Θ=1$\Rightarrow sin^2\Theta = 1$ ⇒sinΘ=1$\Rightarrow sin\Theta = 1$ ⇒sinΘ=sin90°$\Rightarrow sin\Theta = sin90^\circ$ ⇒Θ=90°$\Rightarrow \Theta = 90^\circ$

Q-14. If √3tanΘ–1=0$\sqrt{3}tan\Theta - 1 = 0$, find the value of sin^{2}Θ–cos^{2}Θ $sin^2\Theta - cos^2\Theta$.

Solution.

√3tanΘ–1=0$\sqrt{3}tan\Theta - 1 = 0$ ⇒√3tanΘ=1$\Rightarrow \sqrt{3}tan\Theta = 1$ ⇒√3tanΘ=1√3

$\Rightarrow \sqrt{3}tan\Theta = \frac{1}{\sqrt{3}}$ √3tanΘ=tan30°$\sqrt{3}tan\Theta = tan30^\circ$ Θ=30°$\Theta = 30^\circ$

Now,

sin^{2}Θ–cos^{2}Θ$sin^2\Theta - cos^2\Theta$

= sin^2(30°)–cos^2(30°)$sin^2(30^\circ) - cos^2(30^\circ)$

= (12)^2–(√32)^2 $\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

= 14−34 $\frac{1}{4} - \frac{3}{4}$ = -24 $\frac{-2}{4}$ = -12 $\frac{-1}{2}$