Exercise 15.1: Areas Related to Circles

Q.1: Find the circumference of a circle with a radius of 4.2 cm.

Soln:

Given Data:

Radius = 4.2 cm

The formula to be used:

Circumference of a circle = 2

= 2 x 3.14 x 4.2 = 26.4 cm

Therefore, the circumference of the circle is 26.4 cm

Q.2: Find the circumference of a circle with area 301.84 cm^2 .

Soln:

Given Data: Area = 301.84 cm^2

We know that, Area of a Circle =

301.84 cm^2 = 3.14 x

r = 9.8 cm

Therefore, Radius, = 9.8 cm.

Circumference of a circle = 2

= 2 x 3.14 x 9.8 cm = 61.6 cm

Therefore, the circumference is 61.6 cm.

Q.3: Find the area of a circle whose circumference is 44 cm.

Soln:

Given Data:

Circumference = 44 cm

We know that, Circumference of a circle = 2

44 cm = 2 x 3.14 x r

r = 7 cm

Formula to be used:

Area of a Circle =

= 3.14 x 7 x 7

= 154 cm^2

Therefore, Radius, = 7 cm

Area of a Circle = 154 cm^2

Q.4: The circumference of a circle exceeds its diameter by 16.8 cm. Find the circumference of the circle.

Soln:

Let the radius of the circle be = r cm

Therefore, Diameter (d) = 2r [radius is half the diameter]

We know that, Circumference of a circle (C) = 2

Given Data : circumference of a circle exceeds its diameter by 16.8 cm

C = d + 16.8

2= 2r + 16.8 [d = 2r]

2- 2r = 16.8

2r (– 1) = 16.8

2r (3.14 – 1) = 16.8

r = 3.92 cm

Therefore, Radius, = 3.92 cm

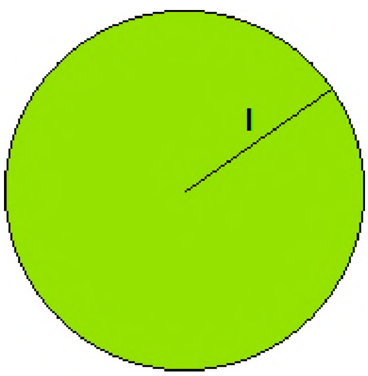
Circumference of a circle (C) = 2

C = 2 x 3.14 x 3.92

= 24.64 cm

Therefore, the circumference of the circle is 24.64 cm.

Q.5: A horse is tied to a pole with 28 m long string. Find the area where the horse can graze.



Soln:

Given Data: Length of the string l = 28 m

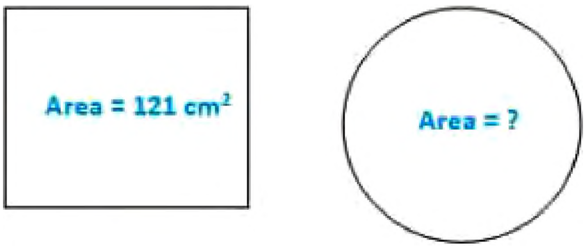
Area the horse can graze is the area of the circle in the figure shown with a radius equal to the length of the string.

We know that, Area of a Circle =   = 3.14 x 28 x 28 = 2464 m^2

Therefore, the Area of the circle and the area the horse can graze is 2464 m^2

Q6. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle. Find the area of the circle.

Soln:



Given Data : Area of the square = 121 cm^2

Area of the circle =?

We know that, area of a square = a^2

121 cm^2 = a^2

a = 11 cm   [$11^2 = 121$]

Therefore, each side of the square ‘a’ = 11 cm

We know that, the perimeter of a square = 4a = 4 x 11 = 44 cm

Perimeter of the square = Circumference of the circle   [in this case only]

We know that, Circumference of a circle (C) = 2

4a = 2

2 = 44

r = 7 cm

We know that, Area of a Circle = = 3.14 x 7 x 7 = 154 cm^2

Therefore, the Area of the circle is 154 cm^2

Q7. The circumference of two circles is in a ratio of 2 : 3. Find the ratio of their areas.

Soln:

Let the radius of two circles C1 and C2 be r1 and r2.

We know that, Circumference of a circle (C) = 2

Hence their circumference will be 2_1 and 2_2 .

Also, their circumference will be in a ratio of = 2_1 : 2_2

Given Data: circumference of two circles is in a ratio of 2 : 3

Therefore, 2_1 : 2_2 = 2 : 3

Also, the ratios of their areas = Πr_{21}:$\Pi r_{22}$$\pi r_1^2 : \pi r_2^2$

= (r1r2)^2$(\frac{r1}{r2})^2$

= (23)^2$(\frac{2}{3})^2$

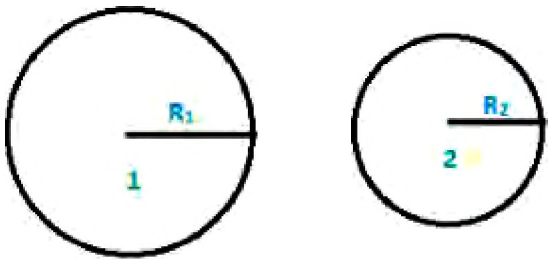
= 916 $\frac{9}{16}$

= 4 : 9

Therefore, ratio of their areas = 4 : 9 .

Q8. The sum of the radii of two circles is 140 cm and the difference of their circumference is 88 cm. Find the diameters of the circles.

Soln:



Let the radius of the circles be r1 and r2.

Let the circumferences of the two circles be C1 and C2.

Given Data:

We know that, Circumference of a circle (C) = 2

Sum of radii of two circles;   r1 + r2 = 140 cm   —- **(1)**

Difference of their circumference,

C1 – C2 = 88 cm

$2_1 - 2_2 = 88$ cm

$2\ (r_1 - r_2) = 88$ cm

$(r1 - r_{2)} = 14$ cm

$r_1 = r_2 + 14$ —- **(2)**

Substituting the value of r_1 in equation **(1),** we have,

$r_2 + r_2 + 14 = 140$

$2\ r_2 = 140 - 14$

$2\ r_2 = 126$

$r_2 = 63$ cm

Substituting the value of r_2 in equation **(2),** we have,

$r_1 = 63 + 14\ \ = 77$ cm

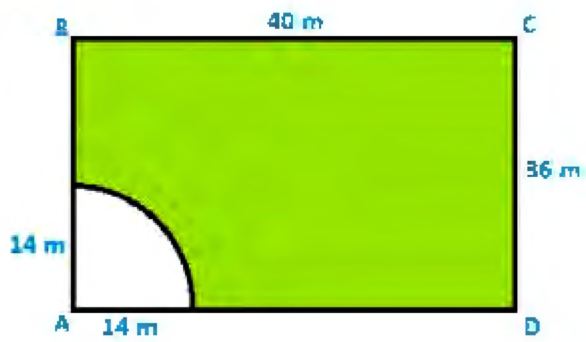
Diameter of circle 1 = $2r_1$ = 2 x 77 = 154 cm

Diameter of circle 2 = 2 r_2 = 2 x 63 = 126 cm

Therefore, Diameter 1 and diameter 2 are 154 cm and 126 cm

Q9. A horse is placed for grazing inside a rectangular field 40 m by 36 m and is tethered to one corner by a rope 14 m long. Over how much area can it graze? (extra question)

Soln:



The figure shows rectangular field ABCD at corner A, a horse is tied with rope length = 14 m. the area it can graze is represented A as shaded region = area of quadrant with (radius = length) of string

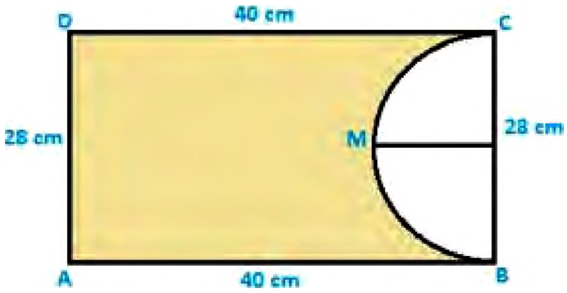
area=14×(areaofcircle)=πr^2=14×227×14×14=(22×7)=154m^2

$$area = \frac{1}{4} \times (areaofcircle) = \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = (22 \times 7) = 154 m^2$$

Therefore, the area it can graze = 154 m^2

Q10. A sheet of paper is in the form of a rectangle ABCD in which AB = 40 cm and AD = 28 cm. A semicircular portion with BC as diameter is cut off. Find the area of the remaining paper.

Soln:



Given Data:

Sheet of paper ABCD, AB = 40 cm and AD = 28 cm

CD = 40 cm and BC = 28 cm [since it is a rectangle]

Semicircle is represented as BMC with BC as the diameter.

Therefore, The radius = 14 cm [diameter is double the radius]

We know that, Area of a Circle =

Area of the remaining (shaded region) = (area of rectangle) – (area of semicircle)

= (AB x BC) – (0.5 x)

= (40 x 28) – (0.5 x 3.14 x 14 x 14)

= 1120 – 308 = 812cm^2

Therefore, the Area of the circle is 812 cm^2

Q.10: The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of two circles.

Soln:

Given Data :

Radii of circles are 6 cm and 8 cm

Area of circle with radius 8 cm = = $(8)^2 = 64$ cm^2

Area of circle with radius 6cm = = $(6)^2 = 36$ cm^2

Areas sum = 64 + 36 = 100 cm^2

Let the radius of circle be x cm

Area = = = 100 cm^2

X^2 = 100

x = $\sqrt{100}\sqrt{100}$= 10 cm

Hence, x = 10 cm

Q11. The radii of two circles are 19 cm and 9 cm respectively. Find the radius and area of the circle which has circumferences is equal to sum of the circumference of two circles.

Soln:

Given Data :

Radius of the 1st circle = 19 cm

Radius of the 2nd circle = 9 cm

Formula used :

Circumference of 1st circle = 2 (19) = 38 cm

Circumference of 2^{nd} circle = 2 (9) = 18 cm

Let radius of required circle be r cm

Circumference of required circle = 2 = C1 + C2

2 = 38 cm + 18 cm

2 = 56 cm

Radius, r = 28 cm

Area of required circle = = 3.14 x 28 x 28 = 2464 cm^2

Hence, the area of required circle = = 2464 cm^2

Q12. The side of a square field is 10 cm. Find the area of the circumscribed and inscribed circles.

Soln:

Circumscribed circle :

Radius = diagonalofsquare2 $\frac{diagonalofsquare}{2}$

= 12∗√2 $\frac{1}{2} * \sqrt{2}$

= 0.5 x 1.414 x 10 = 7.07 cm

Therefore, Radius of the circle, = 7.07 cm

We know that, Area of a Circle =

= 3.14 x 7.07 x 7.07 = 157.41 cm^2

Therefore, the Area of the Circumscribed circle is 157.41 cm^2

Inscribed circle :

Radius = frac12∗side $frac12 * side$

= 12∗side $\frac{1}{2} * side$

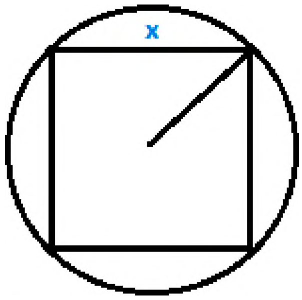
= 0.5 x 10 = 5 m

= 3.14 x 5 x 5 = 78.5 cm^2

Therefore, area of the circle is 78.5 cm^2

Q 13. If a square is inscribed in a circle. Find the ratio of areas of the circle and the square.

Soln:



Let side of square be x cm inscribed in a circle.

Given Data :

Radius of circle (r) = 12 $\frac{1}{2}$ (diagonal of square)

= 12(sqrt2x) $\frac{1}{2}(sqrt2x)$

= x2 $\frac{x}{2}$

We know that, area of a square = x^2

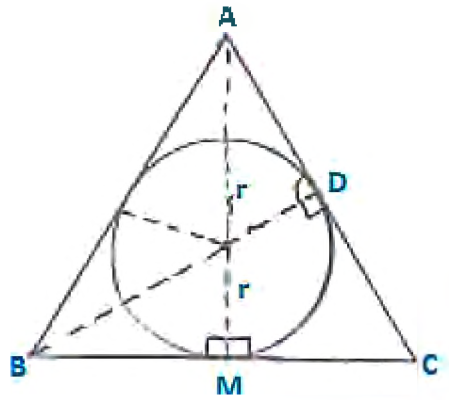
= π(x√2)^2=π∗x²2 $\pi(\frac{x}{\sqrt{2}})^2 = \frac{\pi * x^2}{2}$

areaofcircleareaofsquare = π2 =π:2 $\frac{areaofcircle}{areaofsquare} = \frac{\pi}{2} = \pi : 2$

Hence, obtained.

Q14. The area of circle, inscribed in equilateral triangle is 154 cm^2. Find the perimeter of triangle.

Soln:



Let the circle inscribed in the equilateral triangle be with a centre O and radius r.

Formula used :

We know that, Area of a Circle =but the given that area is 154 cm^2.

= 154

3.14 x = 154

= 7 x 7 = 49

r = 7 cm

From the figure shown above, we infer that at point M, BC side is tangent and also at point M BM is perpendicular to OM. In equilateral triangle, the perpendicular from vertex divides the side into two halves.

BM=12BC$BM = \frac{1}{2}BC$=12x$\frac{1}{2}x$ =x2 $\frac{x}{2}$

OB^2=BM^2+$MO^2$$OB^2 = BM^2 + MO^2$ OB=√r2+x24=√49x24

$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49\frac{x^2}{4}}$

BD=√32(side)=√32x=OB+OD$\frac{\sqrt{3}}{2}(side) = \frac{\sqrt{3}}{2}x = OB + OD$

√32x−r=√49+x24,r=7$\frac{\sqrt{3}}{2}x - r = \sqrt{49 + \frac{x^2}{4}}, r = 7$

After solving the above equations we have,

X = 14√3cm$14\sqrt{3}cm$

perimeter=3x=3∗14√3=42√3cm$perimeter = 3x = 3 * 14\sqrt{3} = 42\sqrt{3}cm$

Hence the perimeter to be found is 42√3cm$42\sqrt{3}cm$.

Q15. A field is in the form of the circle. A fence is to be erected around the field. The cost of fencing would to Rs. 2640 at rate of Rs.12 per meter. Then the field is to be thoroughly plowed at cost of Rs. 0.50 per m^2. What is the amount required to plow the field?

Soln :

Given Data: Total cost of fencing the circuit field = Rs. 2640

Cost per meter fencing = Rs 12

Total cost of fencing = circumference x cost per fencing

2640 = circumference x 12

Therefore, Circumference = 220m

Let radius of field be 'r' m

Circumference of a circle (C) = 2

2

2 x 3.14 x r = 220

r = 35 m

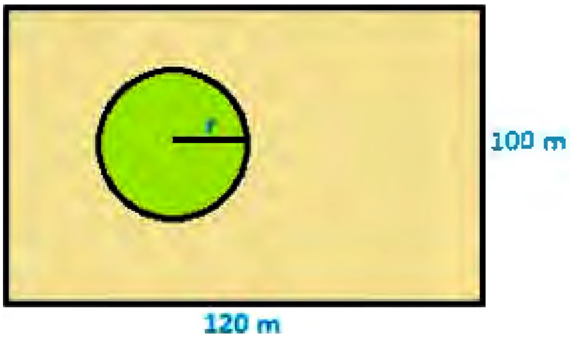
= 3.14 x 35 x 35 = 3850 m^2

Cost of ploughing per m^2 land = Rs. 0.5 x = 0.50 x 3850 m^2 = Rs. 1925

Therefore, Cost of ploughing per m^2 land = Rs. 1925

Q 16. A park is in the form of rectangle 120 m x 100 m. At the center of park, there is a circular lawn. The area of park excluding lawn is 8700 m^2. Find the radius of the circular lawn.

Soln:



Given Data :

Dimensions of rectangular park

length = 120 m

Breadth = 100 m

Area of park = l x b = 120 x 100 = 12000 m^2

Let radius of circular lawn be r

Formula used :

Area of circular lawn =

Area of remaining park excluding lawn = (area of park) x (area of circular lawn)

8700 = 12000 – = 12000 – 8700 = 3300 m^2

r = 32.4 m

Therefore, radius of circular lawn = 32.4 m

Q18. A truck travels 1 km distance in which each wheel makes 450 complete revolutions. Find the radius of the wheel.

Soln:

Given data:

Distance travelled = 1000 m

Number of revolutions made = n = 450

Formula used:

We know that, Circumference of a circle (C) = 2   = 2 x 3.14 x r

Distance for 450 revolutions = (2 x 3.14 x r) 450

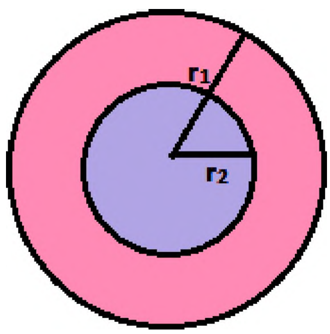
But we have been given with a distance of 1000 m, hence,

(2 x 3.14 x r) 450 = 1000

Radius, r = 10009π $\frac{1000}{9\pi}$ cm

Q19. The area enclosed between the concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm , then find the radius of the inner circle.

Soln:



Given Data :

Radius of outer circle = R1 = 21 cm

Radius of inner circle = R2

Area between concentric circles = area of outer circle – area of inner circle

Formula used :

We know that, Area of a Circle =

770 cm^2 = ($R1^2$ – $R2^2$)

$R1^2$ – $R2^2$ = 245

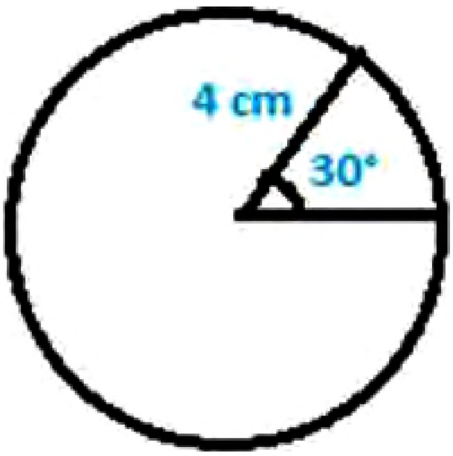
21^2 – $R2^2$ = 245

R2 = 14 cm

Therefore, the radius of the inner circle is 14 cm.

Exercise 15.2: Areas Related to Circles

Q1. Find in terms of π, the length of the arc that subtends an angle of 30 degrees, at the center of 'O' of the circle with a radius of 4 cm.



Soln:

Given Data :

Radius = 4 cm

Angle subtended at the centre ‘O’ = 30°

Formula to be used :

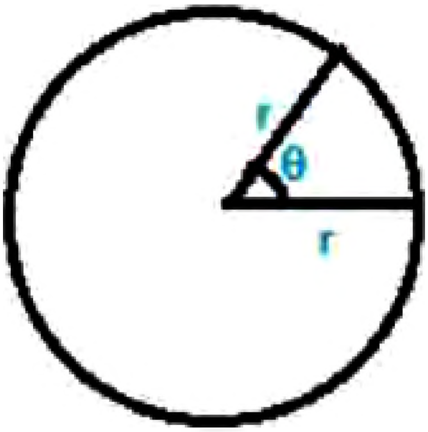
Lengthofarc=θ360∗2πrcm$Length of arc = \frac{\theta}{360} * 2\pi r cm$

Lengthofarc=30360∗2π∗4cm$Length of arc = \frac{30}{360} * 2\pi * 4cm$

= 2π3 cm$\frac{2\pi}{3} cm$

Therefore, the Length of arc the length of the arc that subtends an angle of 60 degrees is 2π3 cm$\frac{2\pi}{3} cm$

Q2. Find the angle subtended at the centre of circle of radius 5 cm by an arc of length 5π3 cm$\frac{5\pi}{3} cm$.



Soln:

Given data:

Radius = 5 cm

Length of arc = 5π3 cm$\frac{5\pi}{3} cm$

Formula to be used:

Length of arc = θ360∗2πrcm$\frac{\theta}{360} * 2\pi r cm$

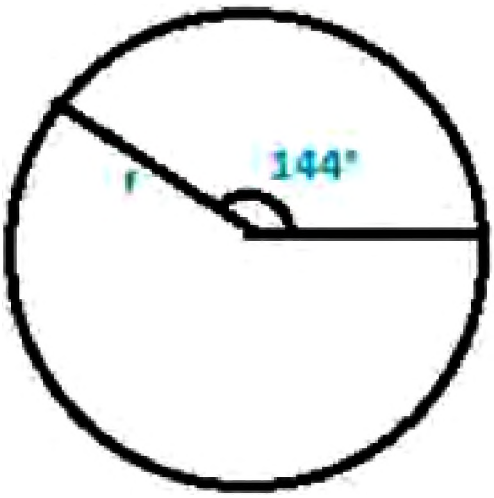
5π3 cm $\frac{5\pi}{3} cm$ = θ360 ∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

Solving the above equation, we have:

θ = 60°

Therefore, angle subtended at the centre of circle is60°

Q3. An arc of length cm subtends an angle of 144° at the center of the circle.



Soln:

Given Data : length of arc = cm

θ = angle subtended at the centre of circle = 144°

Formula to be used :

Length of arc = θ360 ∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

θ360 ∗2πrcm $\frac{\theta}{360} * 2\pi rcm$ = 144360 ast2πrcm $\frac{144}{360} ast2\pi rcm$

= 4π5 ∗rcm $\frac{4\pi}{5} * rcm$

As given in the question, length of arc = cm ,

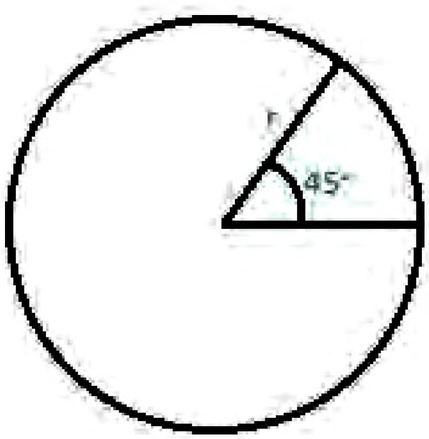
Therefore, cm = 4π5 ∗rcm $\frac{4\pi}{5} * rcm$

Solving the above equation, we have

r = 25 cm.

Therefore the radius of the circle is found to be 25 cm.

Q4. An arc of length 25 cm subtends an angle of 55° at the center of a circle. Find in terms of radius of the circle.



Soln:

Given Data :

length of arc =25 cm

θ = angle subtended at the centre of circle = 55°

Formula to be used :

Length of arc = θ360 ∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

=5536 ∗2πrcm $\frac{55}{36} * 2\pi rcm$

As given in the question length of arc =25 cm ,hence, 25 cm = 55360 *2π*rcm $\frac{55}{360} * 2\pi * rcm$

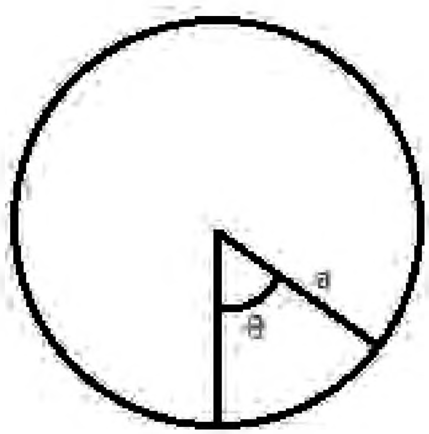
25 = 11πr36 $\frac{11\pi r}{36}$

radius= 25*3611*π $radius = \frac{25*36}{11*\pi}$

= 90011π $\frac{900}{11\pi}$

Therefore, the radius of the circle is 90011π $\frac{900}{11\pi}$

Q5. Find the angle subtended at the center of the circle of radius ‘a’ cm by an arc of fraca pi4 $fraca\ pi4$ **length cm .**



Soln:

Given data :

Radius = a cm

Length of arc = aπ4 $\frac{a\pi}{4}$ cm

θ = angle subtended at the centre of circle

Formula to be used:

Length of arc = θ360∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

Length of arc = θ360∗2πacm $\frac{\theta}{360} * 2\pi acm$

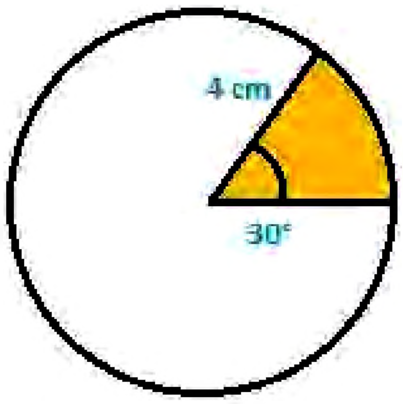
θ360∗2πacm $\frac{\theta}{360} * 2\pi acm$ = aπ4 $\frac{a\pi}{4}$ cm

Solving the above equation, we have

θ = 45°

Therefore, the angle subtended at the centre of circle is 45°

Q6. A sector of the circle of radius 4 cm subtends an angle of 30°. Find the area of the sector.



Soln:

Given Data:

Radius = 4 cm

Angle subtended at the centre ‘O’ = 30°

Formula to be used :

Areaofthesector=Θ360∗πr2 $Area of the sector = \frac{\Theta}{360} * \pi r^2$

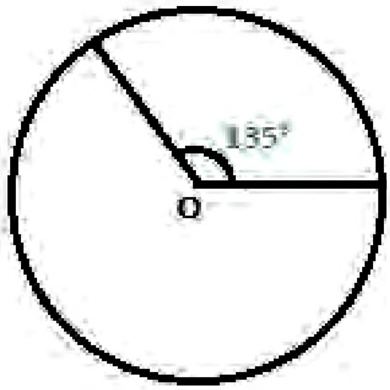
Areaofthesector=30360∗π42 $Area of the sector = \frac{30}{360} * \pi 4^2$

Solving the above equation, we have:

Area of the sector = 4.9 cm^2

Therefore, Area of the sector is found to be 4.9 cm^2

Q7. A sector of a circle of radius 8 cm subtends an angle of 135°. Find the area of sector.



Soln:

Given Data:

Radius = 8 cm

Angle subtended at the centre 'O' = 135°

Formula to be used:

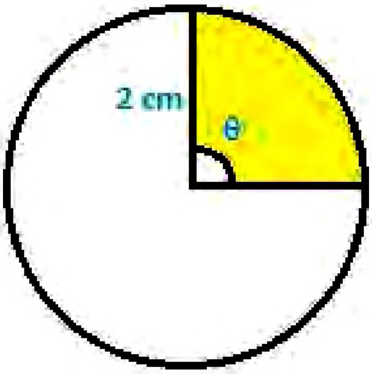
Areaofthesector=Θ360 ∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=135360 ∗π8^2 $Areaofthesector = \frac{135}{360} * \pi 8^2$

=5287 $\frac{528}{7}$ cm^2

Therefore, Area of the sector calculated is 5287 $\frac{528}{7}$ cm^2

Q8. The area of sector of circle of radius 2 cm is cm^2. Find the angle subtended by the sector.



Soln:

Given Data:

Radius = 2 cm

Angle subtended at the centre 'O' =?

Area of sector of circle = cm^2

Formula to be used:

Areaofthesector=Θ360∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=Θ360∗π3^2 $Areaofthesector = \frac{\Theta}{360} * \pi 3^2$

= πΘ90 $\frac{\pi\Theta}{90}$

As given in the question area of sector of circle = cm^2

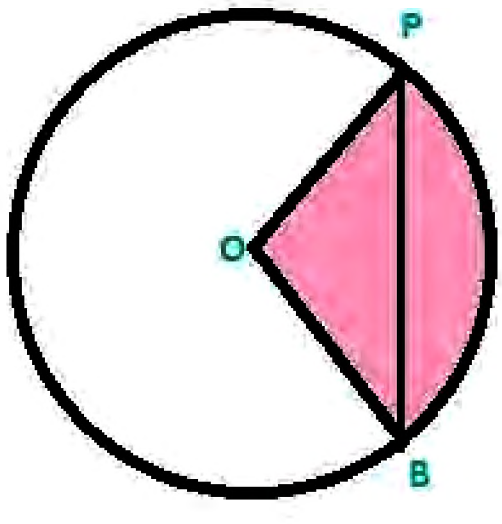
cm^2 = πΘ90 $\frac{\pi\Theta}{90}$

Solving the above equation, we have

θ = 90°

Therefore, the angle subtended at the centre of circle is 90°

Q10. PQ is a chord of circle with centre ‘O’ and radius 4 cm. PQ is of the length 4 cm. Find the area of sector of the circle formed by chord PQ.



Soln:

Given Data: PQ is chord of length 4 cm.

Also, PO = QO= 4 cm

OPQ is an equilateral triangle.

Angle POQ = 60°

Area of sector (formed by the chord (shaded region)) = (area of sector)

Formula to be used:

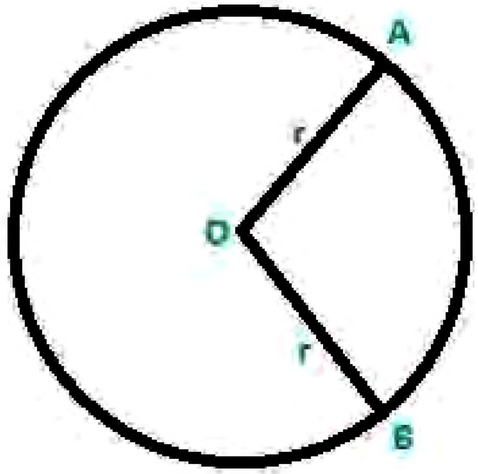
Areaofthesector=Θ360 ∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=60360 ∗π4^2 $Areaofthesector = \frac{60}{360} * \pi 4^2$

= 32π3 $\frac{32\pi}{3}$

Therefore, Area of the sector is 32π3 $\frac{32\pi}{3}$ cm^2

Q11. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of arc and area of sector.



Soln:

Given Data:

Radius = 35 cm

Angle subtended at the centre ‘O’ = 72°

Area of sector of circle =?

Formula to be used:

Lengthofarc=θ360∗2πrcm $Lengthofarc = \frac{\theta}{360} * 2\pi rcm$

Lengthofarc=108360∗2πx42cm $Lengthofarc = \frac{108}{360} * 2\pi x42cm$

Solving the above equation we have,

Length of arc = 44 cm

We know that,

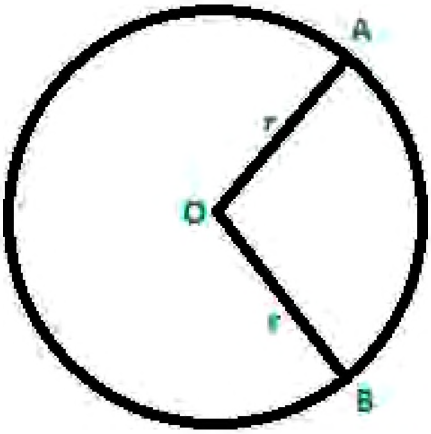
Areaofthesector=Θ360∗πr2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=72360∗π352 $Areaofthesector = \frac{72}{360} * \pi 35^2$

Solving the above equation, we have, Area of the sector = (35 x 22) cm^2

Therefore, Area of the sector is 770 cm^2

Q12. The perimeter of a sector of a circle of radius 5.7 m is 27.2m. find the area of the sector.



Soln:

Given Data:

Radius = 5.7 cm = OA = OB [from the figure shown above]

Perimeter = 27.2 m

Let the angle subtended at the centre be θ

Perimeter = θ360∗2πrcm $\frac{\theta}{360} * 2\pi rcm$ + OA + OB

= θ360∗2πx5.7cm $\frac{\theta}{360} * 2\pi x5.7cm$ + 5.7 + 5.7

Solving the above equation we have,

θ = 158.8°

We know that,

Areaofthesector=θ360∗πr² $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

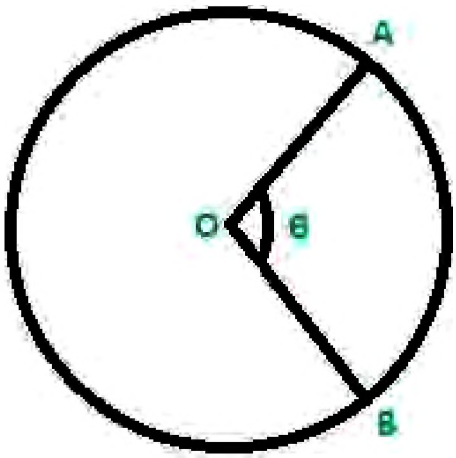
Area of the sector = 158.8360∗π5.7² $\frac{158.8}{360} * \pi 5.7^2$

Solving the above equation we have,

Area of the sector = 45.048 cm^2

Therefore, Area of the sector is 45.048 cm^2

Q13. The perimeter of a certain sector of a circle of radius is 5.6 m and 27.2 m. find the area of a sector.



Soln:

Given data:

Radius of the circle = 5.6 m = OA = OB

(AB arc length) + OA + OB = 27.2

Let the angle subtended at the centre be θ

We know that,

Length of arc = θ360∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

θ360∗2πrcm $\frac{\theta}{360} * 2\pi rcm$ + OA + OB = 27.2 m

θ360∗2πrcm $\frac{\theta}{360} * 2\pi rcm$ + 5.6 + 5.6 = 27.2 m

Solving the above equation, we have,

θ = 163.64°

We know that,

Areaofthesector=Θ360∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

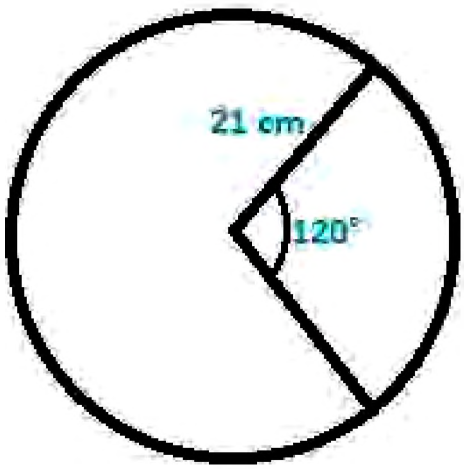
Areaofthesector=163.64360∗π5.6^2 $Areaofthesector = \frac{163.64}{360} * \pi 5.6^2$

On solving the above equation, we have,

Area of the sector = 44.8 cm^2

Therefore, Area of the sector is 44.8 cm^2

Q14. A sector was cut from a circle of radius 21 cm. The angle of sector is 120°. Find the length of its arc and its area.



Soln:

Given data:

Radius of circle (r) = 21 cm

θ = angle subtended at the centre of circle = 120°

Formula to be used:

Lengthofarc=θ360∗2πrcm $Lengthofarc = \frac{\theta}{360} * 2\pi rcm$

Lengthofarc=120360∗2πx21cm $Lengthofarc = \frac{120}{360} * 2\pi x21cm$

On solving the above equation, we get,

Length of arc = 44 cm

We know that,

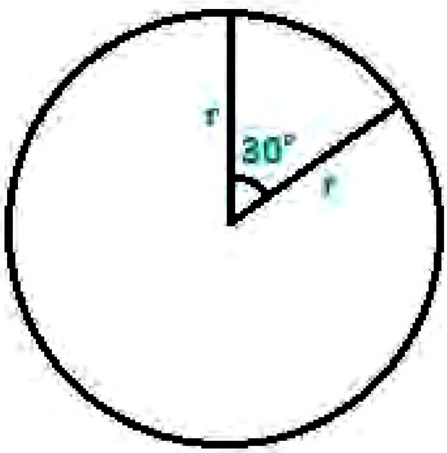
Areaofthesector=Θ360∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=120360∗π21^2 $Areaofthesector = \frac{120}{360} * \pi 21^2$

Area of the sector = (22 x 21) cm^2

Therefore, Area of the sector is 462 cm^2

Q15. The minute hand of a circle is √21 $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of clock between 7:00 a.m to 7:05 a.m.



Soln:

Given data:

Radius of the minute hand (r) =√21 $\sqrt{21}$ cm

Time between 7:00 a.m to 7:05a.m = 5 min

We know that, 1 hr = 60 min, minute hand completes

One revolution = 360°

60 min = 360°

θ = angle subtended at the centre of circle = 5 x 6° = 30°

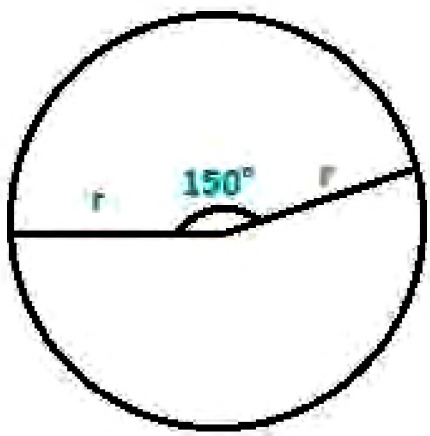
Areaofthesector=Θ360∗πr^2 $Area of the sector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=30360∗ pi35^2 $Area of the sector = \frac{30}{360} * \ pi 35^2$

Area of the sector = 5.5 cm^2

Therefore, Area of the sector is 5.5 cm^2

Q 16. The minute hand of clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 8 a.m to 8:25 a.m.



Soln:

Given data:

Radius of the circle = radius of the clock = length of the minute hand = 10 cm

We know that, 1 hr = 60 min

60 min = 360°

1 min = 6°

Time between 8:00 a.m to 8:25 a.m = 25 min

Therefore, the subtended = 6° x 25 = 150°

Formula to be used :

Areaofthesector=Θ360∗πr²$Area of the sector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=150360∗π10²$Area of the sector = \frac{150}{360} * \pi 10^2$

Area of the sector = 916.6 cm^2= 917 cm^2

Therefore, Area of the sector is 917 cm^2

Q17. A sector of 56° cut out from a circle subtends area of 4.4 cm^2. Find the radius of the circle.

Soln:

Given data:

Angle subtended by the sector at the centreof the circle, θ = 56°

Let the radius of the circle be = ‘r’ cm

Formula to be used:

Areaofthesector=56360∗πr²$Area of the sector = \frac{56}{360} * \pi r^2$

On solving the above equation, we get,

r^2 = √91 $\sqrt{\frac{9}{1}}$ cm

r = 3 cm

Therefore, radius of the circle is r = 3 cm

Q18. In circle of radius 6 cm. Chord of length 10 cm makes an angle of 110° at the centre of circle. Find:

(i) Circumference of the circle

(ii) Area of the circle

(iii) Length of arc

(iv) The area of sector

Soln:

Given data:

Radius of the circle = 6 cm

Chord of length = 10 cm

Angle subtended by chord with the centre of the circle = 110°

Formulae to be used:

Circumference of a circle = 2

Area of a Circle =

Areaofthesector=Θ360∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Lengthofarc=90360∗2πx28cm $Lengthofarc = \frac{90}{360} * 2\pi x 28cm$

Circumference of a circle = 2 = 2 x 3.14 x 8 = 37.7 cm

Area of a Circle = = 3.14 x 6 x 6 = 113.14 cm^2

Areaofthesector=Θ360∗πr^2 $Areaofthesector = \frac{\Theta}{360} * \pi r^2$

Areaofthesector=110360∗π6^2 $Areaofthesector = \frac{110}{360} * \pi 6^2$

On solving the above equation we get,

Area of the sector = 33.1 cm^2

Lengthofarc=θ360∗2πrcm $Lengthofarc = \frac{\theta}{360} * 2\pi rcm$

Lengthofarc=110360∗2π6cm $Lengthofarc = \frac{110}{360} * 2\pi 6cm$

On solving the above equation we get,

Length of arc = 22.34 cm.

Therefore,Circumference = 37.7 cm

Area of a Circle = 113.14 cm^2

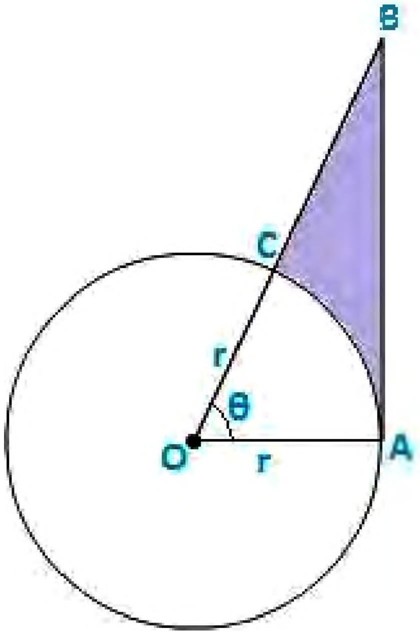
Area of the sector = 33.1 cm^2

Q19. The given figure shows a sector of a circle with centre 'O' subtending an angle θ°. Prove that:

1. Perimeter of shaded region is r(tanθ+secθ+(π theta180)–1)

$r\left(\tan\theta + \sec\theta + \left(\frac{\pi\, theta}{180}\right) - 1\right)$

2. Area of the shaded region is r²2(tanθ−πθ180) $\frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$



Soln:

Given Data: Angle subtended at the centre of the circle = θ°

Angle OAB = 90° [at point of contact, tangent is perpendicular to radius]

OAB is a right angle triangle

Cos θ = adjsidehypotenuse=rOB=OB=rsecθ $\frac{adjside}{hypotenuse} = \frac{r}{OB} = OB = r\sec\theta$

sec θ = oppsideadjside=ABr=AB=rtanθ $\frac{oppside}{adjside} = \frac{AB}{r} = AB = r\tan\theta$

Perimeter of the shaded region = AB + BC + CA (arc)

= r tanθ + (OB – OC) + θ360 ∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

= r(tanθ+secθ+πθ180−1) $r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$

Area of the shaded region = (area of triangle AOB) – (area of sector)

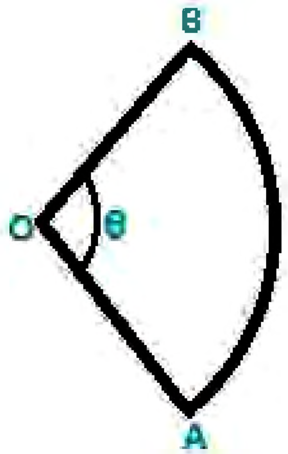
(12∗OA∗AB)−θ360∗πr2 $\left(\frac{1}{2} * OA * AB\right) - \frac{\theta}{360} * \pi r^2$

On solving the above equation we get,

r22[tanθ−πθ180] $\frac{r^2}{2}\left[\tan\theta - \frac{\pi\theta}{180}\right]$

Q 20. The diagram shows a sector of circle of radius ‘r’ cm subtends an angle θ. The area of sector is A cm^2 and perimeter of sector is 50 cm. Prove that θ=360π(25r−1)

$\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$ **and A = 25 r – r^2**



Soln:

Given Data:

Radius of circle = ‘r’ cm

Angle subtended at centre of the circle = θ

Perimeter = OA + OB + (AB arc)

r+r+θ360∗2πr=2r+2r[πθ360] $r + r + \frac{\theta}{360} * 2\pi r = 2r + 2r\left[\frac{\pi\theta}{360}\right]$

As given in the question, perimeter = 50 θ=360π[25r−1]$\theta = \frac{360}{\pi}\left[\frac{25}{r} - 1\right]$

Therefore, θ=360π[25r−1]$\theta = \frac{360}{\pi}\left[\frac{25}{r} - 1\right]$

Areaofthesector=Θ360∗πr^2$Areaofthesector = \frac{\Theta}{360} * \pi r^2$

On solving the above equation, we have

A = 25r – r^2

Hence, proved.

Exercise 15.3: Areas Related to Circles

Q1. AB is a chord of a circle with center O and radius 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment.

Soln:

Given data:

Radius of the circle with center'O', r = 4 cm = OA = OB

Length of the chord AB = 4 cm

OAB is an equilateral triangle and angle AOB = 60° + θ

Angle subtended at centre θ = 60°

Area of the segment (shaded region) = (area of sector) – (area of triangleAOB)

= θ360 ×∏r^2–√34 $(side)^2$ $\frac{\theta}{360} \times \prod r^2 - \frac{\sqrt{3}}{4}(side)^2$

= 60360 ×∏4^2–√34 $(4)^2$ $\frac{60}{360} \times \prod 4^2 - \frac{\sqrt{3}}{4}(4)^2$

On solving the above equation, we get,

= 58.67 – 6.92 = 51.75 cm2

Therefore, the required area of the segment is 51.75 cm2

Q2. A chord PQ of length 12 cm subtends an angle 120 at the center of a circle. Find the area of the minor segment cut off by the chord PQ.

Soln:

We know that,

Area of the segment = θ360 ×∏r2–√34 (side)2 $\frac{\theta}{360} \times \prod r^2 - \frac{\sqrt{3}}{4}(side)^2$

We have,

∠POQ=120andPQ=12cm $\angle POQ = 120 and PQ = 12cm$

PL = PQ x (0.5)

= 12 x 0.5 = 6 cm

Since, ∠POQ $\angle POQ$ = 120

∠POL $\angle POL$ = ∠QOL $\angle QOL$ = 60

In triangle OPQ, we have

sinθ=PLOA $sin\theta = \frac{PL}{OA}$,

sin 60° = 6OA $\frac{6}{OA}$,

OA = 12√3 $\frac{12}{\sqrt{3}}$

Thus ,OA = 12√3 $\frac{12}{\sqrt{3}}$

Now using the value of r and angle θ we will find the area of minor segment.

A=4{4π–3√3}cm2 $A = 4\left\{4\pi - 3\sqrt{3}\right\} cm^2$.

Q 3. A chord of circle of radius 14 cm makes a right angle at the centre. Find the areas of minor and major segments of the circle.

Soln:

Given data:

Radius (r) = 14 cm

Angle subtended by the chord with the centre of the circle, θ = 90°

Area of minor segment (ANB) = (area of ANB sector) – (area of the triangle AOB)

= θ360 ×∏r^2 $\frac{\theta}{360} \times \prod r^2$ – 0.5 x OA x OB

= 90360 ×∏14^2 $\frac{90}{360} \times \prod 14^2$ – 0.5 x 14 x 14 = 154 – 98 = 56 cm2

Therefore the area of the minor segment (ANB) = 56 cm2

Area of the major segment (other than shaded) = area of circle – area of segment ANB

= ∏r^2–56cm^2 $\prod r^2 - 56cm^2$

= 3.14 x 14 x 14 – 56 = 616 – 56 = 560 cm2

Therefore, the area of the major segment = 560 cm2.

Q 4. A chord 10 cm long is drawn in a circle whose radius is 5√2cm $5\sqrt{2}cm$. Find the area of both segments.

Soln:

Given data: Radius of the circle , r = 5√2cm $5\sqrt{2}cm$ = OA = OB

Length of the chord AB = 10cm

In triangle OAB , OA2 +OB2 = (5√2)²+(5√2)² $(5\sqrt{2})^2 + (5\sqrt{2})^2$ = 50 + 50 = 100 =(AB)2

Hence, pythogoras theorem is satisfied.

Therefore OAB is a right angle triangle.

Angle subtended by the chord with the centre of the circle, θ = 90°

Area of segment (minor) = shaded region = area of sector – area of triangle OAB

= θ360×∏r² $\frac{\theta}{360} \times \prod r^2$ – 0.5 x OA x OB

= 90360×∏(5√2)²–0.5x(5√2)²x(5√2)² $\frac{90}{360} \times \prod(5\sqrt{2})^2 - 0.5x(5\sqrt{2})^2x(5\sqrt{2})^2$

= 11007−1007=10007 cm² $\frac{1100}{7} - \frac{100}{7} = \frac{1000}{7}cm^2$

Therefore, Area of segment (minor) = 10007 cm² $\frac{1000}{7}cm^2$.

Q5. A chord AB of circle of radius 14 cm makes an angle of 60° at the centre. Find the area of the minor segment of the circle.

Soln:

Given data: radius of the circle (r) = 14 cm = OA = OB

Angle subtended by the chord with the centre of the circle, θ = 60°

In triangle AOB, angle A = angle B [angle opposite to equal sides OA and OB] = x

By angle sum property, ∠A+∠B+∠O=180 $\angle A + \angle B + \angle O = 180$

X + X + 60° =180°

2X = 120°, X = 60°

All angles are 60°, triangle OAB is equilateral OA = OB = AB

= area of the segment (shaded region in the figure) = area of sector– area of triangle OAB

= θ360×∏r²–√34(−AB)² $\frac{\theta}{360} \times \prod r^2 - \frac{\sqrt{3}}{4}(-AB)^2$

On solving the above equation we get,

= 308–147√33 cm² $\frac{308-147\sqrt{3}}{3}cm^2$

Therefore, area of the segment (shaded region in the figure) = 308–147√33 cm²

$\frac{308-147\sqrt{3}}{3}cm^2$.

Q 6. Ab is the diameter of a circle with centre 'O' . C is a point on the circumference such that ∠COB $\angle COB$ = θ. The area of the minor segmentcut off by AC is equal to

twice the area of sector BOC. Prove that sin θ2 .cos θ2 $\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$ **= ∏(12−θ120)** $\prod\left(\frac{1}{2}-\frac{\theta}{120}\right)$.

Soln:

Given data: AB is a diameter of circle with centre O,

Also, ∠COB$\angle COB$ = θ = Angle subtended

Area of sector BOC = θ360 ×∏r2 $\frac{\theta}{360}\times\prod r^2$

Area of segment cut off by AC = (area of sector) – (area of triangle AOC)

∠AOC$\angle AOC$ = 180 – θ ∠AOCand∠BOC$\angle AOC and \angle BOC$ from linear pair]

Area of sector = (180−θ)360 ×π×r2=π×r2 2−πθr2 360 $\frac{(180-\theta)}{360}\times\pi\times r^2=\frac{\pi\times r^2}{2}-\frac{\pi\theta r^2}{360}$

In triangle AOC , drop a perpendicular AM , this bisects ∠AOC$\angle AOC$ and side AC.

Now, In triangle AMO, sin∠AOM=AMOA=sin(180−θ2)=AMr

$$sin\angle AOM=\frac{AM}{OA}=sin(\frac{180-\theta}{2})=\frac{AM}{r}$$

AM=rsin(90−θ2)=rcos θ2 $AM=r\sin\left(90-\frac{\theta}{2}\right)=r\cos\frac{\theta}{2}$

cos∠AOM=OMOA=cos(90−θ2)=OMr⇒OM=rsin θ2

$$\cos\angle AOM=\frac{OM}{OA}=\cos\left(90-\frac{\theta}{2}\right)=\frac{OM}{r}\Rightarrow OM=r\sin\frac{\theta}{2}$$

Area of segment= πr2 2 − πθr2 360 − 12 (AC×OM)[AC=2AM]

$$\frac{\pi r^2}{2}-\frac{\pi\theta r^2}{360}-\frac{1}{2}(AC\times OM)\,[AC=2AM]$$

= πr2 2 − πθr2 360 − 12 (2rcos θ2 rsin θ2)=r2[π2 − πθ360 −cos θ2 sin θ2]

$$\frac{\pi r^2}{2}-\frac{\pi\theta r^2}{360}-\frac{1}{2}\left(2r\cos\frac{\theta}{2}r\sin\frac{\theta}{2}\right)=r^2\left[\frac{\pi}{2}-\frac{\pi\theta}{360}-\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$$

Area of segment by AC = 2 (Area of sector BOC)

r2[π2 − πθ360 –cos θ2 .sin θ2]=2r2[πθ360] $r^2\left[\frac{\pi}{2}-\frac{\pi\theta}{360}-\cos\frac{\theta}{2}.\sin\frac{\theta}{2}\right]=2r^2\left[\frac{\pi\theta}{360}\right]$

On solving the above equation we get,

cos θ2 ×sin θ2 =π(12 − θ120) $cos\frac{\theta}{2}\times\sin\frac{\theta}{2}=\pi\left(\frac{1}{2}-\frac{\theta}{120}\right)$

Hence proved that, cosθ2.sinθ2=π(12−θ120) $\cos\frac{\theta}{2}.\sin\frac{\theta}{2}=\pi\left(\frac{1}{2}-\frac{\theta}{120}\right)$.

Q 7. A chord a circle subtends an angle θ at the center of the circle. The area of the minor segment cut off by the chord is one-eighth of the area of the circle. Prove that 8sinθ2.cosθ2+π=πθ45 $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2}+\pi=\frac{\pi\theta}{45}$.

Soln:

Let the area of the given circle be = r

We know that, area of a circle = π r2

AB is a chord, OA and OB are joined. Drop a OM such that it is perpendicular to AB, this OM bisects AB as well as ∠AOM $\angle AOM$

∠AOM=∠MOB=12(0)=θ2,AB=2AM $\angle AOM=\angle MOB=\frac{1}{2}(0)=\frac{\theta}{2},AB=2AM$

Area of segment cut off by AB = (area of sector) – (area of the triangle formed)

θ360×πr2–12×AB×OM=r2[πθ360]–12.2rsinθ2.cosθ2

$$\frac{\theta}{360}\times\pi r^2-\frac{1}{2}\times AB\times OM=r^2\left[\frac{\pi\theta}{360}\right]-\frac{1}{2}.2r\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$$

Area of segment = 18 $\frac{1}{8}$ (area of circle)

r2[πθ360−sinθ2.cosθ2]=18πr2 $r^2\left[\frac{\pi\theta}{360}-\sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]=\frac{1}{8}\pi r^2$

On solving the above equation we get,

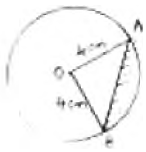
8sinθ2.cosθ2+π=πθ45 $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2}+\pi=\frac{\pi\theta}{45}$

Hence proved, 8sinθ2.cosθ2+π=πθ45 $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2}+\pi=\frac{\pi\theta}{45}$.

Exercise 15.4: Areas Related to Circles

1. AB is a chord of a circle with centre O and radius 4cm. AB is length 4cm and divides circle into two segments. Find the area of minor segment

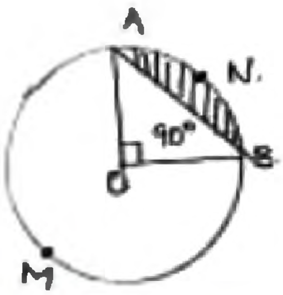
   Sol:

   Radius of circle r = 4cm = OA = OB
   Length of chord AB = 4cm
   OAB is equilateral triangle $\angle AOB = 60° \rightarrow \theta$
   Angle subtended at centre $\theta = 60°$
   Area of segment (shaded region) = (area of sector) – (area of ΔAOB)
   $= \frac{\theta}{360°} \times \pi r^2 = \frac{\sqrt{3}}{4}(side)^2$
   $= \frac{60}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{\sqrt{2}}{4} \times 4 \times 4$
   $= \frac{176}{3} - 4\sqrt{3} = 58.67 - 6.92 = 51.75\ cm^2$

2. A chord of circle of radius 14cm makes a right angle at the centre. Find the areas of minor and major segments of the circle.

   Sol:



Radius (r) = 14cm

$\theta = 90°$

= OA = OB

Area of minor segment (ANB)

$= (area\ of\ ANB\ sector) - (area\ of\ \Delta AOB)$

$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$

$= 154 - 98 = 56cm^2$

Area of major segment (other than shaded)

= area of circle – area of segment ANB

$= \pi r^2 - 56$

$= \frac{22}{7} \times 14 \times 14 - 56$

= 616 – 56

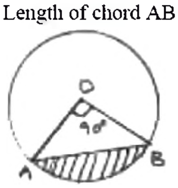
= 560 cm^2.

3. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both segments

Sol:

Given radius = r = $5\sqrt{2}$ cm = OA = OB

Length of chord AB = 10cm



In ΔOAB, OA = OB = $5\sqrt{2}\ cm$ AB = 10cm

$OA^2 + OB^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2 = 50 + 50 = 100 = (AB)^2$

Pythagoras theorem is satisfied OAB is right triangle

θ = angle subtended by chord = ∠AOB = 90°

Area of segment (minor) = shaded region

$= area\ of\ sector - area\ of\ \Delta OAB$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7}(5\sqrt{2})^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

$= \frac{275}{7} - 25 - \frac{100}{7}\ cm^2$

Area of major segment = (area of circle) – (area of minor segment)

$= \pi r^2 2 - \frac{100}{7}$

$= \frac{22}{7} \times (5\sqrt{2})^2 - \frac{100}{7}$

$= \frac{1100}{7} - \frac{100}{7} = \frac{1000}{7}\ cm^2$

4. A chord AB of circle. of radius 14cm makes an angle of 60° at the centre. Find the area of minor segment of circle.

Sol:

Given radius (r) = 14cm = OA = OB

θ = angle at centre = 60°

In ΔAOB, ∠A = ∠B [angles opposite to equal sides OA and OB] = x

By angle sum property ∠A + ∠B + ∠O = 180°

x + x + 60° = 180° ⇒ 2x = 120° ⇒ x = 60°

All angles are 60°. OAB is equilateral OA = OB = AB

Area of segment = area of sector – area Δle OAB

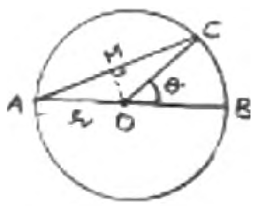
$= \frac{\theta}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (-AB)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{308}{3} - 49\sqrt{3} = \frac{308 - 147\sqrt{3}}{3}\ cm^2$

5. AB is the diameter of a circle. centre O. C is a point on the circumference such that ∠COB $= \theta$. The area of the minor segment cutoff by AC is equal to twice the area of sector BOC. Prove that $\sin\frac{\theta}{2} . \cos\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$

Sol:



Given AB is diameter of circle with centre O

$\angle COB = \theta$

Area of sector BOC $= \frac{\theta}{360°} \times \pi r^2$

Area of segment cut off, by AC = (area of sector) – (area of ΔAOC)

$\angle AOC = 180 - \theta$ [∠AOC and ∠BOC form linear pair]

Area of sector $= \frac{(180-\theta)}{360°} \times \pi r^2 = \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°}$

In ΔAOC, drop a perpendicular AM, this bisects ∠AOC and side AC.

Now, In ΔAMO, $\sin\angle AOM = \frac{AM}{DA} \Rightarrow \sin\left(\frac{180-\theta}{2}\right) = \frac{AM}{R}$

$\Rightarrow AM = R\sin\left(90 - \frac{\theta}{2}\right) = R.\cos\frac{\theta}{2}$

$\cos\ \angle ADM = \frac{OM}{OA} \Rightarrow \cos\left(90 - \frac{\theta}{2}\right) = \frac{OM}{Y} \Rightarrow OM = R.Sin\frac{\theta}{2}$

Area of segment $= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°} - \frac{1}{2}(AC \times OM)\ [AC = 2\ AM]$

$= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°} - \frac{1}{2} \times \left(2\ R\cos\frac{\theta}{2}\ R\sin\frac{\theta}{2}\right)$

$= r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$

Area of segment by AC = 2 (Area of sector BDC)

$r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2} . \sin\frac{\theta}{2}\right] = 2r^2\left[\frac{\pi\theta}{360°}\right]$

$\cos\frac{\theta}{2} . \sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360} - \frac{2\pi\theta}{360°}$

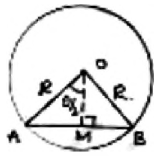
$= \frac{\pi}{2} - \frac{\pi\theta}{360°}[1 + 2]$

$= \frac{\pi}{2} - \frac{\pi\theta}{360°} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$

$\cos\frac{\theta}{2} . \sin\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$

6. A chord of a circle subtends an angle θ at the centre of circle. The area of the minor segment cut off by the chord is one eighth of the area of circle. Prove that $8\sin\frac{\theta}{2} . \cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$

Sol:



Let radius of circle = r

Area of circle = πr^2

AB is a chord, OA, OB are joined drop OM ⊥ AB. This OM bisects AB as well as ∠AOB.

$\angle AOM = \angle MOB = \frac{1}{2}(0) = \frac{\theta}{2}$ AB = 2AM

In ΔAOM, ∠AMO = 90°

$\text{Sin}\frac{\theta}{2} = \frac{AM}{AD} \Rightarrow AM = R.\sin\frac{\theta}{2}$ AB = 2R $\sin\frac{\theta}{2}$

$\text{Cos}\frac{\theta}{2} = \frac{OM}{AD} \Rightarrow OM = R\cos\frac{\theta}{2}$

Area of segment cut off by AB = (area of sector) – (area of triangles)

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB \times OM$

$= r^2\left[\frac{\pi\theta}{360°} - \frac{1}{2}.2r\sin\frac{\theta}{2}.R\cos\frac{\theta}{2}\right]$

$= R^2\left[\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]$

Area of segment $= \frac{1}{2}(\text{area of circle})$

$$r^2\left[\frac{\pi\theta}{360} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right] = \frac{1}{8}\pi r^2$$

$$\frac{8\pi\theta}{360°} - 8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} = \pi$$

$$8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$$