Exercise 14.1: Coordinate Geometry

1. On which axis do the following points lie?

   (i) P(5, 0)

   (ii) Q(0 – 2)

   (iii) R(−4, 0)

   (iv) S(0, 5)

   Sol:

   (i) $P(5,0)$ lies on $x-axis$

   (ii) $Q(0,-2)$ lies on $y-axis$

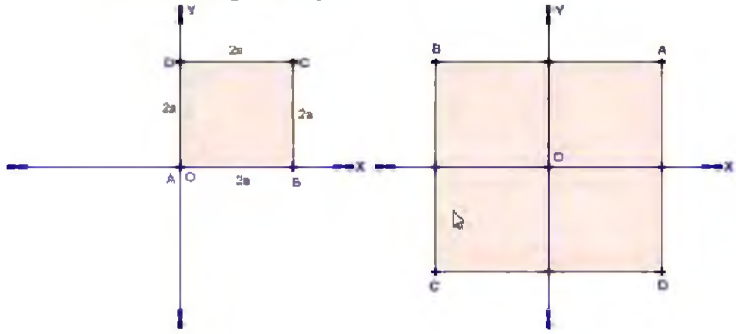
   (iii) $R(-4,0)$ lies on $x-axis$

   (iv) $S(0,5)$ lies on $y-axis$

2. Let ABCD be a square of side 2a. Find the coordinates of the vertices of this square when

(i) A coincides with the origin and AB and AB and coordinate axes are parallel to the sides AB and AD respectively.

(ii) The center of the square is at the origin and coordinate axes are parallel to the sides AB and AD respectively.



Sol:

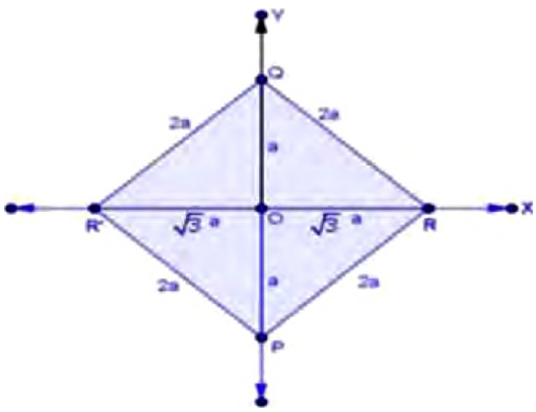
(i) Coordinate of the vertices of the square of side $2a$ are:

$A(0,0), B(2a,0), C(2a,2a)\ and\ D(0,2a)$

(ii) Coordinate of the vertices of the square of side $2a$ are:

$A(a,a), B(-a,a), C(-a,-a)\ and\ (a,-a)$

3. The base PQ of two equilateral triangles PQR and PQR' with side 2a lies along y-axis such that the mid-point of PQ is at the origin. Find the coordinates of the vertices R and R' of the triangles.



Sol:

We have two equilateral triangle PQR and PQR' with side $2a$.

O is the mid-point of PQ.

In ΔQOR, $\angle QOR = 90°$

Hence, by Pythagoras theorem

$OR^2 + OQ^2 = QR^2$

$$OR^2 = (2a)^2 - (a)^2$$

$$OR^2 = 3a^2$$

$$OR = \sqrt{(3)}a$$

Coordinates of vertex R is $(\sqrt{3}a, 0)$ and coordinate of vertex R' is $(-\sqrt{3}a, 0)$

Exercise 14.2: Coordinate Geometry

1. Find the distance between the following pair of points:

   (i) $(-6,7)$ and $(-1,-5)$

   (ii) $(a+b,b+c)$ and $(a-b,c-b)$

   (iii) $(a\sin\alpha,-b\cos\alpha)$ and $(-a\cos\alpha,b\sin\alpha)$

   (iv) $(a,0)$ and $(0,b)$

   Sol:

(i) We have $P(-6,7)$ and $Q(-1,-5)$

Here,

$x_1=-6, y_1=7$ and

$x_2=-1, y_2=-5$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{[-1-(-6)]^2+(-5-7)^2}$$

$$PQ=\sqrt{(-1+6)^2+(-5-7)^2}$$

$$PQ=\sqrt{(5)^2+(-12)^2}$$

$$PQ=\sqrt{25+144}$$

$$PQ=\sqrt{169}$$

$$PQ=13$$

(ii) we have $P(a+b,b+c)$ and $Q(a-b,c-b)$ here,

$x_1=a+b, y_1=b+c$ and $x_2=a-b, y_2=c-b$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{[a-b-(a+b)]^2+(c-b-(b+c))^2}$$

$$PQ=\sqrt{(a-b-a-b)^2+(c-b-b-c)^2}$$

$$PQ=\sqrt{(-2b)^2+(-2b)^2}$$

$$PQ=\sqrt{4b^2+4b^2}$$

$$PQ=\sqrt{8b^2}$$

$$PQ=\sqrt{4\times 2b^2}$$

$$PQ=2\sqrt{2}b$$

(iii) we have $P(a\sin\alpha,-b\cos\alpha)$ and $Q(-a\cos\alpha,b\sin\alpha)$ here

$x_1=a\sin\alpha, y_1=-b\cos\alpha$ and

$x_2=-a\cos\alpha, y_2=b\sin\alpha$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(-a\cos\alpha-a\sin\alpha)^2+\left[-b\sin\alpha-(-b\cos\alpha)\right]^2}$$

$$PQ=\sqrt{(-a\cos\alpha)^2+(-a\sin\alpha)^2+2(-a\cos\alpha)(-a\sin\alpha)+(b\sin\alpha)^2+(-b\cos\alpha)^2-2(b\sin\alpha)(-b\cos\alpha)}$$

$$PQ=\sqrt{a^2\cos^2\alpha+a^2\sin^2\alpha+2a^2\cos\alpha\sin\alpha+b^2\sin^2\alpha+b^2\cos^2\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\left(\cos^2\alpha+\sin^2\alpha\right)+2a^2\cos\alpha\sin\alpha+b^2\left(\sin^2\alpha+\cos^2\alpha\right)+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\times 1+2a^2\cos\alpha\sin\alpha+b^2\times 1+2b^2\sin\alpha\cos\alpha} \qquad \left[\because \sin^2\alpha+\cos^2\alpha=1\right]$$

$$PQ=\sqrt{a^2+b^2+2a^2\cos\alpha\sin\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{\left(a^2+b^2\right)+2\cos\alpha\sin\alpha\left(a^2+b^2\right)}$$

$$PQ=\sqrt{\left(a^2+b^2\right)(1+2\cos\alpha\sin\alpha)}$$

(iv) We have $P(a,0)$ and $Q(0,b)$

Here,

$x_1=a, y_1=0, x_2=0, y_2=b,$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(0-a)^2+(b-0)^2}$$

$$PQ=\sqrt{(-a)^2+(b)^2}$$

$$PQ=\sqrt{a^2+b^2}$$

2. Find the value of a when the distance between the points (3, a) and (4, 1) is $\sqrt{10}$.

Sol:

We have $P(3,a)$ and $Q(4,1)$

Here,

$x_1 = 3, y_1 = a$

$x_2 = 4, y_2 = 1$

$PQ = \sqrt{10}$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$

$\Rightarrow \sqrt{10} = \sqrt{(1)^2 + (1-a)^2}$

$\Rightarrow \sqrt{10} = \sqrt{1+1+a^2-2a}$ $\quad \left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$\Rightarrow \sqrt{10} = \sqrt{2+a^2-2a}$

Squaring both sides

$\Rightarrow \left(\sqrt{10}\right)^2 = \left(\sqrt{2+a^2-2a}\right)^2$

$\Rightarrow 10 = 2 + a^2 - 2a$

$\Rightarrow a^2 - 2a + 2 - 10 = 0$

$\Rightarrow a^2 - 2a - 8 = 0$

Splitting the middle team.

$\Rightarrow a^2 - 4a + 2a - 8 = 0$

$\Rightarrow a(a-4) + 2(a-4) = 0$

$\Rightarrow (a-4)(a+2) = 0$

$\Rightarrow a = 4, a = -2$

3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

Sol:

We have $P(2,1)$ and $Q(1,-2)$ and $R(X,Y)$

Also, $PR = QR$

$PR = \sqrt{(x-2)^2 + (y-1)^2}$

$\Rightarrow PR = \sqrt{x^2 + (2)^2 - 2xx \times 2 + y^2 + (1)^2 - 2 \times y \times 1}$

$\Rightarrow PR = \sqrt{x^2 + 4 - 4x + y^2 + 1 - 2y}$

$$\Rightarrow PR = \sqrt{x^2 + 5 - 4x + y^2 - 2y}$$

$$QR = \sqrt{(x-1)^2 + (y+2)^2}$$

$$\Rightarrow PR = \sqrt{x^2 + 1 - 2x + y^2 + 4 + 4y}$$

$$\Rightarrow PR = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$$

$$\because PR = QR$$

$$\Rightarrow \sqrt{x^2 + 5 - 4x + y^2 - 2y} = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$$

$$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$$

$$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$$

$$\Rightarrow -4x + 2x - 2y - 4y = 0$$

$$\Rightarrow -2x - 6y = 0$$

$$\Rightarrow -2(x + 3y) = 0$$

$$\Rightarrow x + 3y = \frac{0}{-2}$$

$$\Rightarrow x + 3y = 0$$

Hence proved.

4. Find the values of x, y if the distances of the point (x, y) from (−3, 0) as well as from (3,0) are 4.

Sol:

We have $P(x, y), Q(-3, 0)$ and $R(3, 0)$

$$PQ = \sqrt{(x+3)^2 + (y-0)^2}$$

$$\Rightarrow 4 = \sqrt{x^2 + 9 + 6x + y^2}$$

Squaring both sides

$$\Rightarrow (4)^2 = \left(\sqrt{x^2 + 9 + 6x + y^2}\right)^2$$

$$\Rightarrow 16 = x^2 + 9 + 6x + y^2$$

$$\Rightarrow x^2 + y^2 = 16 - 9 - 6x$$

$$\Rightarrow x^2 + y^2 = 7 - 6x \qquad \ldots\ldots\ldots\ldots(1)$$

$$PR = \left(\sqrt{(x-3)^2 + (y-0)^2}\right)$$

$$\Rightarrow 4 = \sqrt{x^2 + 9 - 6x + y^2}$$

Squaring both sides

$$(4)^2=\left(\sqrt{x^2+9-6x+y^2}\right)^2$$

$$\Rightarrow 16=x^2+9-6x+y^2$$

$$\Rightarrow x^2+y^2=16-9+6x$$

$$\Rightarrow x^2+y^2=7+6x \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Equating (1) and (2)

$$7-6x=7+6x$$

$$\Rightarrow 7-7=6x+6x$$

$$\Rightarrow 0=12x$$

$$\Rightarrow x=12$$

Equating (1) and (2)

$$7-6x=7+6x$$

$$\Rightarrow 7-7=6x+6x$$

$$\Rightarrow 0=12x$$

$$\Rightarrow x=12$$

Substituting the value of $x=0$ in (2)

$$x^2+y^2=7+6x$$

$$0+y^2=7+6\times 0$$

$$y^2=7$$

$$y=\pm\sqrt{7}$$

Exercise 14.3: Coordinate Geometry

1. Find the coordinates of the point which divides the line segment joining (— 1, 3) and (4, — 7) internally in the ratio 3 : 4.

Sol:

Let $P(x, y)$ be the required point.

$x = \frac{mx_2 + nx_1}{m+n}$

$y = \frac{my_2 + ny_1}{m+n}$

Here, $x_1 = -1$

$y_1 = 3$

$x_2 = 4$

$y_2 = -7$

$m : n = 3 : 4$

$$x = \frac{3\times4+4\times(-1)}{3+4}3$$

$$x = \frac{12-4}{7}$$

$$x = \frac{8}{7}$$

$$y = \frac{3\times(-7)+4\times3}{3+4}$$

$$y = \frac{-21+12}{7}$$

$$y = \frac{-9}{7}$$

$$\therefore \text{The coordinates of P are } \left(\frac{8}{7}, \frac{-9}{7}\right)$$

2. Find the points of trisection of the line segment joining the points:
(i) (5, —6) and (— 7, 5),
(ii) (3, —2) and (—3, —4)
(iii) (2, —2) and (—7, 4).

Sol:

(i) Let P and Q be the point of trisection of AB i.e., $AP = PQ = QB$

$(5,-6)$ $(-7,5)$

Therefore, P divides AB internally in the ratio of 1:2, thereby applying section formula, the coordinates of P will be

$$\left(\frac{1(-7)+2(5)}{1+2}\right), \left(\frac{1(5)+2(-6)}{1+2}\right) i.e., \left(1, \frac{-7}{3}\right)$$

Now, Q also divides AB internally in the ratio of 2:1 there its coordinates are

$$\left(\frac{2(-7)+1(5)}{2+1}\right), \frac{2(5)+1(-6)}{2+1} i.e., \left(-3, \frac{4}{3}\right)$$

(ii)

Let P, Q be the point of tri section of AB i.e.,

$AP = PQ = QB$

$(3,-2)$ $\qquad\qquad\qquad\qquad$ $(-3,-4)$

Therefore, P divides AB internally in the ratio of 1:2
Hence by applying section formula, Coordinates of P are

$$\left(\left(\frac{1(-3)+2(3)}{1+2}\right), \frac{1(-4)+1(-2)}{1+2}\right) i.e., \left(1, \frac{-8}{3}\right)$$

Now, Q also divides as internally in the ratio of 2:1
So, the coordinates of Q are

$$\left(\left(\frac{2(-3)+1(3)}{2+1}\right), \frac{2(-4)+1(-2)}{2+1}\right) i.e., \left(-1, \frac{-10}{3}\right)$$

Let P and Q be the points of trisection of AB i.e., $AP = PQ = OQ$

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula, are

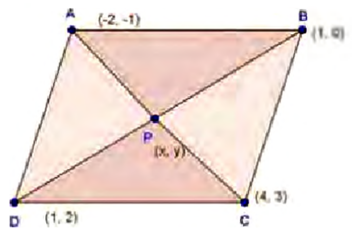
$$\left(\left(\frac{1(-7)+2(2)}{(1+2)}\right), \left(\frac{1(4)+2(-2)}{(1+2)}\right)\right), i.e., (-1, 0)$$

Now, Q also divides AB internally in the ration 2 : 1. So, the coordinates of Q are

$$\left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(2)}{2+1}\right), i.e., (-4, 2)$$

3. Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (—2, —1), (1, 0), (4, 3) and (1, 2) meet.

Sol:



Let P(x, y) be the given points.

We know that diagonals of a parallelogram bisect each other.

$x=\frac{-2+4}{2}$

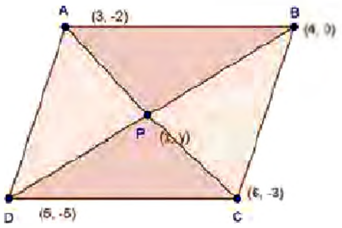
$\Rightarrow x=\frac{2}{2}=1$

$y=\frac{-1+3}{2}=\frac{2}{2}=1$

$\therefore$ Coordinates of P are $(1,1)$

4. Prove that the points (3, —2), (4, 0), (6, —3) and (5, —5) are the vertices of a parallelogram.

Sol:



Let $P(x, y)$ be the point of intersection of diagonals AC and 80 of $ABCD$.

$x=\frac{3+6}{2}=\frac{9}{2}$

$y=\frac{-2-3}{2}=\frac{-5}{2}$

Mid – point of $AC=\left(\frac{9}{5}, \frac{-5}{2}\right)$

Again.

$x=\frac{5+4}{2}=\frac{9}{2}$

$y=\frac{-5+0}{2}=\frac{-5}{2}$

Mid – point of $BD=\left(\frac{9}{2}, -\frac{5}{2}\right)$

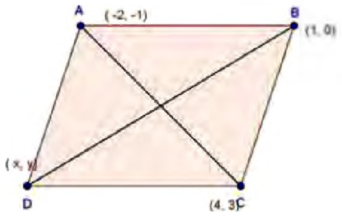
Here mid-point of AC – Mid - point of BD i.e. diagonals AC and BD bisect each other.

We know that diagonals of a parallelogram bisect each other

$\therefore$ $ABCD$ is a parallelogram.

5. Three consecutive vertices of a parallelogram are (—2, —1), (1, 0) and (4, 3). Find the fourth vertex.

Sol:



Let $A(-2,-1), B(1,0), C(4,3)$ and $D(x,y)$ be the vertices of a parallelogram $ABCD$ taken in order.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of the mid - point of AC = Coordinates of the mid-point of BD.

$$\Rightarrow \frac{-2+4}{2} = \frac{1+x}{2}$$

$$\Rightarrow \frac{2}{2} = \frac{x+1}{2}$$

$$\Rightarrow 1 = \frac{x+1}{2}$$

$$\Rightarrow x+1 = 2$$

$$\Rightarrow x = 1$$

And, $\frac{-1+3}{2} = \frac{y+0}{2}$

$$\Rightarrow \frac{2}{2} = \frac{y}{2}$$

$$\Rightarrow y = 2$$

Hence, fourth vertex of the parallelogram is $(1,2)$

Exercise 14.4: Coordinate Geometry

1. Find the centroid of the triangle whose vertices are:

   (i) $(1,4),(-1,-1)$ *and* $(3,-2)$

   Sol:

   We know that the coordinates of the centroid of a triangle whose vertices are $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are

   $$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

   So, the coordinates of the centroid of a triangle whose vertices are $(1,4),(-1,-1)$ *and* $(3,-2)$ are $\left(\frac{1-1+3}{3},\frac{4-1-2}{3}\right)$

   $$=\left(1,\frac{1}{3}\right)$$

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

   Sol:

Let the coordinates of the third vertex bee (x, y). Then

Coordinates of centroid of triangle are

$$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$$

We have centroid is at origin $(0, 0)$

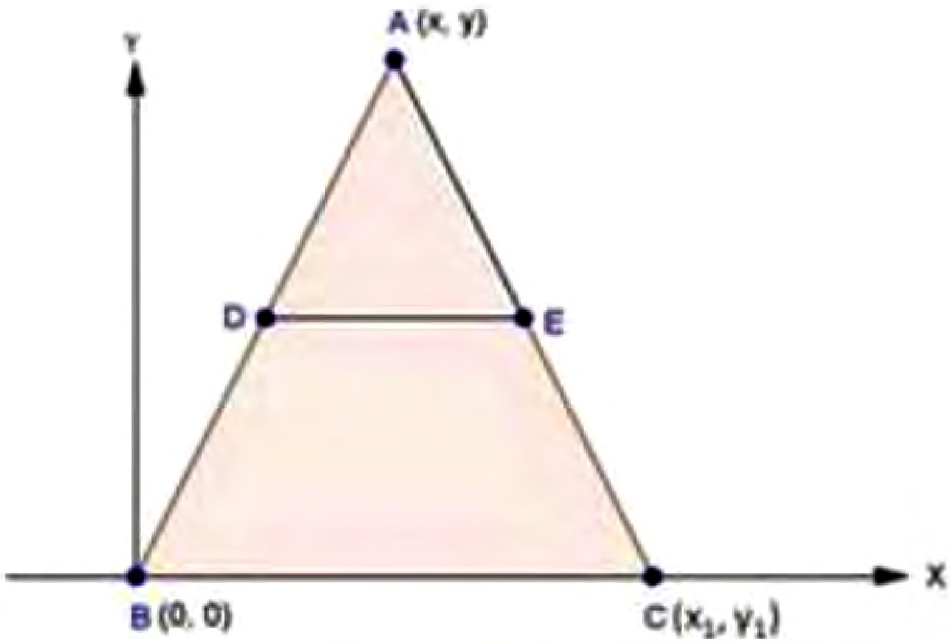
$$\therefore \frac{x+1+3}{3} = 0 \text{ and } \frac{y+2+5}{3} = 0$$

$$\Rightarrow x+4=0 \quad \Rightarrow y+7=0$$

$$\Rightarrow x=-4 \quad \Rightarrow y=-7$$

3. Prove analytically that the line segment joining the middle points of two sides of a triangle is equal to half of the third side.

Sol:



Let ABC be a triangle such that BC is along x-axis

Coordinates of A, B and C are $(x, y), (0, 0)$ and (x_1, y_1)

D and E are the mid-points of AB and AC respectively

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$$= \left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Length of $BC = \sqrt{x_1^2 + y_1^2}$

$$\text{Length of DE} = \sqrt{\left(\frac{x+x_1}{2}-\frac{x}{2}\right)^2+\left(\frac{x+y_1}{2}-\frac{y}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_1}{2}\right)^2+\left(\frac{y_1}{2}\right)^2}$$

$$= \sqrt{\frac{x_1^2}{4}+\frac{y_1^2}{4}}$$

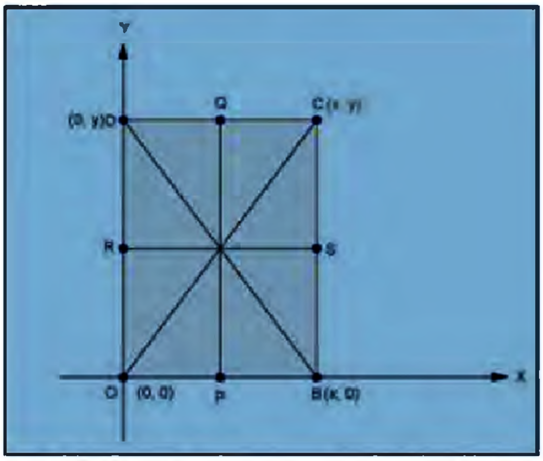
$$= \sqrt{\frac{1}{4}\left(x_1^2+y_1^2\right)}$$

$$= \frac{1}{2}\sqrt{x_1^2+y_1^2}$$

$$= \frac{1}{2}BC$$

4. Prove that the lines joining the middle points of the opposite sides of a quadrilateral and the join of the middle points of its diagonals meet in a point and bisect one another.

Sol:



Let $OBCD$ be the quadrilateral P,Q,R,S be the midpoint off OB,CD,OD and BC.

Let the coordinates of O,B,C,D are $(0,0),(x,0),(x,y)$ and $(0,y)$

Coordinates of P are $\left(\frac{x}{2},0\right)$

Coordinates of Q are $\left(\frac{x}{2},y\right)$

Coordinates of R are $\left(0,\frac{y}{2}\right)$

Coordinates of S are $\left(x, \frac{y}{2}\right)$

Coordinates of midpoint of PQ are

$$\left[\frac{\frac{x}{2}+\frac{x}{2}}{2}, \frac{0+y}{2}\right]=\left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of midpoint of RS are $\left[\frac{(0+x)}{2}, \frac{\frac{y}{2}+\frac{y}{2}}{2}\right]=\left[\frac{x}{2}, \frac{y}{2}\right]$

Since, the coordinates of the mid-point of $PQ =$ coordinates of mid-point of RS

$\therefore PQ$ and RS bisect each other

5. If G be the centroid of a triangle ABC and P be any other point in the plane, prove that $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$.

Sol:

Let $A(0,0), B(a,0)$, and $C(c,d)$ are the o-ordinates of triangle ABC

Hence, $G\left[\frac{c+0+a}{3}, \frac{d}{3}\right]$

i.e., $G\left[\frac{a+c}{3}, \frac{d}{3}\right]$

let $P(x,y)$

To prove:

$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$

Or, $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + GP^2 + GP^2 + GP^2$

Or, $PA^2 - GP^2 + PB^2 - GP^2 + PC^2 + GP^2 = GA^2 + GB^2 + GC^2$

Proof:

$PA^2 = x^2 + y^2$

$GP^2 = \left(x - \frac{a+c}{3}\right)^2 + \left(y - \frac{d}{3}\right)^2$

$PB^2 = (x-a)^2 + y^2$

$PC^2 = (x-c)^2 + (y-d)^2$

L.H.S

$$= x^2 + y^2 - \left[x^2 + \left(\frac{a+c}{3}\right)^2 + 2x\frac{(a+c)}{3} + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-a)^2 + y^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-c)^2 + (y-d)^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= x^2 + y + x^2 + x^2 + a^2 - 2ax + y^2 + x^2 + c^2 - 2xc + y^2 + d^2 - 2yd - 3$$

$$\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= \cancel{3x^2} + \cancel{3y^2} + a^2 + c^2 + d^2 - 2ax - 2xc - 2yd - \cancel{3x^2} - \frac{(a+c)^2}{3} + 2x(a+c) - \cancel{3y^2} - \frac{d^2}{3} + 2yd$$

$$= a^2 + c^2 + d^2 - \cancel{2ax} - \cancel{2xc} - \cancel{2yd} - \frac{a^2 + c^2 + 2ac}{3} + \cancel{2ax} + \cancel{2cx} - \frac{d^2}{3} + \cancel{2yd}$$

$$= \frac{3a^2 + 3c^2 + 3d^2 - a^2 - c^2 - 2ac - d^2}{3} = \frac{2a^2 + 2c^2 + 2d^2 - 2ac}{3} = L.H.S$$

Solving R.H.S

$GA^2 + GB^2 + GC^2$

$$GA^2 = \left(\frac{a+c}{3}\right)^2 + \left(\frac{d}{3}\right)^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9}$$

$$GC^2 = \left(\frac{a+c}{3} = a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{a^2 + 4c^2 - 4ca}{9} + \frac{4d^2}{9}$$

$$GB^2 = \left(\frac{a+c}{3} - a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9} + \frac{a^2 + 4c^2 - 4ac}{9} + \frac{4d^2}{9} + \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$= \frac{a^2 + c^2 + 2ac + d^2 + a^2 + 4c^2 - 4ac + 4d^2 + c^2 + 4a^2 - 4ac + d^2}{9}$$

$$= \frac{6a^2 + 6c^2 + 6d^2 + 6ac}{9} = \frac{2a^2 + 2c^2 + 2d^2 + 2ac}{3}$$

$\therefore L.H.S = R.H.S$

Exercise 14.5: Coordinate Geometry

1. Find the area of a triangle whose vertices are

(i) $(6,3),(-3,5)$ and $(4,-2)$

(ii) $\left[\left(at_1^2,2at_1\right),\left(at_2^2,2at_2\right)\left(at_3^2,2at_3\right)\right]$

(iii) $(a,c+a),(a,c)\, and\, (-a,c-a)$

Sol:

(i) Area of a triangle is given by

$$\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1+y_2\right)\right]$$

Here, $x_1=6, y_1=3, x_2=-3, y_2=5, x_3=4, y_3=-2$

Let $A(6,3), B(-3,5)$ and $C(4,-2)$ be the given points

Area of $\Delta ABC=\frac{1}{2}\left[6(5+2)+(-3)(-2-3)+4(3-5)\right]$

$$=\frac{1}{2}\left[6\times 7-3\times(-5)+4(-2)\right]$$

$$=\frac{1}{2}[42+15-8]$$

$$=\frac{49}{2} sq.units$$

(ii) Let $A=(x_1, y_1)=(at_1^2, 2at_1)$

$B=(x_2, y_2)=(at_2^2, 2at_2)$

$=(x_3, y_3)=(at_3^2, 2at_3)$ be the given points.

The area of ΔABC

$$=\frac{1}{2}\left[at_1^2(2at_2-2at_3)+at_2^2(2at_3-2at_1)+at_3^2(2at_1-2at_2)\right]$$

$$=\frac{1}{2}\left[2a^2t_1^2t_2-2a^2t_1^2t_3+2a^2t_2^2t_3-2a^2t_2^2t_1+2a^2t_3^2t_1-2a^2t_3^2t_2\right]$$

$$=\frac{1}{2}\times 2\left[a^2t_1^2(t_2-t_3)+a^2t_2^2(t_3-t_1)+a^2+t_3^2(t_1-t_2)\right]$$

$$=a^2\left[t_1^2(t_2-t_3)+t_2^2(t_3-t_1)+t_3^2(t_1-t_2)\right]$$

(iii) Let $A=(x_1, y_1)=(a, c+a)$

$B=(x_2, y_2)=(a, c)$

$C=(x_3, y_3)=(-a, c-a)$ be the given points

The area of ΔABC

$$=\frac{1}{2}\left[a(c-\{c-a\})+a(c-a-(c+a))+(-a)(c+a-a)\right]$$

$$=\frac{1}{2}\left[a(c-c+a)+a(c-a-c-a)-a(c+a-c)\right]$$

$$=\frac{1}{2}\left[a\times a+ax(-2a)-a\times a\right]$$

$$=\frac{1}{2}\left[a^2-2a^2-a^2\right]$$

$$=\frac{1}{2}\times(-2a)^2$$

$$=-a^2$$

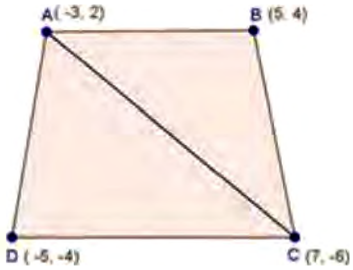
2. Find the area of the quadrilaterals, the coordinates of whose vertices are

(i) (—3, 2), (5, 4), (7, — 6) and (—5, — 4)

(ii) (1, 2), (6, 2), (5, 3) and (3, 4)

(iii) (—4, — 2), (—3, — 5), (3, — 2), (2, 3)

Sol:



Let $A(-3,2), B(5,4), C(7,-6)$ and $D(-5,-4)$ be the given points.

Area of ΔABC

$=\frac{1}{2}[-3(4+6)+5(-6-2)+7(2-4)]$

$=\frac{1}{2}[-3\times 10+5\times(-8)+7(-2)]$

$=\frac{1}{2}[-30-40-14]$

$=-42$

But area cannot be negative

$\therefore$ Area of $\Delta ADC = 42$ square units

Area of ΔADC

$=\frac{1}{2}[-3(-6+4)+7(-4-2)+(-5)(2+6)]$

$=\frac{1}{2}[-3(-2)+7(-6)-5\times 8]$

$=\frac{1}{2}[6-42-40]$

$=\frac{1}{2}\times -76$

$=-38$

But area cannot be negative

$\therefore$ Area of $\Delta ADC = 38$ square units

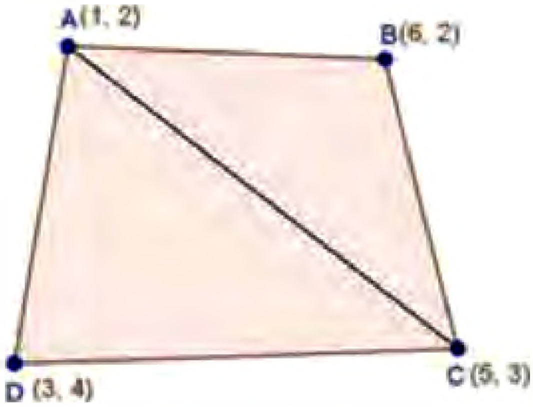
Now, area of quadrilateral $ABCD$

$= Ar.\ of\ ABC + Ar\ of\ ADC$

$=(42+38)$

$=80$ square. units

(i)



Let $A(1,2), B(6,2), C(5,3)$ and $(3,4)$ be the given points

Area of ΔABC

$$=\frac{1}{2}\left[1(2-3)+6(3-2)+5(2-2)\right]$$

$$=\frac{1}{2}\left[-1+6\times(1)+0\right]$$

$$=\frac{1}{2}[-1+6]$$

$$=\frac{5}{2}$$

Area of ΔADC

$$=\frac{1}{2}\left[1(3-4)+5(4-2)+3(2-3)\right]$$

$$=\frac{1}{2}\left[-1\times 5\times 2+3(-1)\right]$$

$=\frac{1}{2}[-1+10-3]$

$=\frac{1}{2}[6]$

$=3$

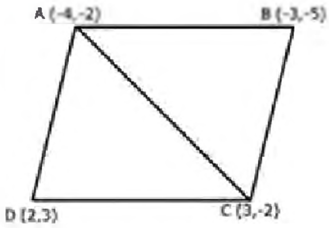
Now, Area of quadrilateral $ABCD$

$=$Area of $ABC+$Area of ADC

$=\left(\frac{5}{2}+3\right)sq.\ units$

$=\frac{11}{2}\ sq.\ units$

(ii)



Let $A(-4,-2), B(-3,-5), C(3,-2)$ and $D(2,3)$ be the given points

Area of $\Delta ABC=\frac{1}{2}\left|(-4)(-5+2)-3(-2+2)+3(-2+5)\right|$

$=\frac{1}{2}\left|(-4)(-3)-3(0)+3(3)\right|$

$=\frac{21}{2}$

Area of $\Delta ACD=\frac{1}{2}\left|(-4)(3+2)+2(-2+2)+3(-2-3)\right|$

$=\frac{1}{2}\left|-4(5)+2(0)+3(-5)\right|=\frac{-35}{2}$

But area can't be negative, hence area of $\Delta ADC=\frac{35}{2}$

Now, area of quadrilateral $(ABCD)=ar(\Delta ABC)+ar(\Delta ADC)$

Area (quadrilateral $ABCD$) $=\frac{21}{2}+\frac{35}{2}$

Area (quadrilateral $ABCD$) $=\frac{56}{2}$

Area (quadrilateral $ABCD$) $=28$ square. Units