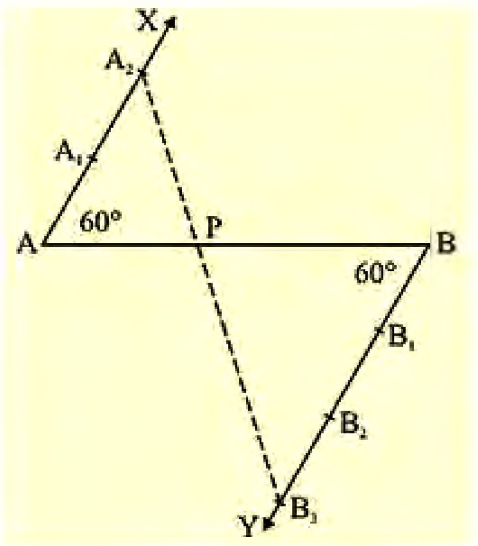
Exercise 11.1: Constructions

Q.1: Determine a point which divides a line segment of length 12 cm internally in the ratio of 2:3. Also, justify your construction.

Solution:



Steps of Construction:

1. Draw a line segment AB of 12 cm

2. Through the points A and B draw two parallel line on the opposite side of AB

3. Cut 2 equal parts on AX and 3 equal parts on BY such that $AX_1=X_1X_2$ $AX_1 = X_1X_2$ and $BX_1=Y_1Y_2=Y_2Y_3$ $BX_1 = Y_1Y_2 = Y_2Y_3$.

4. Join X_2Y_3 X_2Y_3 which intersects AB at P∴ APPB=23.∴ $\frac{AP}{PB} = \frac{2}{3}$.

Justification:

In ΔAX_2P ΔAX_2P and ΔBY_3P ΔBY_3P, we have

$\angle APX_2=\angle BPY_3$ $\angle APX_2 = \angle BPY_3$ { Because they are vertically opposite angle}

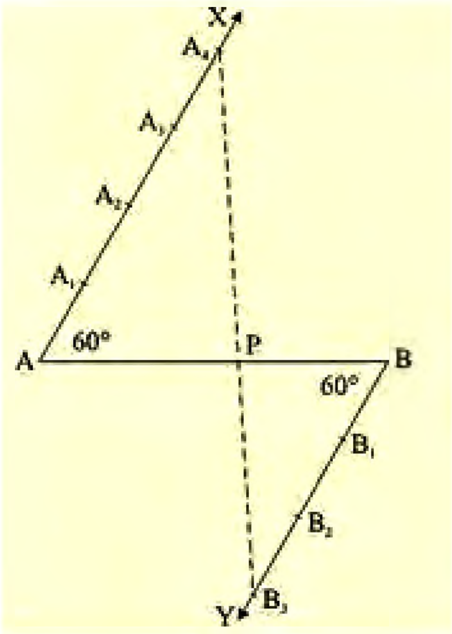
$\angle X_2AP=\angle Y_3BP$ $\angle X_2AP = \angle Y_3BP$ { Because they are alternate interior angles }

ΔAX_2P ΔAX_2P ΔBY_3P ΔBY_3P { Because AA similarity }

∴ APBP = AX_2BY_3=23 $\frac{AP}{BP} = \frac{AX_2}{BY_3} = \frac{2}{3}$ { Because of C.P.C.T }

Q.2: Divide a line segment of length 9 cm internally in the ratio 4:3. Also, give justification for the construction.

Solution:



Steps of construction:

1. Draw a line segment AB of 9 cm

2. Through the points, A and B, draw two parallel lines AX and BY on the opposite side of AB

3. Cut 4 equal parts on AX and 3 equal parts on BY such that: $AX_1=X_1X_2=X_2X_3=X_3X_4$ $AX_1 = X_1X_2 = X_2X_3 = X_3X_4$ and $BY_1=Y_1Y_2=Y_2Y_3$ $BY_1 = Y_1Y_2 = Y_2Y_3$

4. Join X_4Y_3 X_4Y_3 which intersects AB at P

∴ APPB=43 $\therefore \frac{AP}{PB} = \frac{4}{3}$

Justification:

In ΔAPX_4 ΔAPX_4 and ΔBPY_3 ΔBPY_3, we have

$\angle APX_4=\angle BPY_3$ $\angle APX_4 = \angle BPY_3$ { Because they are vertically opposite angles }

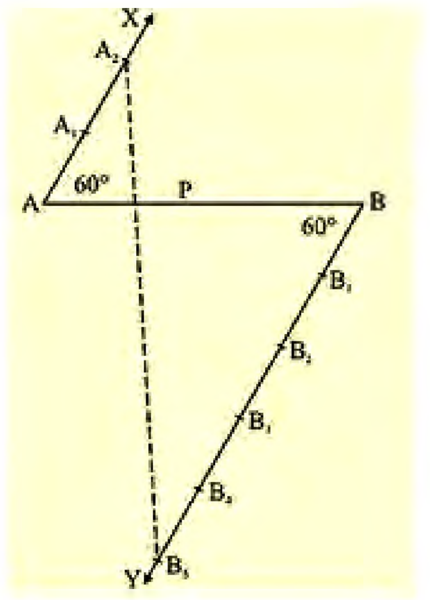
$\angle PAX_4=\angle PBY_3$ $\angle PAX_4 = \angle PBY_3$ { Because they are alternate interior angle}

ΔAPX_4 ΔAPX_4 ΔBPY_3 ΔBPY_3 { Because AA similarity }

∴ PAPB=AX4BY3=43 $\therefore \frac{PA}{PB} = \frac{AX_4}{BY_3} = \frac{4}{3}$ { Because of C.P.C.T }

Q.3: Divide a line segment of length 14 cm internally in the ratio 2:5. Also, give justification for the construction.

Solution:



Steps of construction:

(i) Draw a line segment AB of 14 cm

(ii) Through the points A and B, draw two parallel lines AX and BY on the opposite side of AB

(iii) Starting from A, Cut 2 equal parts on AX and starting from B, cut 5 equal parts on BY such that:

$AX_1=X_1X_2 AX_1 = X_1X_2$ and $BY_1=Y_1Y_2=Y_2Y_3=Y_3Y_4=Y_4Y_5$
$BY_1 = Y_1Y_2 = Y_2Y_3 = Y_3Y_4 = Y_4Y_5$

(iv) Join $X_2Y_5 X_2Y_5$ which intersects AB at P

∴ APPB=25 $\therefore \frac{AP}{PB} = \frac{2}{5}$

Justification:

In $\Delta APX_2 \Delta APX_2$ and $\Delta BPY_5 \Delta BPY_5$, we have

$\angle APX_2 = \angle BPY_5 \angle APX_2 = \angle BPY_5$ { Because they are vertically opposite angles }

$\angle PAX_2 = \angle PBY_5 \angle PAX_2 = \angle PBY_5$ { Because they are alternate interior angles }

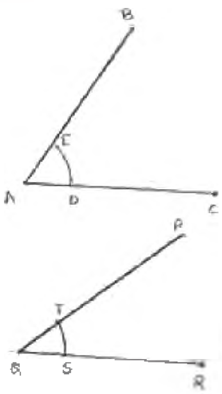
Then, $\Delta APX_2 \Delta APX_2$ $\Delta BPY_5 \Delta BPY_5$ { Because AA similarity }

$\therefore APPB = AX_2BY_5 = 25 \therefore \frac{AP}{PB} = \frac{AX_2}{BY_5} = \frac{2}{5}$ { Because of C.P.C.T }

Exercise 11.2: Constructions

1. Draw an angle and label it as ∠BAC. Construct another angle, equal to ∠BAC.

Sol:

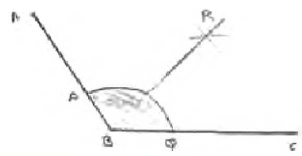


Steps of construction:

1. Draw an angle ABO and a Line segment QR
2. With center A and any radius, draw an arc which intersects $\angle BAC$ at E and O
3. With center Q and same radius draw arc which intersect QR at S.
4. With center S and radius equal to DE, draw an arc which intersect previous arc at T
5. Draw a line segment joining Q and T

$\therefore \angle PQR = \angle BAC$

2. Draw an obtuse angle. Bisect it. Measure each of the angles so obtained.
Sol:

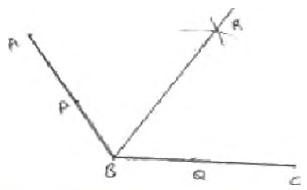


Steps of construction:

1. Draw angle ABC of 120°
2. With center B and any radius, draw an arc which intersects AB at P and BC at Q
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs, with intersect each other at R.
4. Join BR

$\therefore \angle ABR = \angle RBC = 60°$

3. Using your protractor, draw an angle of measure 108°. With this angle as given, draw an angle of 54°.
Sol:

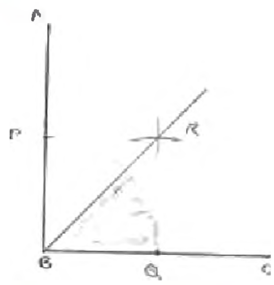


Steps of construction:

1. Draw an angle ABC of 108°
2. With center B and any radius, draw an arc which intersects AB at P and BC at Q
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs, which intersect each other at R.
4. Join BR

$\therefore \angle RBC = 54°$

4. Using protractor, draw a right angle. Bisect it to get an angle of measure 45°.
Sol:



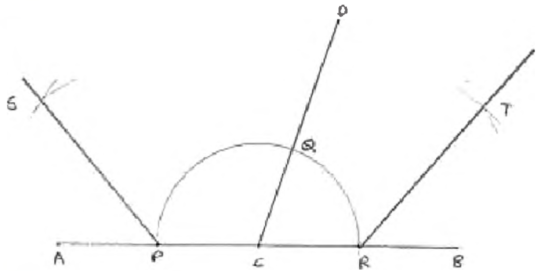
Steps of construction:

1. Draw an angle ABC of 90°
2. With center B and any radius, draw an arc which intersects AB at P and BC at Q
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs, which intersect each other at R.
4. Join RB

$\therefore \angle RBC = 45°$

5. Draw a linear pair of angles. Bisect each of the two angles. Verify that the two bisecting rays are perpendicular to each other.

Sol:



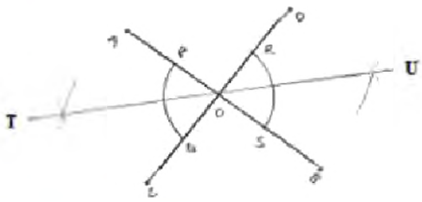
Steps of construction:

1. Draw two angle DCA and DCB forming Linear pair
2. With center C and any radius, draw an arc which intersects AC at P, CD at Q and CB at R.
3. With center P and Q and any radius draw two arcs which interest each other at S
4. Join SC
5. With center Q and R any radius draw two arcs, which intersect each other at T.
6. Join TC

$\angle SCT = 90°$ [By using protractor]

6. Draw a pair of vertically opposite angles. Bisect each of the two angles. Verify that the bisecting rays are in the same line.

Sol:



Steps of construction:

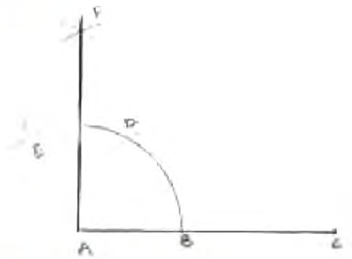
1. Draw a pair of vertically opposite angle AOC and DOB
2. With center O and any radius drawn two arcs which intersect OA at P, $Q \cdot OB$ at S and OD at R.
3. With center P and Q and radius more than $\frac{1}{2}PQ$, draw two arcs which intersect each other at 7.
4. Join to

5. With center R and S radius more than $\frac{1}{2}RS$, draw two arcs which intersect each other at U.
6. Join OU.

$\therefore TOU$ is a straight line

7. Using ruler and compasses only, draw a right angle.

Sol:



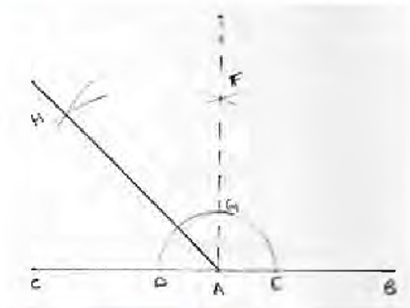
Steps of construction:

1. Draw a line segment AB
2. With center A and any radius draw arc which intersect AB at C.
3. With center C and same radius draw an arc which intersects AB at C.
4. With center D and same radius draw arc which intersect arc in (2) at E.
5. With centers E and C and any radius, draw two arcs which intersect each other at F.
6. Join FA

$\angle FAB = 90°$

8. Using ruler and compasses only, draw an angle of measure 135°.

Sol:

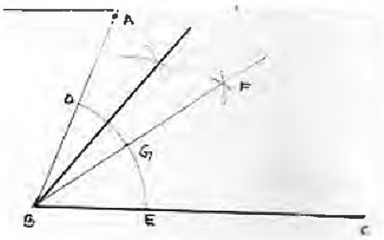


Steps of construction:

1. Draw a line segment AB and produce BA to point C.
2. With center A and any radius draw arc which intersect AC at D and AB at E.

3. With center D and E and radius more than $\frac{1}{2}DE$, draw two arcs which intersect each other at F.
4. Join FA which intersect the arc in (2) at G.
5. With centers G and D and radius more than $\frac{1}{2}GD$, draw two arcs which intersect each other at H.
6. Join HA
   $\therefore \angle HAB = 135°$

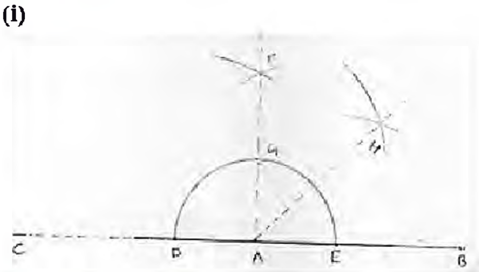
9. Using a protractor, draw an angle of measure 72°. With this angle as given, draw angles of measure 36° and 54°.
Sol:



Steps of construction:
1. Draw an angle ABC of 72° with the help of protractor.
2. With center B and any radius, draw an arc which intersect AB at D and BC at E.
3. With center D and E and radius more than $\frac{1}{2}DE$, draw two arcs which intersect each other at F.
4. Join FB which intersect the arc in (2) at G.
5. With centers D and G and radius more than $\frac{1}{2}DE$, draw two arcs which intersect each other at F.
6. With centers D and G and radius more than $n\frac{1}{2}DG$ draw two arcs which intersect each other at H
7. Join HB

$\therefore \angle HBC = 54°$
$\angle FBC = 36°$

10. Construct the following angles at the initial point of a given ray and justify the construction:
(i) 45° (ii) 90°
Sol:
(i)

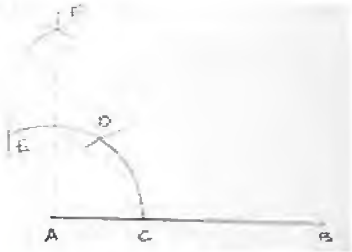


Steps of construction:
1. Draw a line segment AB and produce BA to point C.
2. With center A and any radius drawn an arc which intersect AC at D and AB at E.
3. With center D and E and radius more than $\frac{1}{2}DE$, draw arcs cutting each other at F.
4. Join FA which intersect arc in (2) at G.
5. With centers G and E and radius more than $\frac{1}{2}GE$, draw arcs cutting each other at H.

6. Join HA

$\therefore \angle HAB = 45°$

(ii)



Steps of construction:

1. Draw a line segment AB.
2. With center A and any radius draw in arc which intersect AB at C.
3. With center C and same radius draw an arc which intersects previous arc at D.
4. With centers D same radius draw an arc which intersects are in (2) at E.
5. With centers E and D same radius more than $\frac{1}{2}$ ED draw an arc cutting each other at F.
6. Join FA

$\angle FAB = 90°$

Exercise 11.3: Constructions

Q.1: Draw a circle of radius 6 cm. From a point 10 cm away from its center, construct a pair of tangents to the circle and measure their lengths.

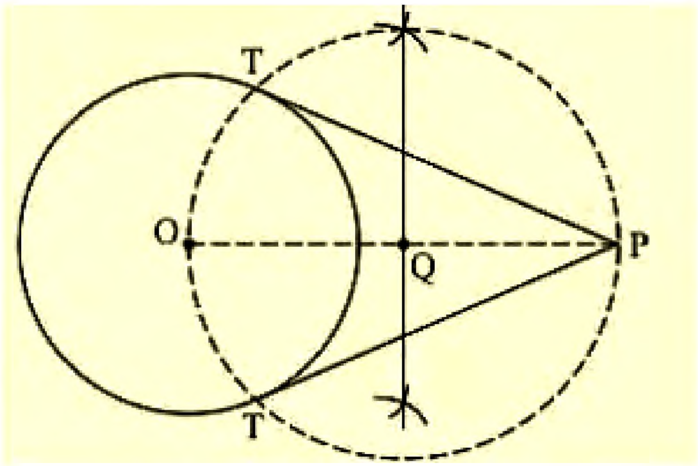
Solutions:

Given that:

Construct a circle of radius 6 cm, and let a point P = 10 cm from its centre, construct a pair of tangents to the circle.

Find the length of the tangents.

We follow the following steps to construct the given:



Steps of construction:

1. First of all, we draw a circle of radius AB = 6 cm.

2. Make a point P at a distance of OP = 10 cm, and join OP.

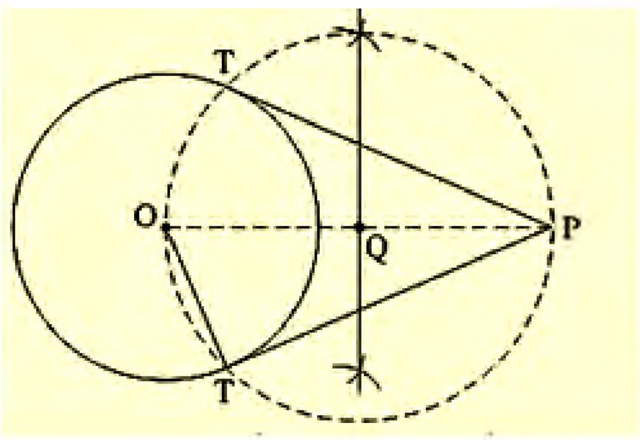
3. Draw a right bisector of P, intersecting OP at Q.

4. Taking Q as center and radius OQ = PQ, draw a circle to intersect the given circle at T and T`.

5. Join PT and PT to obtain the required tangents.

Thus, PT and PT are the required tangents.

Find the length of the tangents.



As we know that OT⊥PTandΔOPT $OT \perp PT \text{ and } \Delta OPT$ is the right triangle.

Therefore,

OT = 6 cm and PO = 10 cm.

In ΔOPT ΔOPT,

$PT^2 = OP^2 - OT^2$ $PT^2 = OP^2 - OT^2$

= $(10)^2 - (6)^2$ $(10)^2 - (6)^2$

= 100 – 36

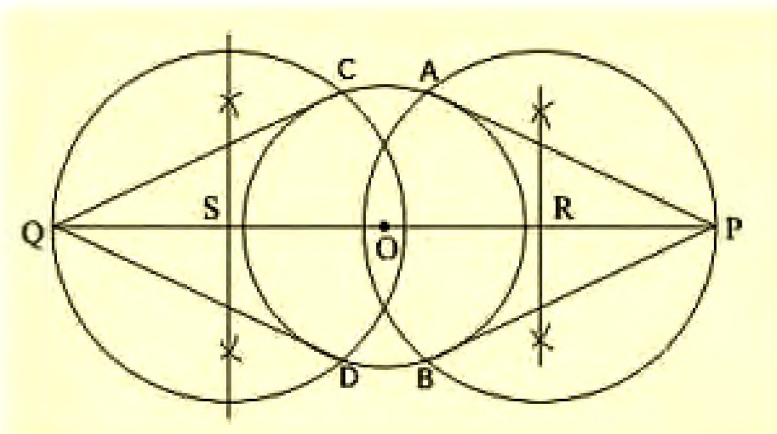
= 64

PT = 8 cm

Thus, length of tangents = 8 cm.

Q.2: Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its center. Draw tangents to the circle from these points P and Q.

Solutions:



Steps of construction:

(i) Draw a line segment PQ of 14 cm.

(ii) Take the midpoint O of PQ.

(iii) Draw the perpendicular bisectors of PO and OQ which intersects at points R and S.

(iv) With center R and radius RP draw a circle.

(v) With center S and radius, SQ draw a circle.

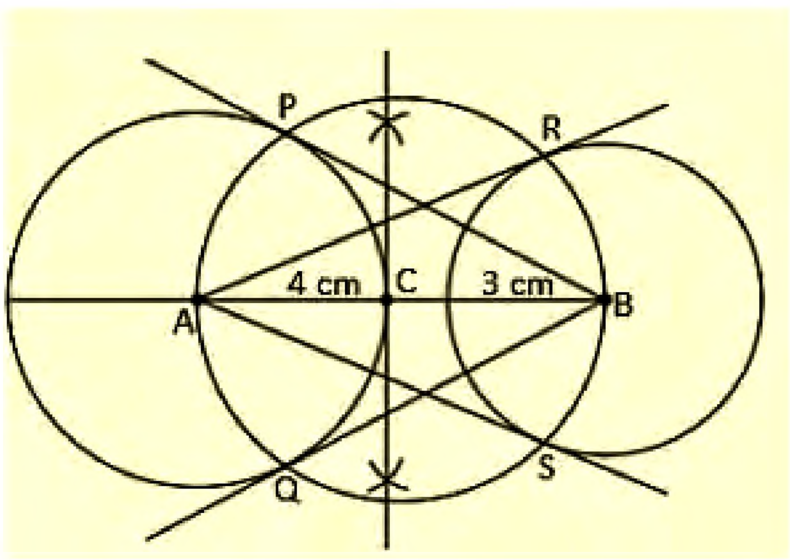
(vi) With center O and radius 3 cm draw another circle which intersects the previous circles at the points A, B, C, and D.

(vii) Join PA, PB, QC, and QD.

So, PA, PB, QC, and QD are the required tangents.

Q.3: Draw a line segment AB of length 8 cm. Taking A as the center, draw a circle of radius 4 cm and taking B as the center, draw another circle of radius 3 cm. Construct tangents to each circle from the center of the other circle.

Solution.



Steps of construction:

(i) Draw a line segment AB of 8 cm.

(ii) Draw the perpendicular of AB which intersects it at C.

(iii) With the center, C and radius CA draw a circle.

(iv) With centers A and B radius 4 cm and 3 cm, draw two circle which intersects the previous at the points P, Q, R and S.

(v) Join AR, AS, BP and BQ

So, AR, AS, BP and BQ are the required tangents.

Q.4: Draw two tangents to a circle of radius 3.5 cm from a point P at a distance of 6.2 cm from its center.

Solution:

Steps of construction:

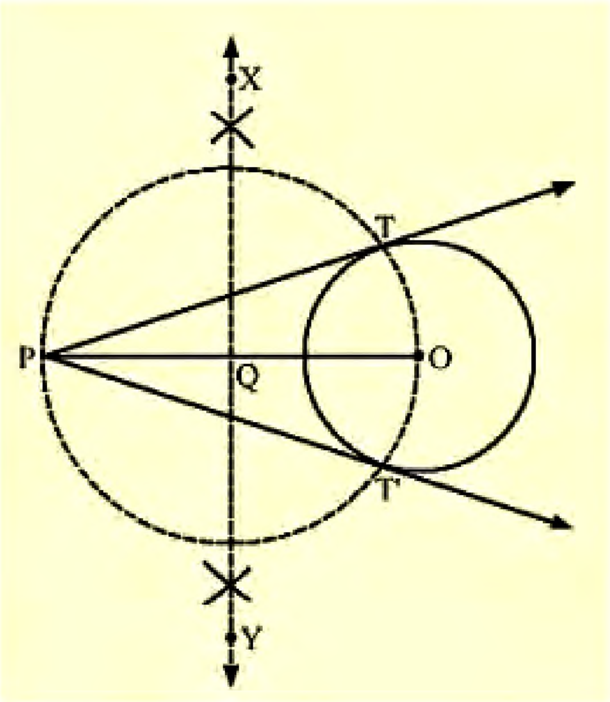
1. Draw a circle with O as a center and radius 3.5 cm.

2. Mark a point P outside the circle such that OP = 6.2 cm

3. Join OP. Draw the perpendicular bisector XY of OP, cutting OP at Q.

4. Draw a circle with Q as center and radius PQ(or OQ), to intersect the given circle at the points T and T`.

5. Join PT and PT`.



Here, PT and PT` are the required tangents.

Q.5: Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

Solution:

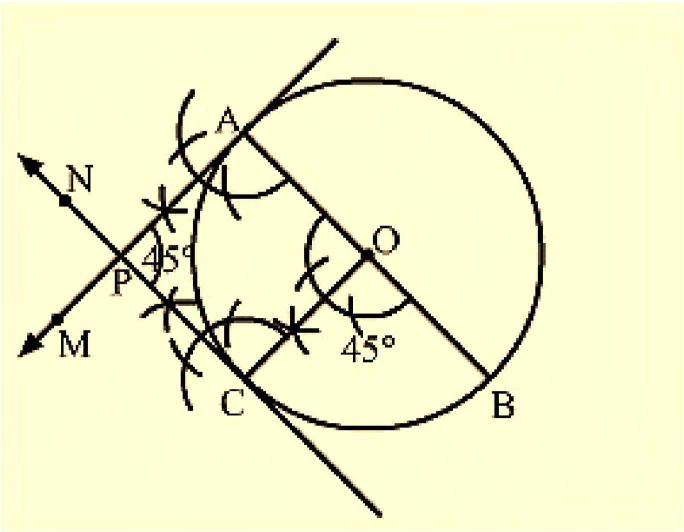
Steps of Construction:

1. Draw a circle with center O and radius 4.5 cm.

2. Draw any diameter AOB of the circle.

3. Construct $\angle BOC = 45^\circ$ such that, radius OC cuts the circle at C.

4. Draw $AM \perp AB$ and $CN \perp OC$. Suppose AM and CN intersect each other at P.



Here, AP and CP are the pairs of tangents to the circle inclined to each other at an angle of $45^{\text{circ}}45^{circ}$.

Q.6: Draw a right triangle ABC in which AB = 6 cm, BC = 6 cm and $\angle$B=90°$\angle B = 90^\circ$. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

Solution:

Steps of construction:

1. Draw a line segment AB = 6 cm

2. At B, draw $\angle$ABX=90°$\angle ABX = 90^\circ$.

3. With B as center and radius 8 cm, draw an arc cutting ray BX at C.

4. Join AC. Thus, ΔABCΔABC is the required triangle.

5. From B, draw BD⊥AC$BD \perp AC$.

6. Draw the perpendicular bisector of BC, cutting BC at O.

7. With O as center and radius OB (or OC), draw a circle. This circle passes through B, C and D.

8. Thus, this is the required circle.

9. Join OA.

10. Draw the perpendicular bisector of OA, cutting OA at E.

11. With E as a center and radius AE (or OE), draw a circle intersecting the circle with center O at B and F.

12. Join AF.