

## Exercise 1.1: Real Numbers

**Q.1:** If  $a$  and  $b$  are two odd positive integers such that  $a > b$ , then prove that one of the two numbers  $a+b$  and  $a-b$  is odd and the other is even.

**Sol:**

**Given:** If  $a$  and  $b$  are two odd positive integers such that  $a > b$ .

**To Prove:** That one of the two numbers  $a+b$  and  $a-b$  is odd and the other is even.

**Proof:** Let  $a$  and  $b$  be any odd positive integer such that  $a > b$ . Since any positive integer is of the form  $2q + 1$

Let,  $a = 2q + 1$  and  $b = 2m + 1$ , where,  $q$  and  $m$  are some whole numbers

$$a+b = (2q+1)+(2m+1) = 2\frac{(2q+1)+(2m+1)}{2}$$

$$a+b = 2((q+m)+1) = 2\frac{2((q+m)+1)}{2}$$

$$a+b = 2(q + m + 1) \text{ which is a positive integer.}$$

Also,

$$a-b = (2q+1)-(2m+1) = 2\frac{(2q+1)-(2m+1)}{2}$$

$$a-b \mid 2 \frac{a-b}{2} = 2(q+m) \mid 2 \frac{2(q+m)}{2}$$

$$a-b \mid 2 \frac{a-b}{2} = (q - m)$$

Given,  $a > b$

$$2q + 1 > 2m + 1$$

$$2q > 2m$$

$$q > m$$

$$\text{Therefore, } a-b \mid 2 \frac{a-b}{2} = (q - m) > 0$$

Thus,  $a-b \mid 2 \frac{a-b}{2}$  is a positive integer.

Now we need to prove that one of the two numbers  $a+b \mid 2 \frac{a+b}{2}$  and  $a-b \mid 2 \frac{a-b}{2}$  is odd and other is even.

$$\text{Consider, } a+b \mid 2 \frac{a+b}{2} - a-b \mid 2 \frac{a-b}{2} = (a+b) - (a-b) \mid 2 \frac{(a+b) - (a-b)}{2} = 2b \mid 2 \frac{2b}{2} = b$$

Also, we know that from the proof above that  $a+b \mid 2 \frac{a+b}{2}$  and  $a-b \mid 2 \frac{a-b}{2}$  are positive integers.

We know that the difference of two integers is an odd number if one of them is odd and another is even. (Also, difference between two odd and two even integers is even)

Hence it is proved that if  $a$  and  $b$  are two odd positive integers is even.

Hence, it is proved that if  $a$  and  $b$  are two odd positive integers such that  $a > b$  then one of the two number  $a+b \mid 2 \frac{a+b}{2}$  and  $a-b \mid 2 \frac{a-b}{2}$  is odd and the other is even.

**Q.2: Prove that the product of two consecutive positive integers is divisible by 2.**

**Sol:**

**To Prove:** that the product of two consecutive integers is divisible by 2.

**Proof:** Let  $n - 1$  and  $n$  be two consecutive positive integers.

Then their product is  $n(n - 1) = n^2 - n$

We know that every positive integer is of the form  $2q$  or  $2q + 1$  for some integer  $q$ .

So let  $n = 2q$

So,  $n^2 - n = (2q)^2 - (2q)$

$n^2 - n = (2q)^2 - (2q)$

$n^2 - n = 4q^2 - 2q$

$n^2 - n = 2q(2q - 1)$

$n^2 - n = 2r$  [where  $r = q(2q - 1)$ ]

$n^2 - n$  is even and divisible by 2

Let  $n = 2q + 1$

So,  $n^2 - n = (2q + 1)^2 - (2q + 1)$

$n^2 - n = (2q + 1)(2q + 1) - 1$

$n^2 - n = (2q + 1)(2q)$

$n^2 - n = 2r$  [ $r = q(2q + 1)$ ]

$n^2 - n$  is even and divisible by 2

Hence it is proved that the product of two consecutive integers is divisible by 2.

**Q.3: Prove that the product of three consecutive positive integer is divisible by 6.**

**Sol:**

**To Prove:** the product of three consecutive positive integers is

**Proof:** Let  $n$  be any positive integer.

Since any positive integer is of the form  $6q$  or  $6q + 1$  or  $6q + 2$  or  $6q + 3$  or  $6q + 4$ ,  $6q + 5$ .

If  $n = 6q$ ,

$$n(n+1)(n+2) = 6q(6q+1)(6q+2), \text{ which is divisible by 6}$$

If  $n = 6q + 1$

$$n(n+1)(n+2) = (6q+1)(6q+2)(6q+3)$$

$$n(n+1)(n+2) = 6(6q+1)(3q+1)(2q+1) \text{ Which is divisible by 6}$$

If  $n = 6q + 2$

$$n(n+1)(n+2) = (6q+2)(6q+3)(6q+4)$$

$$n(n+1)(n+2) = 12(3q+1)(2q+1)(2q+3),$$

Which is divisible by 6.

Similarly we can prove others.

Hence it is proved that the product of three consecutive positive integers is divisible by 6.

**Q.4: For any positive integer  $n$ , prove that  $n^3 - n$  divisible by 6.**

**Sol:**

**To Prove:** For any positive integer  $n$ ,  $n^3 - n$  is divisible by 6.

$$\textbf{Proof:} \text{ Let } n \text{ be any positive integer. } n^3 - n = (n-1)(n)(n+1)$$

Since any positive integer is of the form  $6q$  or  $6q + 1$  or  $6q + 2$  or  $6q + 3$  or  $6q + 4$ ,  $6q + 5$

If  $n = 6q$ ,

$$\text{Then, } (n-1)n(n+1) = (6q-1)6q(6q+1)$$

Which is divisible by 6

If  $n = 6q + 1$ ,

Then,  $(n - 1) n (n + 1) = (6q) (6q + 1) (6q + 2)$

Which is divisible by 6.

If  $n = 6q + 2$ ,

Then,  $(n - 1) n (n + 1) = (6q + 1) (6q + 2) (6q + 3)$

$(n - 1) n (n + 1) = 6 (6q + 1) (3q + 1) (2q + 1)$

Which is divisible by 6.

Similarly we can prove others.

Hence it is proved that for any positive integer  $n$ ,  $n^3 - n$  is divisible by 6.

**Q.5: Prove that if a positive integer is of form  $6q + 5$ , then it is of the form  $3q + 2$  for some integer  $q$ , but not conversely.**

**Sol:**

**To Prove:** That if a positive integer is of the form  $6q + 5$  then it is of the form  $3q + 2$  for some integer  $q$ , but not conversely.

**Proof:** Let  $n = 6q + 5$

Since any positive integer  $n$  is of the form of  $3k$  or  $3k + 1$ ,  $3k + 2$

If  $q = 3k$ ,

Then,  $n = 6q + 5$

$n = 18k + 5$  ( $q = 3k$ )

$n = 3 (6k + 1) + 2$

$n = 3m + 2$  (where  $m = (6k + 1)$ )

If  $q = 3k + 1$ ,

Then,  $n = (6q + 5)$

$n = (6 (3k + 1) + 5)$  ( $q = 3k + 1$ )

$$n = 18k + 6 + 5$$

$$n = 18k + 11$$

$$n = 3(6k + 3) + 2$$

$$n = 3m + 2 \text{ (where } m = (6k + 3)\text{)}$$

$$\text{If } q = 3k + 2,$$

$$\text{Then, } n = (6q + 5)$$

$$n = (6(3k + 2) + 5) \text{ (} q = 3k + 2\text{)}$$

$$n = 18k + 12 + 5$$

$$n = 18k + 17$$

$$n = 3(6k + 5) + 2$$

$$n = 3m + 2 \text{ (where } m = (6k + 5)\text{)}$$

Consider here 8 which is the form  $3q + 2$  i.e.  $3 \times 2 + 2$  but it can't be written in the form  $6q + 5$ . Hence the converse is not true.

**Q.6: Prove that square of any positive integer of the form  $5q + 1$  is of same form.**

**Sol:**

**To Prove:** That the square of a positive integer of the form  $5q + 1$  is of the same form

**Proof:** Since positive integer  $n$  is of the form  $5q + 1$

$$\text{If } n = 5q + 1$$

$$\text{Then } n^2 = (5q + 1)^2$$

$$n^2 = (5q)^2 + 2(1)(5q) + 1^2 = 25q^2 + 10q + 1$$

$$n^2 = 5m + 1 \text{ (where } m = (5q^2 + 2q)\text{)}$$

Hence  $n^2$  integer is of the form  $5m + 1$ .

**Q.7: Prove that the square of any positive integer is of the form  $3m$  or  $3m + 1$  but not of the form  $3m + 2$ .**

**Sol:**

**To Prove:** that the square of an positive integer is of the form  $3m$  or  $3m + 1$  but not of the form  $3m + 2$ .

**Proof:** Since positive integer  $n$  is of the form of  $3q$ ,  $3q + 1$  and  $3q + 2$

If  $n = 3q$

$$n^2 = (3q)^2$$

$$n^2 = 9q^2$$

$$n^2 = 3 (3q)^2$$

$$n^2 = 3m \text{ (} m = 3q \text{)}^2$$

If  $n = 3q + 1$

$$\text{Then, } n^2 = (3q + 1)^2$$

$$n^2 = (3q)^2 + 6q + 1$$

$$n^2 = 9q^2 + 6q + 1$$

$$n^2 = 3q (3q + 1) + 1$$

$$n^2 = 3m + 1 \text{ (where } m = (3q + 1) \text{)}$$

If  $n = 3q + 2$

$$\text{Then, } n^2 = (3q + 2)^2 = (3q)^2 + 12q + 4$$

$$n^2 = 9q^2 + 12q + 4$$

$$n^2 = 3 (3q + 4q + 1) + 1$$

$$n^2 = 3m + 1 \text{ (where } q = (3q + 4q + 1) \text{)}$$

Hence,  $n^2$  integer is of the form  $3m$ ,  $3m + 1$  but not of the form  $3m + 2$ .

**Q.8: Prove that the Square of any positive integer is of the form  $4q$  or  $4q + 1$  for some integer  $q$ .**

**Sol:**

**To Prove:** that the square of any positive integer is of the form  $4q$  or  $4q + 1$  for some integer  $q$ .

**Proof:** Since positive integer  $n$  is of the form of  $2q$  or  $2q + 1$

If  $n = 2q$

Then,  $n^2 = (2q)^2$

$$n^2 = 4q^2$$

$$n^2 = 4m \text{ (where } m = q^2\text{)}$$

If  $n = 2q + 1$

Then,  $n^2 = (2q + 1)^2$

$$n^2 = (2q)^2 + 4q + 1$$

$$n^2 = 4q^2 + 4q + 1$$

$$n^2 = 4q(q + 1) + 1$$

$$n^2 = 4q + 1 \text{ (where } m = q(q + 1)\text{)}$$

Hence it is proved that the square of any positive integer is of the form  $4q$  or  $4q + 1$ , for some integer  $q$ .

**Q.9: Prove that the Square of any positive integer is of the form  $5q$  or  $5q + 1$ ,  $5q + 4$  for some integer  $q$ .**

**Sol:**

**To Prove:** that the square of any positive integer is of the form  $5q$  or  $5q + 1$ ,  $5q + 4$  for some integer  $q$ .



**Proof:** Since positive integer  $n$  is of the form of  **$5q$  or  $5q + 1$ ,  $5q + 4$ .**

$$\text{If } n = 5q$$

$$\text{Then, } n^2 = (5q)^2$$

$$n^2 = 25q^2$$

$$n^2 = 5(5q)$$

$$n^2 = 5m \text{ (Where } m = 5q\text{)}$$

$$\text{If } n = 5q + 1$$

$$\text{Then, } n^2 = (5q + 1)^2$$

$$n^2 = (5q)^2 + 10q + 1$$

$$n^2 = 25q^2 + 10q + 1$$

$$n^2 = 5q(5q + 2) + 1$$

$$\text{It } n^2 = 5q(5q + 2) + 1$$

$$n^2 = 5m + 1 \text{ (where } m = q(5q + 2)\text{)}$$

$$\text{If } n = 5q + 2$$

$$\text{Then, } n^2 = (5q + 2)^2$$

$$n^2 = (5q)^2 + 20q + 4$$

$$n^2 = 25q^2 + 20q + 4$$

$$n^2 = 5q(5q + 4) + 4$$

$$n^2 = 5m + 4 \text{ (where } m = q(5q + 4)\text{)}$$

$$\text{If } n = 5q + 4$$

$$\text{Then, } n^2 = (5q + 4)^2$$

$$n^2 = (5q)^2 + 40q + 16$$

$$n^2 = 25q^2 + 40q + 16$$

$$n^2 = 5 (5q^2 + 8q + 3) + 1$$

$$n^2 = 5m + 1 \text{ (where } m = 5q^2 + 8q + 3 \text{ )}$$

Hence it is proved that the square of any positive integer is of the form **5q or 5q + 1, 5q + 4 for some integer q.**

**Q.10: Show that the Square of odd integer is of the form 8q + 1, for some integer q.**

**Sol:**

**To Prove:** the square of any positive integer is of the form **8q + 1 for some integer q.**

**Proof:** Since any positive integer n is of the form 4m + 1 and 4m + 3

$$\text{If } n = m + 1$$

Then,

$$n^2 = (4m + 1)^2$$

$$n^2 = (4m)^2 + 8m + 1$$

$$n^2 = 16m^2 + 8m + 1$$

$$n^2 = 8m (2m + 1) + 1$$

$$n^2 = 8q + 1 \text{ (where } q = m (2m + 1) \text{)}$$

$$\text{If } n = 4m + 3$$

$$\text{Then, } n^2 = (4m + 3)^2$$

$$n^2 = (4m)^2 + 24m + 9$$

$$n^2 = 16m^2 + 24m + 9$$

$$n^2 = 8 (2m^2 + 3m + 1) + 1$$

$$n^2 = 8q + 1 \text{ (where } q = (2m^2 + 3m + 1) \text{)}$$

Hence,  $n^2$  integer is of the form  $8q + 1$ , for some integer  $q$ .

**Q.11: Show that any positive odd integer is of the form  $6q + 1$  or  $6q + 3$  or  $6q + 5$ , where  $q$  is some integer.**

**Sol:**

**To Show:** That any positive odd integer is of the form  $6q + 1$  or  $6q + 3$  or  $6q + 5$  where  $q$  is any some integer.

**Proof:** Let 'a' be any odd positive integer and  $b = 6$ .

Then, there exists integers  $q$  and  $r$  such that  $a = 6q + r$ ,  $0 \leq r < 6$  (by division algorithm)

$a = 6q$  or  $6q + 1$  or  $6q + 2$  or  $6q + 3$  or  $6q + 4$

But  $6q$  or  $6q + 2$  or  $6q + 4$  are even positive integers.

So,  $a = 6q + 1$  or  $6q + 3$  or  $6q + 5$

Hence it is proved that any positive odd integer is of the form  $6q + 1$  or  $6q + 3$  or  $6q + 5$ , where  $q$  is any some integer.

## **Exercise 1.2: Real Numbers**

**Q.1: Define HCF of two positive integers and find the HCF of the following pairs of number:**

**(i) 32 and 54**

**(ii) 18 and 24**

**(iii) 70 and 30**

**(iv) 56 and 88**

**(v) 475 and 495**

**(vi) 75 and 243**

**(vii) 240 and 6552**

**(viii) 155 and 1385**

**(ix) 100 and 190**

**(x) 105 and 120**

**Sol:**

**(i)** We need to find H.C.F. of 32 and 54.

By applying division lemma  $54 = 32 \times 1 + 22$

Since remainder  $\neq 0$ , apply division lemma on 32 and remainder 22

$$32 = 22 \times 1 + 10$$

Since remainder  $\neq 0$ , apply division lemma on 22 and remainder 10

$$22 = 10 \times 2 + 2$$

Since remainder  $\neq 0$ , apply division lemma on 10 and 2

$$10 = 2 \times 5 + 0$$

Therefore, H.C.F. of 32 and 54 is

**(ii)** We need to find H.C.F. of 18 and 24.

By applying division lemma

$$24 = 18 \times 1 + 6.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 18 and remainder 6

$$18 = 6 \times 3 + 0.$$

Therefore, H.C.F. of 18 and 24 is 6

**(iii)** We need to find H.C.F. of 70 and 30.

By applying Euclid's Division lemma

$$70 = 30 \times 2 + 10.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 30 and remainder 10

$$30 = 10 \times 3 + 0.$$

Therefore, H.C.F. of 70 and 30 = 10

**(iv)** We need to find H.C.F. of 56 and 88.

By applying Euclid's Division lemma

$$88 = 56 \times 1 + 32.$$

Since remainder  $\neq 0$ , apply division lemma on 56 and remainder 32

$$56 = 32 \times 1 + 24.$$

Since remainder  $\neq 0$ , apply division lemma on 32 and remainder 24

$$32 = 24 \times 1 + 8.$$

Since remainder  $\neq 0$ , apply division lemma on 24 and remainder 8

$$24 = 8 \times 3 + 0. \text{ Therefore, H.C.F. of 56 and 88} = 8$$

**(v)** We need to find H.C.F. of 475 and 495.

By applying Euclid's Division lemma,

$$495 = 475 \times 1 + 20.$$

Since remainder  $\neq 0$ , apply division lemma on 475 and remainder 20

$$475 = 20 \times 23 + 15.$$

Since remainder  $\neq 0$ , apply division lemma on 20 and remainder 15

$$20 = 15 \times 1 + 5.$$

Since remainder  $\neq 0$ , apply division lemma on 15 and remainder 5

$$15 = 5 \times 3 + 0.$$

Therefore, H.C.F. of 475 and 495 = 5

**(vi)** We need to find H.C.F. of 75 and 243.

By applying Euclid's Division lemma

$$243 = 75 \times 3 + 18.$$

Since remainder  $\neq 0$ , apply division lemma on 75 and remainder 18

$$75 = 18 \times 4 + 3.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 18 and remainder 3

$$18 = 3 \times 6 + 0.$$

Therefore, H.C.F. of 75 and 243 = 3

**(vii)** We need to find H.C.F. of 240 and 6552.

By applying Euclid's Division lemma

$$6552 = 240 \times 27 + 72.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 240 and remainder 72

$$240 = 72 \times 3 + 24.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 72 and remainder 24

$$72 = 24 \times 3 + 0.$$

Therefore, H.C.F. of 240 and 6552 = 24

**(viii)** We need to find H.C.F. of 155 and 1385.

By applying Euclid's Division lemma

$$1385 = 155 \times 8 + 145.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 155 and remainder 145.

$$155 = 145 \times 1 + 10.$$

Since remainder  $\neq 0$  apply division lemma on divisor 145 and remainder 10

$$145 = 10 \times 14 + 5.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 10 and remainder 5

$$10 = 5 \times 2 + 0.$$

Therefore, H.C.F. of 155 and 1385 = 5

**(ix)** We need to find H.C.F. of 100 and 190.

By applying Euclid's division lemma

$$190 = 100 \times 1 + 90.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 100 and remainder 90

$$100 = 90 \times 1 + 10.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 90 and remainder 10

$$90 = 10 \times 9 + 0.$$

Therefore, H.C.F. of 100 and 190 = 10

**(x)** We need to find H.C.F. of 105 and 120.

By applying Euclid's division lemma

$$120 = 105 \times 1 + 15.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 105 and remainder 15

$$105 = 15 \times 7 + 0.$$

Therefore, H.C.F. of 105 and 120 = 15.

**Q.2: Use Euclid's division algorithm to find the HCF of**

**(i) 135 and 225**

**(ii) 196 and 38220**

**(iii) 867 and 255**

**(iv) 184, 230 and 276 [not available]**



**(v) 136, 170 and 255 [not available]**

**Sol.**

(i) Given integers are 225 and 135.

Clearly  $225 > 135$ .

So we will apply Euclid's division lemma to 225 and 135, we get,

$$225 = (135) (1) + 90$$

Since the remainder  $\neq 0$ . So we apply the division lemma to the divisor 135 and remainder 90. We get,

$$135 = (90) (1) + 45$$

Now we apply the division lemma to the new divisor 90 and remainder 45. We get,

$$90 = (45) (2) + 0$$

The remainder at this stage is 0. So the divisor at this stage is the H.C.F.

So, the H.C.F of 225 and 135 is 45

(ii) Given integers are 38220 and 196. Clearly  $38220 > 196$ .

So we will apply Euclid's division lemma to 38220 and 196, we get,

$$38220 = (196) (195) + 0$$

The remainder at this stage is 0. So the divisor at this stage is the H.C.F.

So the H.C.F of 38220 and 196 is 196

(iii) Given integers are 867 and 255. Clearly  $867 > 255$ .

So we will apply Euclid's division lemma to 867 and 255, we get,

$$867 = (255) (3) + 192$$

Since the remainder  $192 \neq 0$ . So we apply the division lemma to the divisor 225 and remainder 192. We get,

$$225 = (192) (1) + 33$$

Now we apply the division lemma to the new divisor 192 and remainder 33. We get,

$$192 = (33) (5) + 27$$

Now we apply the division lemma to the new divisor 33 and remainder 27. We get,

$$33 = (27) (1) + 6$$

Now we apply the division lemma to the new divisor 27 and remainder 6. We get,

$$27 = (6) (4) + 3$$

Now we apply the division lemma to the new divisor 27 and remainder 6. We get,

$$6 = (3) (2) + 0$$

The remainder at this stage is 0. So the divisor at this stage is the H.C.F.

So the H.C.F of 867 and 255 is 3.

**Q.3: Find the HCF of the following pair of integers and express it as a linear combination of them,**

**(i) 963 and 657**

**(ii) 592 and 252**

**(iii) 506 and 1155**

**(iv) 1288 and 575**

**Sol:**

**(i)** We need to find the H.C.F. of 963 and 657 and express it as a linear combination of 963 and 657. By applying Euclid's division lemma,  $963 = 657 \times 1 + 306$ .

Since remainder  $\neq 0$ , apply division lemma on divisor 657 and remainder 306

$$657 = 306 \times 2 + 45.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 306 and remainder 45

$$306 = 45 \times 6 + 36.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 45 and remainder 36

$$45 = 36 \times 1 + 9.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 36 and remainder 9

$$36 = 9 \times 4 + 0.$$

Therefore, H.C.F. = 9.

$$\text{Now, } 9 = 45 - 36 \times 1$$

$$= 45 - [306 - 45 \times 6] \times 1 = 45 - 306 \times 1 + 45 \times 6$$

$$= 45 \times 7 - 306 \times 1 = [657 - 306 \times 2] \times 7 - 306 \times 1$$

$$= 657 \times 7 - 306 \times 14 - 306 \times 1$$

$$= 657 \times 7 - 306 \times 15$$

$$= 657 \times 7 - [963 - 657 \times 1] \times 15$$

$$= 657 \times 7 - 963 \times 15 + 657 \times 15$$

$$= \underline{657 \times 22 - 963 \times 15}.$$

Hence, obtained.

**(ii)** We need to find the H.C.F. of 592 and 252 and express it as a linear combination of 592 and 252.

By applying Euclid's division lemma

$$592 = 252 \times 2 + 88$$

Since remainder  $\neq 0$ , apply division lemma on divisor 252 and remainder 88

$$252 = 88 \times 2 + 76$$

Since remainder  $\neq 0$ , apply division lemma on divisor 88 and remainder 76

$$88 = 76 \times 1 + 12$$

Since remainder  $\neq 0$ , apply division lemma on divisor 76 and remainder 12

$$76 = 12 \times 6 + 4$$

Since remainder  $\neq 0$ , apply division lemma on divisor 12 and remainder 4

$$12 = 4 \times 3 + 0.$$

Therefore, H.C.F. = 4.

$$\text{Now, } 4 = 76 - 12 \times 6$$

$$= 76 - 88 - 76 \times 1 \times 6$$

$$= 76 - 88 \times 6 + 76 \times 6$$

$$= 76 \times 7 - 88 \times 6$$

$$= 252 - 88 \times 2 \times 7 - 88 \times 6$$

$$= 252 \times 7 - 88 \times 14 - 88 \times 6$$

$$= 252 \times 7 - 88 \times 20$$

$$= 252 \times 7 - 592 - 252 \times 2 \times 20$$

$$= 252 \times 7 - 592 \times 20 + 252 \times 40$$

$$= 252 \times 47 - 592 \times 20$$

$$= \underline{252 \times 47 + 592 \times (-20)}$$

Hence obtained.

**(iii)** We need to find the H.C.F. of 506 and 1155 and express it as a linear combination of 506 and 1155. By applying Euclid's division lemma

$$1155 = 506 \times 2 + 143.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 506 and remainder 143

$$506 = 143 \times 3 + 77.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 143 and remainder 77

$$143 = 77 \times 1 + 66.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 77 and remainder 66

$$77 = 66 \times 1 + 11.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 66 and remainder 11

$$66 = 11 \times 6 + 0.$$

Therefore, H.C.F. = 11.

$$\text{Now, } 11 = 77 - 66 \times 1 = 77 - [143 - 77 \times 1] \times 1$$

$$= 77 - 143 \times 1 + 77 \times 1$$

$$= 77 \times 2 - 143 \times 1$$

$$= [506 - 143 \times 3] \times 2 - 143 \times 1$$

$$= 506 \times 2 - 143 \times 6 - 143 \times 1$$

$$= 506 \times 2 - 143 \times 7 = 506 \times 2 - [1155 - 506 \times 2] \times 7 = 506 \times 2 - 1155 \times 7 + 506 \times 14$$

$$= \underline{506 \times 16 - 1155 \times 7}$$

Hence obtained.

**(iv)** We need to find the H.C.F. of 1288 and 575 and express it as a linear combination of 1288 and 575. By applying Euclid's division lemma

$$1288 = 575 \times 2 + 138.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 506 and remainder 143

$$575 = 138 \times 4 + 23.$$

Since remainder  $\neq 0$ , apply division lemma on divisor 143 and remainder 77

$$138 = 23 \times 6 + 0.$$

Therefore, H.C.F. = 23.

$$\text{Now, } 23 = 575 - 138 \times 4 = 575 - [1288 - 575 \times 2] \times 4$$

$$= \underline{575 - 1288 \times 4 + 575 \times 8}$$

Hence, obtained.

**Q.4: Find the largest number which divides 615 and 963 leaving remainder 6 in each case.**

**Sol:**

We need to find the largest number which divides 615 and 963 leaving remainder 6 in each case.

The required number when divides 615 and 963, leaves remainder 6, this means  $615 - 6 = 609$  and  $963 - 6 = 957$  are completely divisible by the number.

Therefore,

The required number = H.C.F. of 609 and 957.

By applying Euclid's division lemma

$$957 = 609 \times 1 + 348$$

$$609 = 348 \times 1 + 261$$

$$348 = 216 \times 1 + 87$$

$$261 = 87 \times 3 + 0.$$

Therefore, H.C.F. = 87.

Hence, the required number is 87

**Q.5: If the HCF of 408 and 1032 is expressible in the form  $1032m - 408 \times 5$ , find m.**

**Sol:**

We need to find m if the H.C.F of 408 and 1032 is expressible in the form  $1032m - 408 \times 5$

Given integers are 408 and 1032 where  $408 < 1032$

By applying Euclid's division lemma, we get  $1032 = 408 \times 2 + 216$ .

Since the remainder  $\neq 0$ , so apply division lemma on divisor 408 and remainder 216

$$408 = 216 \times 1 + 192.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 216 and remainder 192

$$216 = 192 \times 1 + 24.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 192 and remainder 24

$$192 = 24 \times 8 + 0.$$

We observe that remainder is 0. So the last divisor is the H.C.F of 408 and 1032.

Therefore,

$$24 = 1032m - 408 \times 5$$

$$1032m = 24 + 408 \times 5$$

$$1032m = 24 + 2040$$

$$1032m = 2064$$

$$m = \frac{2064}{1032}$$

$$m = 2$$

Therefore,  $m = 2$ .

**Q.6: If the HCF of 657 and 963 is expressible in the form  $657x + 963y - 15$ , find  $x$ .**

**Sol:**

We need to find  $x$  if the H.C.F of 657 and 963 is expressible in the form  $657x + 963y - 15$ .

Given integers are 657 and 963.

By applying Euclid's division lemma, we get,

$$963 = 657 \times 1 + 306.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 657 and remainder 306

$$657 = 306 \times 2 + 45.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 306 and remainder 45

$$306 = 45 \times 6 + 36.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 45 and remainder 36

$$45 = 36 \times 1 + 9.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 36 and remainder 9

$$36 = 9 \times 4 + 0.$$

Therefore, H.C.F. = 9.

$$\text{Given H.C.F} = 657x + 936 (-15).$$

$$\text{Therefore, } 9 = 657x - 14445$$

$$9 + 14445 = 657x$$

$$14454 = 657x$$

$$x = \frac{14454}{657}$$

On solving the above, we have,

$$x = 22.$$

Hence obtained.

**Q.7: An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to in the same number of columns. What is the maximum number of columns in which they can march?**

**Sol.**

We are given that an army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. We need to find the maximum number of columns in which they can march.

$$\text{Members in army} = 616$$

$$\text{Members in band} = 32.$$

Therefore, Maximum number of columns = H.C.F of 616 and 32



By applying Euclid's division lemma

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0.$$

Therefore, H.C.F. = 8

Hence, the maximum number of columns in which they can march is 8

**Q.8: A merchant has 120 liters of oil of one kind, 180 liters of another and 240 liters of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?**

**Sol:**

The merchant has 3 different oils of 120 liters, 180 liters and 240 liters respectively.

So the greatest capacity of the tin for filling three different types of oil is given by the H.C.F. of 120, 180 and 240.

So first we will calculate H.C.F of 120 and 180 by Euclid's division lemma.

$$180 = (120) (1) + 60$$

$$120 = (60) (2) + 0$$

The divisor at the last step is 60. So the H.C.F of 120 and 180 is 60.

Now we will find the H.C.F. of 60 and 240,

$$240 = (60) (4) + 0$$

The divisor at the last step is 60. So the H.C.F of 240 and 60 is 60.

Therefore, the tin should be of 60 liters.

**Q.9: During a sale, color pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?**

**Sol:**

We are given that during a sale, color pencils were being sold in packs of 24 each and crayons in packs of 32 each. If we want full packs of both and the same number of pencils and crayons, we need to find the number of each we need to buy.

Given that, Number of color pencils in one pack = 24

Number of crayons in pack = 32.

Therefore, the least number of both colors to be purchased

L.C.M of 24 and 32 =  $2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$

Hence, the number of packs of pencils to be bought

$$96 \div 24 = 4 \frac{96}{24} = 4,$$

And number of packs of crayon to be bought

$$96 \div 32 = 3 \frac{96}{32} = 3$$

**Q.10: 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and is to contain cartons of same drink, what would be the greatest number of cartons each stack would have?**

**Sol:**

Given that 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and contains cartons of the same drink We need to find the greatest number of cartons, each stack would have

Given that,

Number of cartons of coke cans = 144

Number of cartons of Pepsi cans = 90.

Therefore, the greatest number of cartons in one stack = H.C.F. of 144 and 90.

By applying Euclid's division lemma  $144 = 90 \times 1 + 54$

$$90 = 54 \times 1 + 36$$

$$54 = 36 \times 1 + 18$$

$$36 = 18 \times 2 + 0$$

$$\text{H.C.F.} = 18.$$

Hence, the greatest number cartons in one stack 18

**Q.11: Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.**

**Sol:**

We need to find the greatest number which divides 285 and 1249 leaving remainder 9 and 7 respectively.

The required number when divides 285 and 1249, leaves remainder 9 and 7, this means

$285 - 9 = 276$  and  $1249 - 7 = 1242$  are completely divisible by the number.

Therefore, the required number = H.C.F. of 276 and 1242.

By applying Euclid's division lemma,

$$1242 = 276 \times 4 + 138$$

$$276 = 138 \times 2 + 0.$$

$$\text{Therefore, H.C.F.} = 138$$

Hence, required number is 138

**Q.12: Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.**

**Sol:**

We need to find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.

The required number when divides 280 and 1245, leaves remainder 4 and 3, this means

$280 - 4 = 276$  and  $1245 - 3 = 1242$  are completely divisible by the number.

Therefore, the required number = H.C.F. of 276 and 1242.

By applying Euclid's division lemma  $1242 = 276 \times 4 + 138$

$276 = 138 \times 2 + 0$ .

Therefore, H.C.F. = 138.

Hence, the required number is 138

**Q.13: What is the largest number which that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively?**

**Sol:**

We need to find the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.

The required number when divides 626, 3127 and 15628 leaves remainders 1, 2 and 3 this means

$626 - 1 = 625$ ,

$3127 - 2 = 3125$ ,

And  $15628 - 3 = 15625$  are completely divisible by the number.

Therefore, the required number = H.C.F. of 625, 3125 and 15625.

First we consider 625 and 3125.

By applying Euclid's division lemma

$3125 = 625 \times 5 + 0$ .

H.C.F. of 625 and 3125 = 625

Now, consider 625 and 15625.

By applying Euclid's division lemma  $15625 = 625 \times 25 + 0$ .

Therefore, H.C.F. of 625, 3125 and 15625 = 625

Hence, the required number is 625

**Q.14: Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.**

**Sol:**

To find the greatest number that divides 445, 572 and 699 and leaves remainders of 4, 5 and 6 respectively. The required number when divides 445, 572 and 699 leaves remainders 4, 5 and 6 this means

$445 - 4 = 441$ ,  $572 - 5 = 567$  and  $699 - 6 = 693$  are completely divisible by the number.

Therefore, the required number = H.C.F. of 441, 567 and 693.

First consider 441 and 567.

By applying Euclid's division lemma

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0.$$

Therefore, H.C.F. of 441 and 567 = 63

Now, consider 63 and 693

By applying Euclid's division lemma

$$693 = 63 \times 11 + 0.$$

Therefore, H.C.F. of 441, 567 and 693 = 63

Hence, the required number is 63

**Q.15: Find the greatest number which divides 2011 and 2623 leaving remainders 9 and 5 respectively.**

**Sol:**

To find the greatest number which divides 2011 and 2623 leaving remainder 9 and 5 respectively.

The required number when divides 2011 and 2623 leaves remainders 9 and 5 this means

$2011 - 9 = 2002$  and  $2623 - 5 = 2618$  are completely divisible by the number.

Therefore, the required number = H.C.F. of 2002 and 2618

By applying Euclid's division lemma

$$2618 = 2002 \times 1 + 616$$

$$2002 = 616 \times 3 + 154$$

$$616 = 154 \times 4 + 0.$$

$$\text{H.C.F. of } 2002 \text{ and } 2618 = 154$$

Hence, the required number is 154

**Q.16: Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?**

**Sol:**

We are given that two brands of chocolates are available in packs of 24 and 15 respectively. If lie needs to buy an equal number of chocolates of both kinds, then find least number of boxes of each kind he would need to buy.

Given that,

Number of chocolates of 1st brand in one pack = 24

Number of chocolates of 2nd brand in one pack = 15.

Therefore, the least number of chocolates he need to purchase is

$$\text{L.C.M. of 24 and 15} = 2 \times 2 \times 2 \times 3 \times 5 = 120$$

Therefore, the number of packet of 1<sup>st</sup> brand is

$$120 \div 24 = 5,$$

And the number of packet of 2<sup>nd</sup> brand is

$$120 \div 15 = 8$$

**Q.17: A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10ft by 8ft. what would be the size in inches of the tile required that has to be cut and how many such tiles are required?**

**Sol:**

**Given:**

Size of bathroom = 10 ft by 8 ft

= (10 x 12) inch by (8 x 12) inch

= 120 inch by 96 inch

The largest size of tile required = HCF of 120 and 96

By applying Euclid's division lemma

$$120 = 96 \times 1 + 24$$

$$96 = 24 \times 4 + 0$$

Therefore, HCF = 24

Therefore, Largest size of tile required = 24 inches

$$\text{no. of tiles required} = \frac{\text{area of bathroom}}{\text{area of 1 tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20 \text{ tiles}$$

$$\text{no. of tiles required} = \frac{\text{area of bathroom}}{\text{area of 1 tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20 \text{ tiles}$$

**Q.18: 15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuits packets in each. How many biscuit packets and how many pastries will each box contain?**

**Sol:**

**Given:**

Number of pastries = 15

Number of biscuit packets = 12

Therefore, the required no of boxes to contain equal number = HCF of 15 and 12

By applying Euclid's division lemma  $15 = 12 \times 1 + 3$

$12 = 2 \times 3 + 0$

Therefore, No. of boxes required = 3

Hence each box will contain  $15 \div 3 = 5$  pastries and  $12 \div 3 = 4$  biscuit packs.

**Q.19: 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?**

**Sol:**

**Given:**

Number of goats = 105

Number of donkey = 140

Number of cows = 175



Therefore, The largest number of animals in one trip = HCF of 105, 140 and 175.

First consider 105 and 140

By applying Euclid's division lemma

$$140 = 105 \times 1 + 35$$

$$105 = 35 \times 3 + 0$$

Therefore, HCF of 105 and 140 = 35

Now consider 35 and 175

By applying Euclid's division lemma

$$175 = 35 \times 5 + 0$$

HCF of 105, 140 and 175 = 35

**Q.20: The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.**

**Sol:**

Length of room = 8m 25 cm = 825 cm

Breadth of room = 6m 75cm = 675 cm

Height of room = 4m 50cm = 450 cm

The required longest rod = HCF of 825, 675 and 450

First consider 675 and 450

By applying Euclid's division lemma

$$675 = 450 \times 1 + 225$$

$$450 = 225 \times 2 + 0$$

Therefore, HCF of 675 and 450 = 225

Now consider 625 and 825

By applying Euclid's division Lemma:

$$825 = 225 \times 3 + 150$$

$$225 = 150 \times 1 + 75$$

$$150 = 75 \times 2 + 0$$

Therefore, HCF of 825, 675 and 450 = 75

**Q.21: Express the HCF of 468 and 222 as  $468x + 222y$  where  $x, y$  are integers in two different ways.**

**Sol:**

We need to express the H.C.F. of 468 and 222 as  $468x + 222y$

Where  $x, y$  are integers in two different ways.

Given integers are 468 and 222, where  $468 > 222$

By applying Euclid's division lemma, we get  $468 = 222 \times 2 + 24$ .

Since the remainder  $\neq 0$ , so apply division lemma on divisor 222 and remainder 24

$$222 = 24 \times 9 + 6.$$

Since the remainder  $\neq 0$ , so apply division lemma on divisor 24 and remainder 6

$$24 = 6 \times 4 + 0.$$

We observe that remainder is 0. So the last divisor 6 is the H.C.F. of 468 and 222 from we have

$$6 = 222 - 24 \times 9$$

$$6 = 222 - (468 - 222 \times 2) \times 9$$

$$6 = 222 - 468 \times 9 + 222 \times 18$$

$$6 = 222 \times 19 - 468 \times 9 \text{ [Substituting } 24 = 468 - 222 \times 2]$$

$$6 = 222y + 468x, \text{ where } x = -9 \text{ and } y = 19.$$

Hence, obtained.

## Exercise 1.3: Real Numbers

**Q.1:** Express each of the following integers as a product of its prime.

**1. 420**

**2. 468**

**3. 945**

**4. 7325**

**Sol:**

**To express:** each of the following numbers as a product of their prime factors

**1. 420**

$$420 = 2 \times 2 \times 3 \times 5 \times 7$$

**2. 468**

$$468 = 2 \times 2 \times 3 \times 3 \times 13$$

**3. 945**

$$945 = 3 \times 3 \times 3 \times 5 \times 7$$

**4. 7325**

$$7325 = 5 \times 5 \times 293$$

**Q.2: Determine the prime factorization of each of the following positive integer :**

**1. 20570**

**2. 58500**

**3. 45470971**

**Sol:**

**TO EXPRESS:** each of the following numbers as a product of their prime factors.

**1. 20570**

$$20570 = 2 \times 5 \times 11 \times 11 \times 17$$

**2. 58500**

$$58500 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 13$$

**3. 45470971**

$$45470971 = 7 \times 7 \times 13 \times 13 \times 17 \times 17 \times 19$$

**Q.3: Explain why  $7 \times 11 \times 13 + 13$  and  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$  are composite numbers.**

**Sol:**

**Explanation:**

Why  $7 \times 11 \times 13 + 13$  and  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$  are composite numbers.

We can see that both the numbers have common factor 7 and 1.

$$7 \times 11 \times 13 + 13 = (77 + 1) \times 13 = 78 \times 13$$

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = (7 \times 6 \times 4 \times 3 \times 2 + 1) \times 5 = 1008 \times 5$$

And we know that composite numbers are those numbers which have at least one more factor other than 1.

Hence after simplification we see that both numbers are even and therefore the given two numbers are composite numbers

**Q.4: Check whether  $6^n$  can end with the digit 0 for any natural number n.**

**Sol:**

**TO CHECK:** Whether  $6^n$  can end with the digit 0 for any natural number n.

We know that  $6^n = (2 \times 3)^n$

$$6^n = 2^n \times 3^n$$

Therefore, prime factorization of  $6^n$  does not contain 5 and 2 as a factor together. Hence  $6^n$  can never end with the digit 0 for any natural number n.

## **Exercise 1.4: Real Numbers**

**Q.1: Find the LCM and HCF of the following pairs of integers and verify that  $\text{LCM} \times \text{HCF} = \text{product of the integers}$ :**

**(i) 26 and 91**

**(ii) 510 and 92**

**(iii) 336 and 54**

**Sol:**

**TO FIND:** LCM and HCF of following pairs of integers

**TO VERIFY:** L.C.M  $\times$  H.C.F = product of the numbers

**(i) 26 and 91**

Let us first find the factors of 26 and 91

$$26 = 2 \times 13$$

$$91 = 7 \times 13$$

L.C.M of 26, and 91 =  $2 \times 7 \times 13$

L.C.M of 26, and 91 = 182

H.C.F of 26, and 91 = 13

We know that, L.C.M x H.C.F = First number x Second number

$$182 \times 13 = 26 \times 91 = 2366 = 2366$$

Hence verified

**(ii) 510 and 92**

Let us first find the factors of 510 and 92

$$510 = 2 \times 3 \times 5 \times 17$$

$$92 = 2 \times 2 \times 23$$

$$\text{L.C.M of 510 and 92} = 2 \times 2 \times 3 \times 5 \times 23 \times 17$$

$$\text{L.C.M of 510 and 92} = 23460$$

$$\text{H.C.F of 510 and 92} = 2$$

We know that, L.C.M x H.C.F = First Number x Second Number

$$23460 \times 2 = 510 \times 92$$

$$46920 = 46920$$

Hence verified.

**(iii) 336 and 54**

Let us first find the factors of 336 and 54

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$$

$$54 = 2 \times 3 \times 3 \times 3$$

$$\text{L.C.M of 336 and 54} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$$



L.C.M of 336 and 54 = 3024

H.C.F of 336 and 54 = 6

We know that, L.C.M x H.C.F = First Number x Second Number

$$3024 \times 6 = 336 \times 54$$

$$18144 = 18144$$

Hence verified

**Q.2: Find the LCM and HCF of the following integers by applying the prime factorization method:**

**(i) 12, 15 and 21**

**(ii) 17, 23 and 29**

**(iii) 8, 9 and 25**

**(iv) 40, 36 and 126**

**(v) 84, 90 and 120**

**(vi) 24, 15 and 36**

**Sol:**

**TO FIND:** LCM and HCF of following pairs of integers

**(i) 15, 12 and 21**

Let us first find the factors of 15, 12 and 21

$$12 = 2 \times 2 \times 3$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\text{L.C.M of 12, 15 and 21} = 2 \times 2 \times 3 \times 5 \times 7$$

$$\text{L.C.M of 12, 15 and 21} = 420$$

H.C.F of 12, 15 and 21 = 3

**(ii)** 17, 23 and 29

Let us first find the factors of 17, 23 and 29

$$17 = 1 \times 17$$

$$23 = 1 \times 23$$

$$29 = 1 \times 29$$

$$\text{L.C.M of 17, 23 and 29} = 1 \times 17 \times 23 \times 29$$

$$\text{L.C.M of 17, 23 and 29} = 11339$$

$$\text{H.C.F of 17, 23 and 29} = 1$$

**(iii)** 8, 9 and 25

Let us first find the factors of 8, 9 and 25

$$8 = 2 \times 2 \times 2$$

$$9 = 3 \times 3$$

$$25 = 5 \times 5$$

$$\text{L.C.M of 8, 9 and 25} = 2^3 \times 3^2 \times 5^2$$

$$\text{L.C.M of 8, 9 and 25} = 1800$$

$$\text{H.C.F of 8, 9 and 25} = 1$$

**(iv)** 40, 36 and 126

Let us first find the factors of 40, 36 and 126

$$40 = 2^3 \times 5$$

$$36 = 2^2 \times 3^2$$

$$126 = 2 \times 3 \times 3 \times 7$$

$$\text{L.C.M of } 40, 36 \text{ and } 126 = 2^3 \times 3^2 \times 5 \times 7$$

$$\text{L.C.M of } 40, 36 \text{ and } 126 = 2520$$

$$\text{H.C.F of } 40, 36 \text{ and } 126 = 2$$

**(v) 84, 90 and 120**

Let us first find the factors of 84, 90 and 120

$$84 = 2 \times 2 \times 3 \times 7$$

$$90 = 2 \times 3 \times 3 \times 5$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{L.C.M of } 84, 90 \text{ and } 120 = 2^3 \times 3^2 \times 5 \times 7$$

$$\text{L.C.M of } 84, 90 \text{ and } 120 = 2520$$

$$\text{H.C.F of } 84, 90 \text{ and } 120 = 6$$

**(vi) 24, 15 and 36**

Let us first find the factors of 24, 15 and 36

$$24 = 2^3 \times 3$$

$$15 = 3 \times 5$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$\text{LCM of } 24, 15 \text{ and } 36 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

$$\text{LCM of } 24, 15 \text{ and } 36 = 360$$

$$\text{HCF of } 24, 15 \text{ and } 36 = 3$$

**Q.3: Given that HCF (306, 657) = 9 , find LCM ( 306, 657 )**

**Sol:**

**Given:**

HCF of two numbers 306 and 657 is 9

**To find:** LCM of number

We know that,

LCM x HCF = first number x second number

LCM x 9 = 306 x 657

$$LCM = \frac{306 \times 657}{9} = 22338$$

**Q.4: Can two numbers have 16 as their HCF and 380 as their as their LCM? Give reason.**

**Sol:**

**To find:** can two numbers have 16 as their HCF and 380 as their LCM

On dividing 380 by 16 we get 23 as the quotient and 12 as the remainder

Since LCM is not exactly divisible by the HCF, two number cannot have 16 as their HCF and 380 as their LCM

**Q.5: The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.**

**Sol:**

**Given:**

LCM and HCF of two numbers 145 and 2175 respectively.

If one number is 725

To find: other number

We know that,

LCM x HCF = first number x second number

$2175 \times 145 = 725 \times \text{second number}$

$$LCM = \frac{2175 \times 145}{725} = 435$$

**Q.6: The HCF of two numbers is 16 and their LCM is 3072. Find the LCM.**

**Sol:**

**Given:** HCF of two numbers is 16. If the product of the numbers is 3072

**To find:** LCM of number

We know that,

LCM x HCF = first number x second number

$LCM \times 16 = 3072$

$$LCM = \frac{3072}{16} = 192$$

**Q.7: The LCM and HCF of two numbers are 180 and 6 respectively. If one of the numbers is 30, find the other number.**

**Sol:**

**Given:**

LCM and HCF of two numbers 180 and 6 respectively.

If one number is 30

To find: other number

We know that,

LCM x HCF = first number x second number

$180 \times 6 = 30 \times \text{second number}$

$$\text{second number} = \frac{180 \times 6}{30} = 36$$

**Q.8: Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.**

**Sol:**

**TO FIND:** Smallest number which when increased by 17 is exactly divisible by both 520 and 468. L.C.M OF 520 and 468

$$520 = 2^3 \times 5 \times 13$$

$$468 = 2^2 \times 3^2 \times 13$$

$$\text{LCM of 520 and 468} = 2^3 \times 3^2 \times 5 \times 13 = 4680$$

Hence 4680 is the least number which exactly divides 520 and 468 i.e. we will get a remainder of 0 in this case. But we need the Smallest number which when increased by 17 is exactly divided by 520 and 468.

$$\text{Therefore} = 4680 - 17 = 4663$$

Hence 4663 is Smallest number which when increased by 17 is exactly divisible by both 520 and 468.

**Q.9: Find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.**

**Sol:**

**TO FIND:** The smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

L.C.M of 28 and 32.

$$28 = 2 \times 2 \times 7$$

$$32 = 2^5$$

$$\text{L.C.M of 28, and 32} = 2^5 \times 7 = 224$$

Hence 224 is the least number which exactly divides 28 and 32 i.e. we will get a remainder of 0 in this case. But we need the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

$$\text{Therefore} = 224 - 8 - 12 = 204$$

Hence 2041 is the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively

**Q.10: What is the smallest number that, when divided by 35, 56 and 91 leaves remainders of 7 in each case?**

**Sol:**

**TO FIND:** Smallest number that. When divided by 35, 56 and 91 leaves remainder of 7 in each case L.C.M OF 35, 56 and 91

$$35 = 5 \times 7$$

$$56 = 2^3 \times 7$$

$$91 = 13 \times 7$$

$$\text{L.C.M of 35, 56 and 91} = 2^3 \times 7 \times 5 \times 13 = 3640$$

Hence 84 is the least number which exactly divides 28, 42 and 84 i.e. we will get a remainder of 0 in this case. But we need the smallest number that, when divided by 35, 56 and 91 leaves remainder of 7 in each case.

$$\text{Therefore} = 3640 + 7 = 3647$$

Hence 36471 is smallest number that, when divided by 35, 56 and 91 leaves remainder of 7 in each case.

**Q.11: A rectangular courtyard is 18m 72cm long and 13m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.**

**Sol:**

**GIVEN:** A rectangular yard is 18 m 72 cm long and 13 m 20 cm broad .It is to be paved with square tiles of the same size.

**TO FIND:** Least possible number of such tiles.

Length of the yard = 18 m 72 cm = 1800 cm + 72 cm = 1872 cm (therefore, 1 m = 100 cm)

Breadth of the yard = 13 in 20 cm = 1300 cm + 20 cm = 1320 cm

The size of the square tile of same size needed to the pave the rectangular yard is equals the HCF of the length and breadth of the rectangular yard.

Prime factorization of 1872 =  $2^4 \times 3^2 \times 13$

Prime factorization of 1320 =  $2^3 \times 3 \times 5 \times 11$

HCF of 1872 and 1320 =  $2^3 \times 3 = 24$

Therefore, Length of side of the square tile = 24 cm

Number of tiles required = Area of the courtyard, Area of each tile = Length x Breadth

$\text{Side}^2 = 1872 \text{ cm} \times 1320 \text{ cm} \div 24 \text{ cm} = 101280$

Thus, the least possible number of tiles required is 101280.

**Q.12: Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.**

**Sol:**

**TO FIND:** Greatest number of 6 digits exactly divisible by 24, 15 and 36



The greatest 6 digit number be 999999 24, 15 and 36

$$24 = 2 \times 2 \times 2 \times 3$$

$$15 = 3 \times 5$$

$$36 = 2 \times 2 \times 3 \times 3$$

L.C.M of 24, 15 and 36 = 360

$$\text{Since, } 999999 \div 360 = 2777 \times 360 + 279 \text{ Since, } \frac{999999}{360} = 2777 \times 360 + 279$$

Therefore, the remainder is 279. Hence the desired number is =  $999999 - 279 = 9997201$

Hence 9997201 is the greatest number of 6 digits exactly divisible by 24, 15 and 36.

**Q.13: Determine the number nearest to 110000 but greater 100000 which is exactly divisible by each of 8, 15 and 21.**

**Sol:**

**TO FIND:** The number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

L.C.M of 8, 15 and 21.

$$8 = 2 \times 2 \times 2$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\text{L.C.M of 8, 15 and 21} = 2^3 \times 3 \times 5 \times 7 = 840$$

When 110000 is divided by 840, the remainder is obtained as 800.

Now,  $110000 - 800 = 109200$  is divisible by each of 8, 15 and 21.

Also,  $110000 + 40 = 110040$  is divisible by each of 8, 15 and 21.

109200 and 110040 are greater than 100000.

Hence, 110040 is the number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

**Q.14: Find the least number that is divisible by all the numbers between 1 and 10 (both inclusive)**

**Sol:**

**TO FIND:** Least number that is divisible by all the numbers between 1 and 10 (both inclusive)  
Let us first find the L.C.M of all the numbers between 1 and 10 (both inclusive)

$$1 = 1$$

$$2 = 2$$

$$3 = 3$$

$$4 = 2 \times 2$$

$$5 = 5$$

$$6 = 2 \times 3$$

$$7 = 7$$

$$8 = 2 \times 2 \times 2$$

$$9 = 3 \times 3$$

$$10 = 2 \times 5$$

L.C.M 2520

Hence 2520 is the least number that is divisible by all the numbers between 1 and 10 (both inclusive)

**Q.15: A circular field has a circumference of 360km. three cyclists start together and can cycle 48, 60 and 72 km a day, round the field. When will they meet again?**

**Sol:**

**GIVEN:** A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60, and 72 km a day, round the field.

**TO FIND:** When they meet again.

In order to calculate the time when they meet, we first find out the time taken by each cyclist in covering the distance.

Number of days 1<sup>st</sup> cyclist took to cover 360 km =  $\frac{\text{Total distance covered in 1 day}}{48} = \frac{360}{48} = 7.5$   
 $= 7\frac{1}{2} = 15\frac{1}{2}$  days

Similarly, number of days taken by 2<sup>nd</sup> cyclist to cover same distance =  $\frac{360}{60} = 6$  days

Also, number of days taken by 3<sup>rd</sup> cyclist to cover this distance =  $\frac{360}{72} = 5$  days

Now, LCM of 15 $\frac{1}{2}$ , 6 and 5 = LCM of numerators

HCF of denominators = 301 = 30 days

Thus, all of them will take 30 days to meet again.

**Q.16: In a morning walk three persons step off together, their steps measure 80cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?**

**Sol:**

**GIVEN:** In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm.

**TO FIND:** minimum distance each should walk so that all can cover the same distance in complete steps.

The distance covered by each of them is required to be same as well as minimum. The required distance each should walk would be the L.C.M of the measures of their steps i.e. 80 cm, 85 cm, and 90 cm,

So we have to find the L.C.M of 80 cm, 85 cm, and 90 cm.

$$80 = 2^4 \times 5$$

$$85 = 17 \times 5$$

$$90 = 2 \times 3 \times 3 \times 5$$

$$\text{L.C.M of 80, 85 and 90} = 2^4 \times 3 \times 3 \times 5 \times 17 = 12240 \text{ cm}$$

Hence minimum 12240 cm distance each should walk so that all can cove the same distance in complete steps.

## Exercise 1.5: Real Numbers

**Q.1: Show that the following numbers are irrational.**

(i)  $7\sqrt{57}\sqrt{5}$

Let us assume that  $7\sqrt{57}\sqrt{5}$  is rational. Then, there exist positive co primes a and b such that

$$7\sqrt{57}\sqrt{5} = ab \frac{a}{b}$$

$$\sqrt{57}\sqrt{5} = a^7b \frac{a}{7b}$$

We know that  $\sqrt{57}\sqrt{5}$  is an irrational number

Here we see that  $\sqrt{57}\sqrt{5}$  is a rational number which is a contradiction.

(ii)  $6+\sqrt{26} + \sqrt{2}$

Let us assume that  $6+\sqrt{26} + \sqrt{2}$  is rational. Then, there exist positive co primes a and b such that

$$6+\sqrt{26} + \sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 6\frac{a}{b} - 6$$

$$\sqrt{2}\sqrt{2} = a - 6bb \frac{a-6b}{b}$$

Here we see that  $\sqrt{2}\sqrt{2}$  is a rational number which is a contradiction as we know that  $\sqrt{2}\sqrt{2}$  is an irrational number

Hence  $6 + \sqrt{2}6 + \sqrt{2}$  is an irrational number

(iii)  $3 - \sqrt{5}3 - \sqrt{5}$

Let us assume that  $3 - \sqrt{5}3 - \sqrt{5}$  is rational. Then, there exist positive co primes a and b such that

$$3 - \sqrt{5}3 - \sqrt{5} = ab \frac{a}{b}$$

$$\sqrt{5}\sqrt{5} = 3 - ab3 - \frac{a}{b}$$

$$\sqrt{5}\sqrt{5} = 3b - ab \frac{3b-a}{b}$$

Here we see that  $\sqrt{5}\sqrt{5}$  is a rational number which is a contradiction as we know that  $\sqrt{5}\sqrt{5}$  is an irrational number

Hence  $3 - \sqrt{5}3 - \sqrt{5}$  is an irrational number.

**Q.2: Prove that the following numbers are irrationals.**

**Sol:**

(i)  $2\sqrt{7} \frac{2}{\sqrt{7}}$

Let us assume that  $2\sqrt{7}2\sqrt{7}$  is rational. Then, there exist positive co primes a and b such that

$$2\sqrt{7}2\sqrt{7} = ab \frac{a}{b}$$

$$\sqrt{7}\sqrt{7} = 2ba \frac{2b}{a}$$

$\sqrt{7}\sqrt{7}$  is rational number which is a contradiction

Hence  $2\sqrt{7}2\sqrt{7}$  is an irrational number

**(ii)  $32\sqrt{5} \frac{3}{2\sqrt{5}}$**

Let us assume that  $32\sqrt{5} \frac{3}{2\sqrt{5}}$  is rational. Then, there exist positive co primes a and b such that

$$32\sqrt{5} \frac{3}{2\sqrt{5}} = ab \frac{a}{b}$$

$$\sqrt{5}\sqrt{5} = 3b2a \frac{3b}{2a}$$

$\sqrt{5}\sqrt{5}$  is rational number which is a contradiction

Hence  $32\sqrt{5} \frac{3}{2\sqrt{5}}$  is irrational.

**(iii)  $4+\sqrt{2}4 + \sqrt{2}$**

Let us assume that  $4+\sqrt{2}4 + \sqrt{2}$  is rational. Then, there exist positive co primes a and b such that

$$4+\sqrt{2}4 + \sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 4 \frac{a}{b} - 4$$

$$\sqrt{2}\sqrt{2} = a-4bb \frac{a-4b}{b}$$

$\sqrt{2}\sqrt{2}$  is rational number which is a contradiction

Hence  $4 + \sqrt{24} + \sqrt{2}$  is irrational.

(iv)  $5\sqrt{25}\sqrt{2}$

Let us assume that  $5\sqrt{25}\sqrt{2}$  is rational. Then, there exist positive co primes a and b such that

$$5\sqrt{25}\sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 5\frac{a}{b} - 5$$

$$\sqrt{2}\sqrt{2} = a - 5bb \frac{a - 5b}{b}$$

$\sqrt{2}\sqrt{2}$  is rational number which is a contradiction

Hence  $5\sqrt{25}\sqrt{2}$  is irrational

**Q.3: Show that  $2 - \sqrt{32} - \sqrt{3}$  is an irrational number.**

**Sol:**

Let us assume that  $2 - \sqrt{32} - \sqrt{3}$  is rational. Then, there exist positive co primes a and b such that

$$2 - \sqrt{32} - \sqrt{3} = ab \frac{a}{b}$$

$$\sqrt{3}\sqrt{3} = 2 - ab - \frac{a}{b}$$

Here we see that  $\sqrt{3}\sqrt{3}$  is a rational number which is a contradiction

Hence  $2 - \sqrt{32} - \sqrt{3}$  is irrational

**Q.4: Show that  $3 + \sqrt{23} + \sqrt{2}$  is an irrational number.**



**Sol:**

Let us assume that  $3 + \sqrt{23} + \sqrt{2}$  is rational. Then, there exist positive co primes a and b such that

$$3 + \sqrt{23} + \sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 3\frac{a}{b} - 3$$

$$\sqrt{2}\sqrt{2} = a - 3b \frac{a - 3b}{b}$$

Here we see that  $\sqrt{2}\sqrt{2}$  is a irrational number which is a contradiction

Hence  $3 + \sqrt{23} + \sqrt{2}$  is irrational

**Q.5: Prove that  $4 - 5\sqrt{24} - 5\sqrt{2}$  is an irrational number.**

**Sol:**

Let us assume that  $4 - 5\sqrt{24} - 5\sqrt{2}$  is rational. Then, there exist positive co primes a and b such that

$$4 - 5\sqrt{24} - 5\sqrt{2} = ab \frac{a}{b}$$

$$5\sqrt{2}5\sqrt{2} = ab - 4\frac{a}{b} - 4$$

$$\sqrt{2} = ab - 4\frac{a}{b} - 4 = \frac{\frac{a}{b} - 4}{5}$$

$$\sqrt{2}\sqrt{2} = a - 4b \frac{a - 4b}{5b}$$

This contradicts the fact that  $\sqrt{2}\sqrt{2}$  is an irrational number

Hence  $4 - 5\sqrt{24} - 5\sqrt{2}$  is irrational

**Q.6: Show that  $5-2\sqrt{35} - 2\sqrt{3}$  is an irrational number.**

**Sol.**

Let us assume that  $5-2\sqrt{35} - 2\sqrt{3}$  is rational. Then, there exist positive co primes a and b such that

$$5-2\sqrt{35} - 2\sqrt{3} = ab \frac{a}{b}$$

$$2\sqrt{3}2\sqrt{3} = ab - 5\frac{a}{b} - 5$$

$$\sqrt{3} = ab - 52\sqrt{3} = \frac{\frac{a}{b} - 5}{2} \sqrt{3} = a - 5b2b\sqrt{3} = \frac{a - 5b}{2b}$$

This contradicts the fact that  $\sqrt{3}\sqrt{3}$  is an irrational number

Hence  $5-2\sqrt{35} - 2\sqrt{3}$  is irrational

**Q.7: Prove that  $2\sqrt{3}-12\sqrt{3} - 1$  is an irrational number.**

**Sol:**

Let us assume that  $2\sqrt{3}-12\sqrt{3} - 1$  is rational. Then, there exist positive co primes a and b such that

$$2\sqrt{3}-12\sqrt{3} - 1 = ab \frac{a}{b}$$

$$2\sqrt{3}2\sqrt{3} = ab + 1\frac{a}{b} + 1$$

$$\sqrt{3} = ab + 12\sqrt{3} = \frac{\frac{a}{b} + 1}{2} \sqrt{3} = a + b2b\sqrt{3} = \frac{a + b}{2b}$$

This contradicts the fact that  $\sqrt{3}\sqrt{3}$  is an irrational number

Hence  $5-2\sqrt{35} - 2\sqrt{3}$  is irrational

**Q.8: Prove that  $2-3\sqrt{5}2 - 3\sqrt{5}$  is an irrational number.**

**Sol:**

Let us assume that  $2-3\sqrt{5}2 - 3\sqrt{5}$  is rational. Then, there exist positive co primes a and b such that

$$2-3\sqrt{5}2 - 3\sqrt{5} = ab \frac{a}{b}$$

$$3\sqrt{5}3\sqrt{5} = ab - 2\frac{a}{b} - 2$$

$$3\sqrt{5} = ab - 23\sqrt{5} = \frac{\frac{a}{b} - 2}{3} \sqrt{5} = a - 3b3b \sqrt{5} = \frac{a-3b}{3b}$$

This contradicts the fact that  $\sqrt{5}\sqrt{5}$  is an irrational number

Hence  $2-3\sqrt{5}2 - 3\sqrt{5}$  is irrational

**Q.9: Prove that  $\sqrt{5}+\sqrt{3}\sqrt{5} + \sqrt{3}$  is irrational.**

**Sol:**

Let us assume that  $\sqrt{5}+\sqrt{3}\sqrt{5} + \sqrt{3}$  is rational. Then, there exist positive co primes a and b such that

$$\sqrt{5}+\sqrt{3}\sqrt{5} + \sqrt{3} = ab \frac{a}{b}$$

$$\sqrt{5} = ab - \sqrt{3}\sqrt{5} = \frac{a}{b} - \sqrt{3} (\sqrt{5})^2 = (ab - \sqrt{3})^3 (\sqrt{5})^2 = \left(\frac{a}{b} - \sqrt{3}\right)^3 5 = (ab)^2 - 2a\sqrt{3}b + 3$$

$$5 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} + 3 \Rightarrow 5-3 = (ab)^2 - 2a\sqrt{3}b \Rightarrow 5-3 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow 2 = (ab)^2 - 2a\sqrt{3}b$$

$$\Rightarrow 2 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow (ab)^2 - 2 = 2a\sqrt{3}b \Rightarrow \left(\frac{a}{b}\right)^2 - 2 = \frac{2a\sqrt{3}}{b} \Rightarrow a^2 - 2b^2b^2 = 2a\sqrt{3}b$$

$$\Rightarrow \frac{a^2 - 2b^2}{b^2} = \frac{2a\sqrt{3}}{b} \Rightarrow (a^2 - 2b^2b^2)(b^2a) = \sqrt{3} \Rightarrow \left(\frac{a^2 - 2b^2}{b^2}\right) \left(\frac{b}{2a}\right) = \sqrt{3} \Rightarrow (a^2 - 2b^22ab) = \sqrt{3}$$

$$\Rightarrow \left(\frac{a^2 - 2b^2}{2ab}\right) = \sqrt{3}$$

Here we see that  $\sqrt{3}\sqrt{3}$  is a rational number which is a contradiction as we know that  $\sqrt{3}\sqrt{3}$  is an irrational number

Hence  $\sqrt{5}+\sqrt{3}\sqrt{5} + \sqrt{3}$  is an irrational number

**Q.10: Prove that  $\sqrt{3}+\sqrt{4}\sqrt{3} + \sqrt{4}$  is irrational.**

**Sol:**

Let us assume that  $\sqrt{3}+\sqrt{4}\sqrt{3} + \sqrt{4}$  is rational. Then, there exist positive co primes a and b such that

$$\sqrt{3}+\sqrt{4}\sqrt{3} + \sqrt{4} = ab \frac{a}{b}$$

$$\sqrt{4} = ab - \sqrt{3}\sqrt{4} = \frac{a}{b} - \sqrt{3} (\sqrt{4})^2 = (ab - \sqrt{3})^3 (\sqrt{4})^2 = \left(\frac{a}{b} - \sqrt{3}\right)^3 4 = (ab)^2 - 2a\sqrt{3}b + 3$$

$$4 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} + 3 \Rightarrow 4-3 = (ab)^2 - 2a\sqrt{3}b \Rightarrow 4-3 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow 1 = (ab)^2 - 2a\sqrt{3}b$$

$$\Rightarrow 1 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow (ab)^2 - 1 = 2a\sqrt{3}b \Rightarrow \left(\frac{a}{b}\right)^2 - 1 = \frac{2a\sqrt{3}}{b} \Rightarrow a^2 - b^2 = 2a\sqrt{3}b$$

$$\Rightarrow \frac{a^2 - b^2}{b^2} = \frac{2a\sqrt{3}}{b} \Rightarrow (a^2 - b^2)(b^2a) = \sqrt{3} \Rightarrow \left(\frac{a^2 - b^2}{b^2}\right) \left(\frac{b}{2a}\right) = \sqrt{3} \Rightarrow (a^2 - b^2)2ab = \sqrt{3}$$

$$\Rightarrow \left(\frac{a^2 - b^2}{2ab}\right) = \sqrt{3}$$

Here we see that  $\sqrt{3}\sqrt{3}$  is a rational number which is a contradiction as we know that  $\sqrt{3}\sqrt{3}$  is an irrational number

Hence  $\sqrt{3}+\sqrt{4}\sqrt{3} + \sqrt{4}$  is an irrational number

**Q.11: Prove that for any prime positive integer p,  $\sqrt{p}\sqrt{p}$  is an irrational number.**

**Sol:**

Let us assume that  $\sqrt{p}\sqrt{p}$  is rational. Then, there exist positive co primes a and b such that

$$\sqrt{p}\sqrt{p} = ab \frac{a}{b}$$

$$pp = (ab)^2 \left(\frac{a}{b}\right)^2$$

$$\Rightarrow pp = a^2 b^2 \frac{a^2}{b^2}$$

$$\Rightarrow pb^2 = a^2 \Rightarrow pb^2 = a^2 \Rightarrow p|a^2 \Rightarrow p|a^2 \Rightarrow p|a \Rightarrow p|a \Rightarrow a = pc \text{ for some positive integer } c$$

$$\Rightarrow a = pc \text{ for some positive integer } c$$

$$\Rightarrow b^2 p \Rightarrow b^2 p = a^2 a^2$$

$$\Rightarrow b^2 p \Rightarrow b^2 p = p^2 c^2 p^2 c^2 \quad (\because a = pc)$$

$$\Rightarrow p|b^2 (\text{since } p|c^2 p) \Rightarrow p|b^2 \quad (\text{since } p|c^2 p) \Rightarrow p|b \Rightarrow p|b \Rightarrow p|a \text{ and } p|b \Rightarrow p|a \text{ and } p|b$$

This contradicts the fact that a and b are co primes

Hence  $\sqrt{p}\sqrt{p}$  is irrational

**Q.12: If p, q are prime positive integers, prove that  $\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q}$  is an irrational number.**

**Sol:**

Let us assume that  $\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q}$  is rational. Then, there exist positive co primes a and b such that

$$\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q} = ab \frac{a}{b}$$

$$\sqrt{p} = ab - \sqrt{q}\sqrt{p} = \frac{a}{b} - \sqrt{q} \quad (\sqrt{p})^2 = (ab - \sqrt{q})^2 \quad (\sqrt{p})^2 = \left(\frac{a}{b} - \sqrt{q}\right)^2 \quad p = (ab)^2 - 2a\sqrt{q}b + q$$

$$p = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b} + q \quad p - q = (ab)^2 - 2a\sqrt{q}b - q = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b} \quad (ab)^2 - (p - q) = 2a\sqrt{q}b$$

$$\left(\frac{a}{b}\right)^2 - (p - q) = \frac{2a\sqrt{q}}{b} \quad a^2 - b^2(p - q)b^2 = 2a\sqrt{q}b \quad \frac{a^2 - b^2(p - q)}{b^2} = \frac{2a\sqrt{q}}{b} \quad (a^2 - b^2(p - q)b^2)(b^2a) = \sqrt{q}$$

$$\left(\frac{a^2 - b^2(p - q)}{b^2}\right) \left(\frac{b}{2a}\right) = \sqrt{q} \quad \sqrt{q} = \frac{a^2 - b^2(p - q)}{2ab} \quad \sqrt{q} = \frac{a^2 - b^2(p - q)}{2ab}$$

Here we see that  $\sqrt{q}\sqrt{q}$  is a rational number which is a contradiction as we know that  $\sqrt{q}\sqrt{q}$  is an irrational number

Hence  $\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q}$  is an irrational number

## Exercise 1.6: Real Numbers

**Q.1:** Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i)  $238 \frac{23}{8}$

(ii)  $125441 \frac{125}{441}$

(iii)  $3550 \frac{35}{50}$

(iv)  $77210 \frac{77}{210}$

(v)  $1292^2 \times 5^7 \times 7^{17} \frac{129}{2^2 \times 5^7 \times 7^{17}}$

**Sol:**

(i) The given number is  $238\frac{23}{8}$

Here,  $8 = 2^3$  and 2 is not a factor of 23.

So, the given number is in its simplest form.

Now,  $8 = 2^3$  is of the form  $2^m \times 5^n$ , where  $m = 3$  and  $n = 0$ .

So, the given number has a terminating decimal expansion.

(ii) The given number is  $125441\frac{125}{441}$

Here,  $441 = 3^2 \times 7^2$  and none of 3 and 7 is a factor of 125.

So, the given number is in its simplest form.

Now,  $441 = 3^2 \times 7^2$  is not of the form  $2^m \times 5^n$

So, the given number has a non-terminating repeating decimal expansion.

(iii) The given number is  $3550\frac{35}{50}$  and  $\text{HCF}(35, 50) = 5$ .

$$\therefore 3560 = 35/550/5 = 710. \therefore \frac{35}{60} = \frac{35/5}{50/5} = \frac{7}{10}$$

Here,  $710\frac{7}{10}$  is in its simplest form.

Now,  $10 = 2 \times 5$  is of the form  $2^m \times 5^n$ , where  $m = 1$  and  $n = 1$ .

So, the given number has a terminating decimal expansion.

(iv) The given number is  $77210\frac{77}{210}$  and  $\text{HCF}(77, 210) = 7$ .



$$\therefore 77:7210:7 = 1130 \therefore \frac{77:7}{210:7} = \frac{11}{30}$$

Here,  $1130 \frac{11}{30}$  is in its simplest form. 30

Now,  $30 = 2 \times 3 \times 5$  is not of the form  $2^m \times 5^n$ .

So, the given number has a non-terminating repeating decimal expansion.

(v) The given number is  $1292^2 \times 5^7 \times 7^{17} \frac{129}{2^2 \times 5^7 \times 7^{17}}$

Clearly, none of 2, 5 and 7 is a factor of 129.

So, the given number is in its simplest form.

**Q.2: Write down the decimal expansions of the following rational numbers by writing their denominators in the form of  $2^m \times 5^n$ , where m, and n, are the non- negative integers.**

(i)  $38 \frac{3}{8}$

(ii)  $13125 \frac{13}{125}$

(iii)  $780 \frac{7}{80}$

(iv)  $14588625 \frac{14588}{625}$

(v)  $1292^4 \times 5^7 \frac{129}{2^4 \times 5^7}$

**Sol:**

(i) The given number is  $38 \frac{3}{8}$

Clearly,  $8 = 2^3$  is of the form  $2^m \times 5^n$ , where  $m = 3$  and  $n = 0$ .

So, the given number has terminating decimal expansion.

$$\therefore 3 \times 5^3 2^3 \times 5^3 = 3 \times 125 (2 \times 5)^3 = 375 (10)^3 = 3751000 = 0.375$$

$$\therefore \frac{3 \times 5^3}{2^3 \times 5^3} = \frac{3 \times 125}{(2 \times 5)^3} = \frac{375}{(10)^3} = \frac{375}{1000} = 0.375$$

(ii) The given number is  $13125 \frac{13}{125}$ .

Clearly,  $125 = 5^3$  is of the form  $2^m \times 5^n$ , where  $m = 0$  and  $n = 3$ .

So, the given number has terminating decimal expansion.

$$38 = 3 \times 5^3 (2 \times 5)^3 = 3751000 \frac{3}{8} = \frac{3 \times 5^3}{(2 \times 5)^3} = \frac{375}{1000}$$

(iii) The given number is  $780 \frac{7}{80}$ .

Clearly,  $80 = 2^4 \times 5$  is of the form  $2^m \times 5^n$ , where  $m = 4$  and  $n = 1$ .

So, the given number has terminating decimal expansion.

$$\therefore 780 = 7 \times 5^3 2^4 \times 5 \times 5^3 = 7 \times 125 (2 \times 5)^4 = 87510^4 = 87510000 = 0.0875$$

$$\therefore \frac{7}{80} = \frac{7 \times 5^3}{2^4 \times 5 \times 5^3} = \frac{7 \times 125}{(2 \times 5)^4} = \frac{875}{10^4} = \frac{875}{10000} = 0.0875$$

(iv) The given number is  $14588625 \frac{14588}{625}$

Clearly,  $625 = 5^4$  is of the form  $2^m \times 5^n$ , where  $m = 0$  and  $n = 4$ .

So, the given number has terminating decimal expansion.

$$\therefore 14588625 \therefore \frac{14588}{625} = 14588 \times 2^4 2^4 \times 5^4 \frac{14588 \times 2^4}{2^4 \times 5^4} = 23340810^4 \frac{233408}{10^4} = 23340810000 \frac{233408}{10000} =$$

$$23.340823.3408$$

(v) The given number is  $1292^4 \times 5^7 \frac{129}{2^4 \times 5^7}$

Clearly,  $2^2 \times 5^7$  is of the form  $2^m \times 5^n$ , where  $m = 2$  and  $n = 7$ .

So, the given number has terminating decimal expansion.

$$\therefore 1292^4 \times 5^7 \therefore \frac{129}{2^4 \times 5^7} = 129 \times 2^5 \times 2^2 \times 5^7 \times 2^5 \frac{129 \times 2^5}{2^4 \times 5^7 \times 2^5} = 129 \times 32 (2 \times 5)^7 \frac{129 \times 32}{(2 \times 5)^7} = 418210^7 \frac{4182}{10^7} =$$
$$418210000000 \frac{4182}{10000000} = 0.00041820.0004182$$

**Q.4: what can you say about the prime factorization of the denominators of the following rational:**

(i) **43.123456789**

(ii) **43. ————— 123456789 43.123456789**

(iii) **27. ————— 142857 27.142857**

(iv) **0.120120012000120000**

**Sol:**

(i) Since 43.123456789 has terminating decimal expansion. So, its denominator is of the form  $2^m \times 5^n$ , where  $m, n$  are non-negative integers.

(ii) Since 43. ————— 123456789 43.123456789 has non-terminating decimal expansion. So, its denominator has factors other than 2 or 5.

(iii) Since 27. ————— 142857 27.142857 has non-terminating decimal expansion. So, its denominator has factors other than 2 or 5.

**(iv)** Since  $0.120120012000120000 \dots$  has non-terminating decimal expansion. So, its denominator has factors other than 2 or 5.