NCERT Solutions for Class 10 Math Chapter 6 – Triangles

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Question 1:

Fill in the blanks using correct word given in the brackets:−

(i) All circles are __________. (congruent, similar)

(ii) All squares are __________. (similar, congruent)

(iii) All __________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are __________. (equal, proportional)

Answer:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

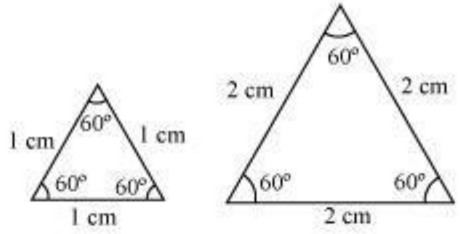
Question 2:

Give two different examples of pair of

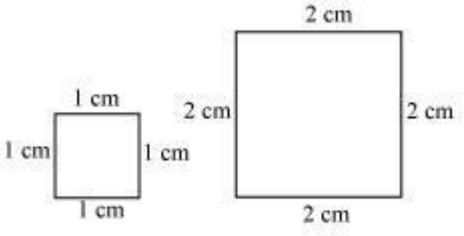
(i) Similar figures

(ii)Non-similar figures
Answer:

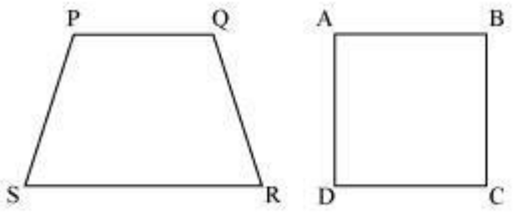
(i) Two equilateral triangles with sides 1 cm and 2 cm



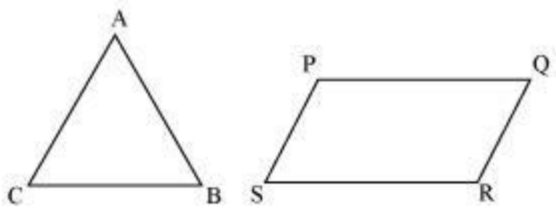
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

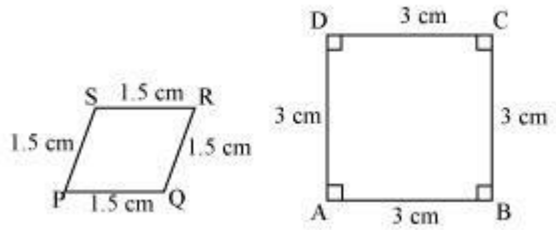


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer:

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

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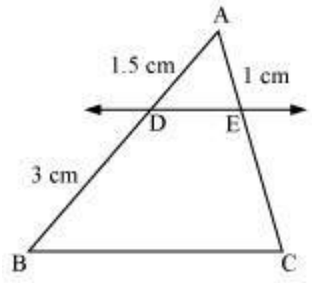
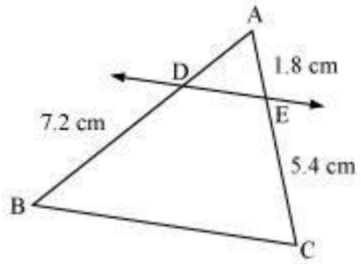
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

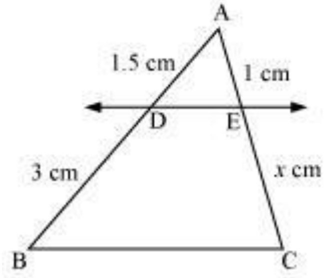


(ii)



Answer:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$$

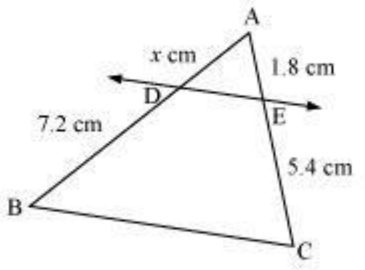
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore \text{EC} = 2 \text{ cm}$$

(ii)



Let AD = *x* cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{x}{7.2} = \frac{1.8}{5.4}$$
$$x = \frac{1.8 \times 7.2}{5.4}$$
$$x = 2.4$$
$$\therefore AD = 2.4 \text{ cm}$$

Question 2:

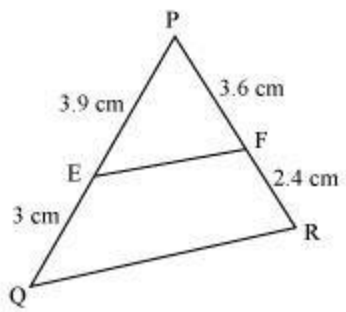
E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm
Answer:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

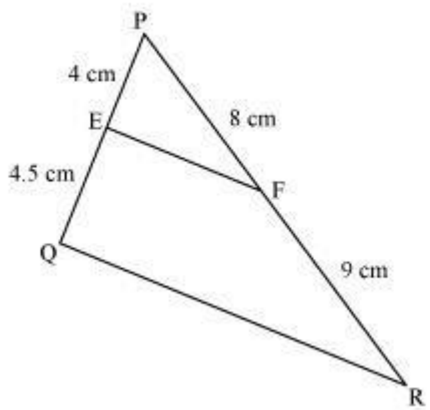
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

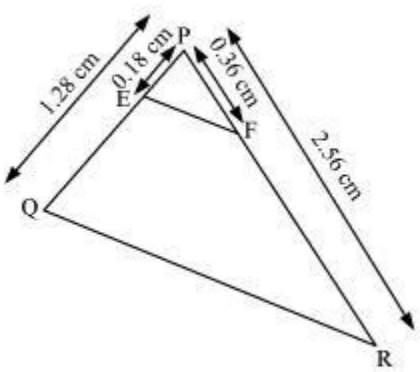
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

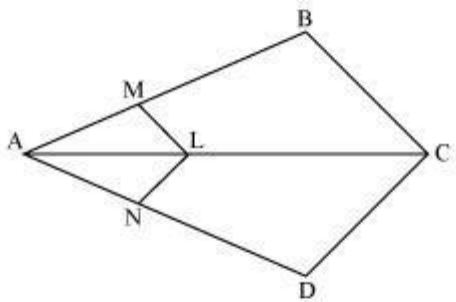
Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore, EF is parallel to QR.

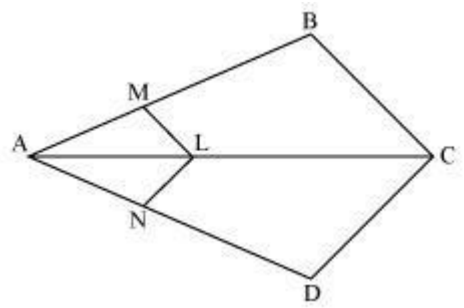
Question 3:

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}.$$



Answer:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad (i)$$

Similarly, LN || CD

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \qquad (ii)$$

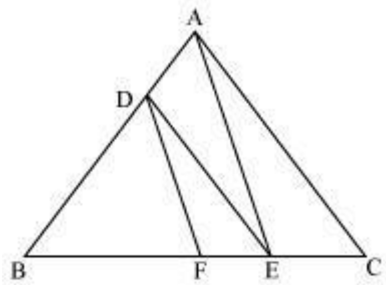
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

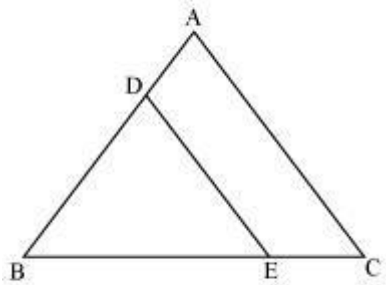
Question 4:

In the following figure, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

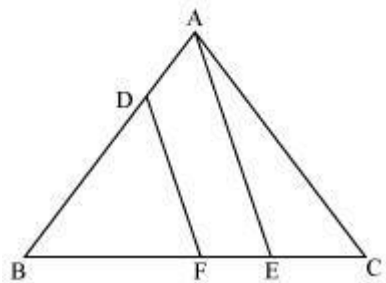


Answer:



In ΔABC, DE || AC

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (Basic Proportionality Theorem) (i)



In ΔBAE, $DF \parallel AE$

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ (Basic Proportionality Theorem) (ii)

From (i) and (ii), we obtain

$\frac{BE}{EC} = \frac{BF}{FE}$

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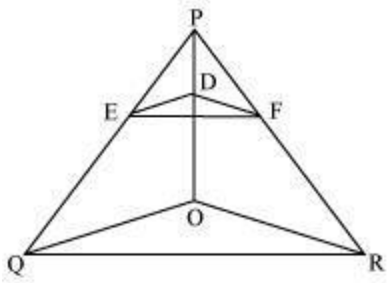
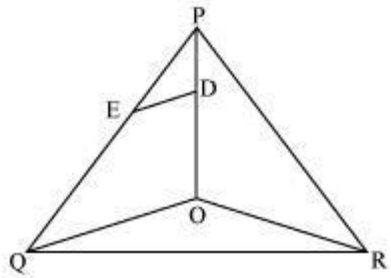
Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

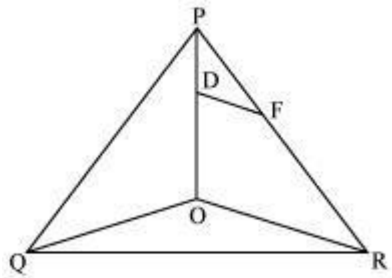


Answer:



In Δ POQ, DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ (Basic proportionality theorem) (i)



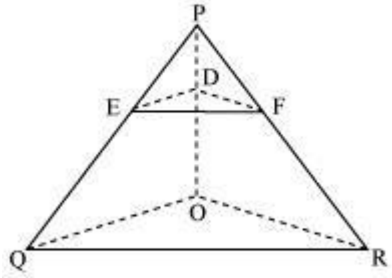
In ΔPOR, DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

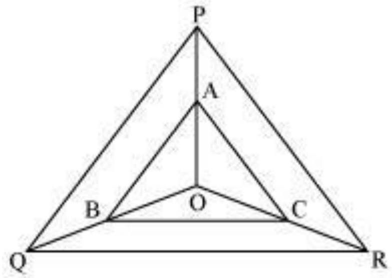
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

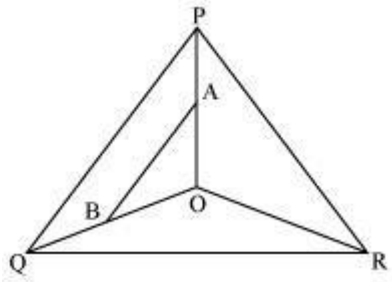


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

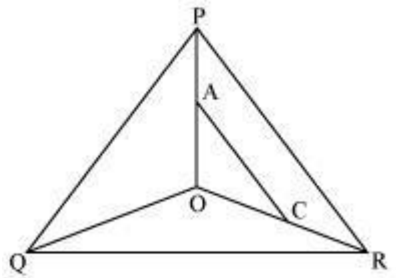


Answer:



In Δ POQ, AB || PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (Basic proportionality theorem) (i)



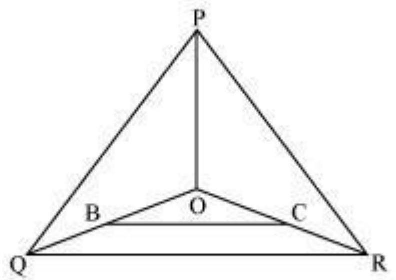
In ΔPOR, AC || PR

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (By basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$\frac{OB}{BQ} = \frac{OC}{CR}$

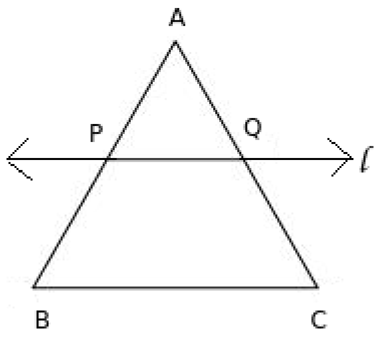
$\therefore$ BC || QR (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Consider the given figure in which l is a line drawn through the mid-point P of line segment AB meeting AC at Q, such that $PQ \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad \left(\text{P is the mid-point of AB. } \therefore \ AP = PB\right)$$

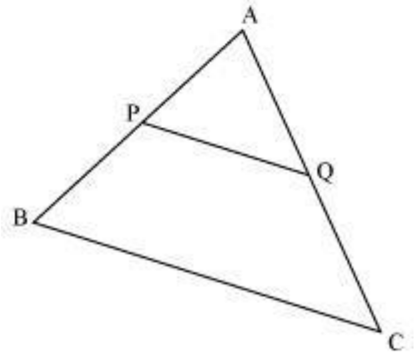
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

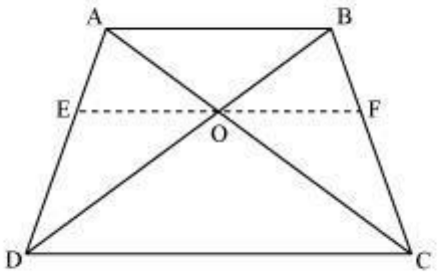
$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain

$PQ \| BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.
Answer:



Draw a line EF through point O, such that $EF \| CD$

In ΔADC, $EO \| CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, $OE \| AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

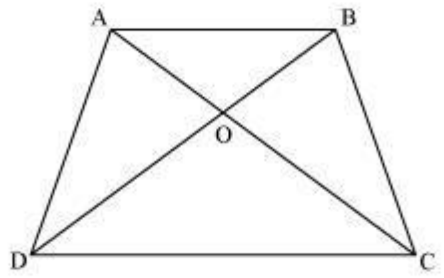
$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

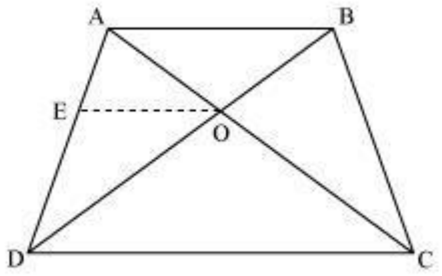
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.
Answer:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

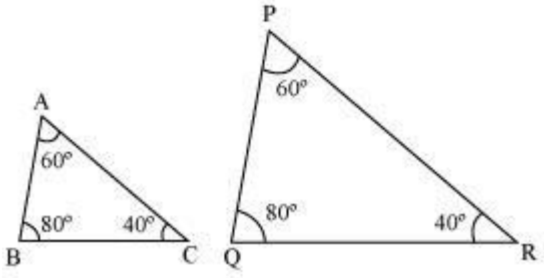
⇒ AB || CD

∴ ABCD is a trapezium.

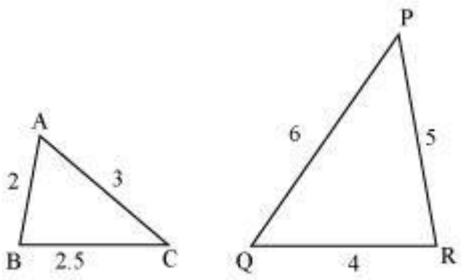
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

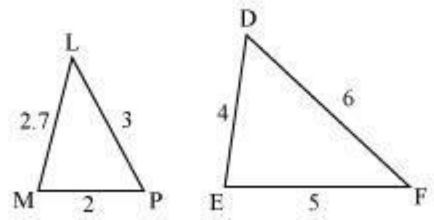
(i)



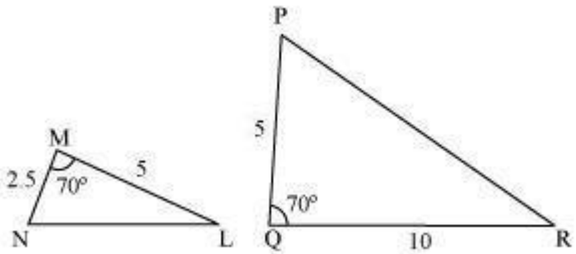
(ii)



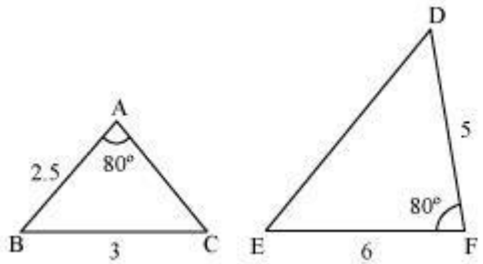
(iii)



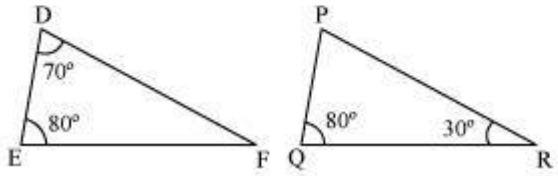
(iv)



(v)



(vi)



Answer:

(i) ∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, ΔABC ~ ΔPQR [By AAA similarity criterion]

(ii) $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

∴ ΔABC ~ ΔQRP [By SSS similarity criterion]

(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv) In âˆ† MNL and âˆ† QPR, we observe that,

MNQP = MLQR = 12∠M = ∠Q = 70°∴ΔMNL ~ ΔQPR By SAS similarity criterion

(v)The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

∠D +∠E +∠F = 180º

(Sum of the measures of the angles of a triangle is 180º.)

$70^\circ + 80^\circ + \angle F = 180^\circ$

$\angle F = 30^\circ$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80^\circ + 30^\circ = 180^\circ$

$\angle P = 70^\circ$

In ΔDEF and ΔPQR,

$\angle D = \angle P$ (Each 70°)

$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

$\therefore \Delta DEF \sim \Delta PQR$ [By AAA similarity criterion]

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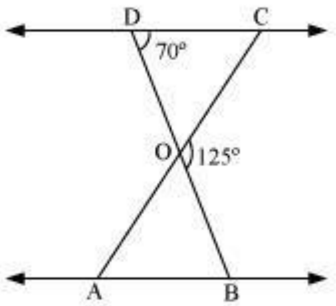
Question 2:

In the following figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer:

DOB is a straight line.

∴ ∠DOC + ∠COB = 180°

⇒ ∠DOC = 180° − 125°

= 55°

In ΔDOC,

∠DCO + ∠CDO + ∠DOC = 180°

(Sum of the measures of the angles of a triangle is 180º.)

⇒ ∠DCO + 70º + 55º = 180°

⇒ ∠DCO = 55°

It is given that ΔODC ~ ΔOBA.

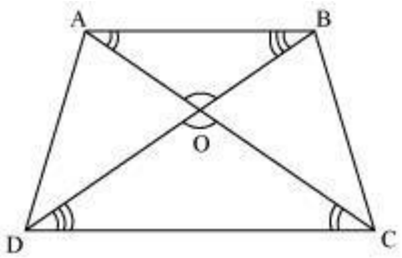
∴ ∠OAB = ∠ OCD [Corresponding angles are equal in similar triangles.]

⇒ ∠OAB = 55°

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer:



In ΔDOC and ΔBOA,

∠CDO = ∠ABO [Alternate interior angles as AB || CD]

∠DCO = ∠BAO [Alternate interior angles as AB || CD]

∠DOC = ∠BOA [Vertically opposite angles]

∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

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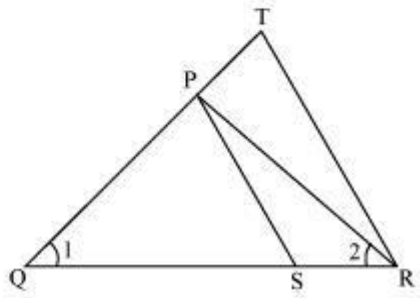
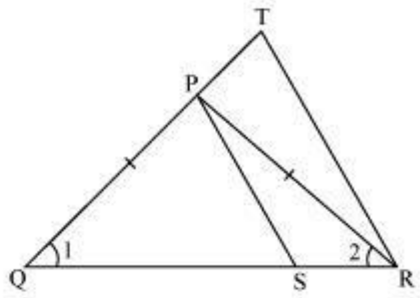
Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$



Answer:



In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad [\text{Using } (ii)]$$

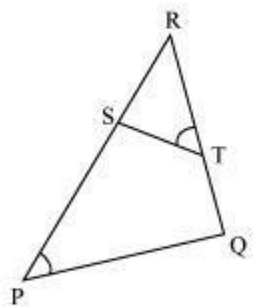
$$\angle Q = \angle Q$$

$$\therefore \ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer:



In ΔRPQ and ΔRST,

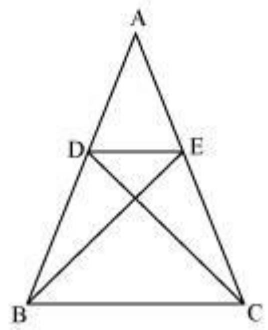
∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Answer:

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

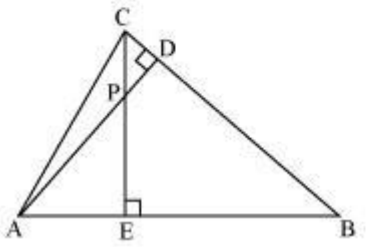
$\frac{AD}{AB} = \frac{AE}{AC}$ [Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



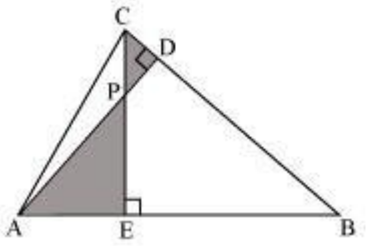
(i) ΔAEP ~ ΔCDP

(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(v) ΔPDC ~ ΔBEC
Answer:

(i)



In ΔAEP and ΔCDP,

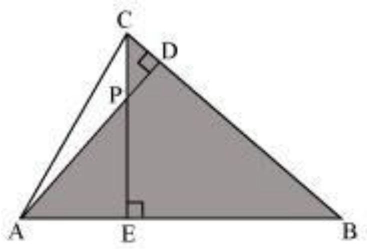
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

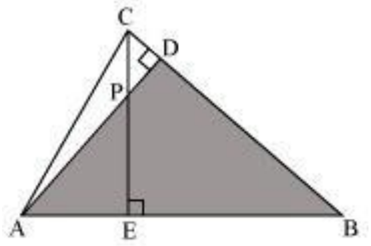
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

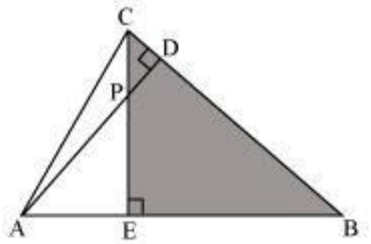
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC,

∠PDC = ∠BEC (Each 90°)

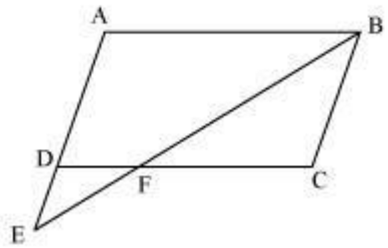
∠PCD = ∠BCE (Common angle)

Hence, by using AA similarity criterion,

ΔPDC ~ ΔBEC

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB
Answer:



In ΔABE and ΔCFB,

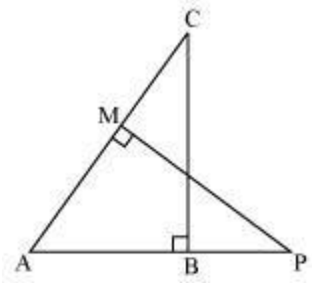
∠A = ∠C (Opposite angles of a parallelogram)

∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer:

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

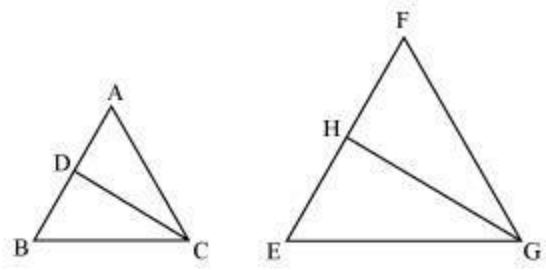
Question 10:

CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF
Answer:



It is given that ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD ~ ΔFGH (By AA similarity criterion)

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

In ΔDCA and ΔHGF,

∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

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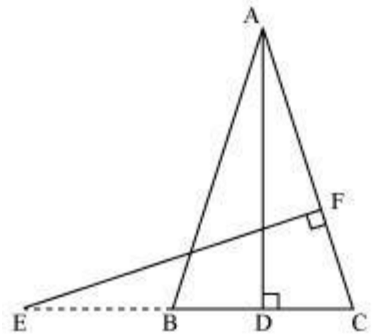
Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF



Answer:

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

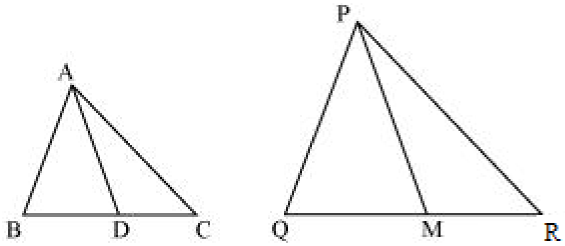
∠ABD = ∠ECF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.

Answer:



Median divides the opposite side.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

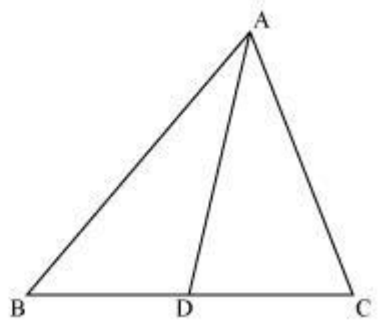
∠ABD = ∠PQM (Proved above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB.CD.$
Answer:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

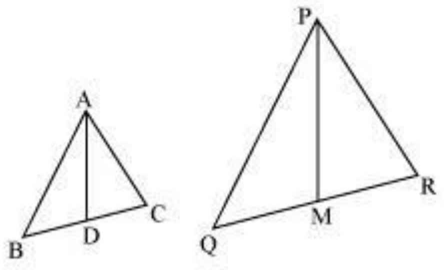
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

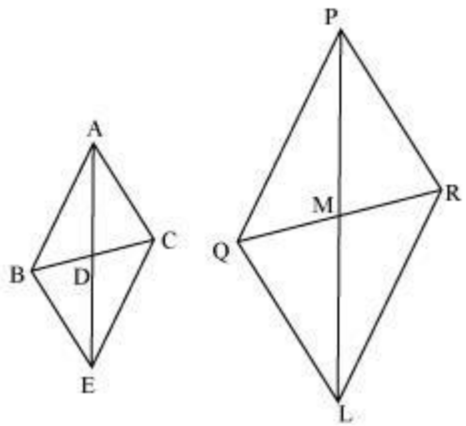
Answer:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

∴ ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC ~ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

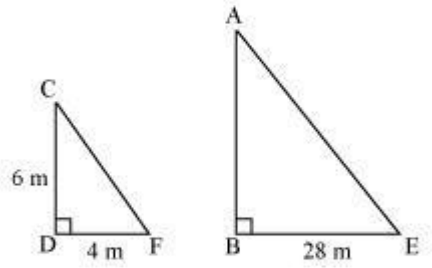
$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.
Answer:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

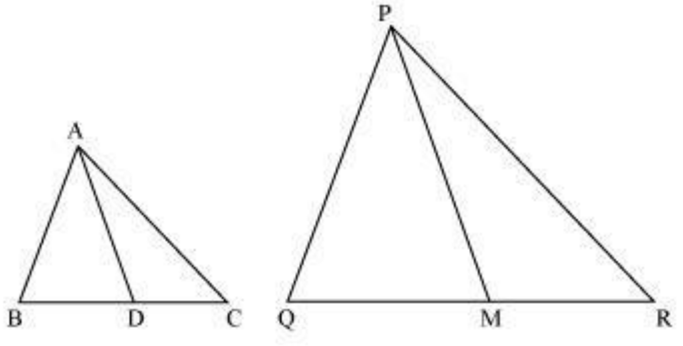
$\therefore \Delta ABE \sim \Delta CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$
$$\Rightarrow \frac{AB}{6\text{ m}} = \frac{28}{4}$$
$$\Rightarrow AB = 42\text{ m}$$

Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$
Answer:



It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

∴ $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ … (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ}=\frac{BD}{QM}$ ... (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

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Question 1:
Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.
Answer:

It is given that ΔABC ~ ΔDEF.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)}=\left(\frac{AB}{DE}\right)^2=\left(\frac{BC}{EF}\right)^2=\left(\frac{AC}{DF}\right)^2$$

Given that,

EF = 15.4 cm,

$ar(\Delta ABC)=64\ cm^2$,

$ar(\Delta DEF)=121\ cm^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64\ cm^2}{121\ cm^2}\right) = \frac{BC^2}{(15.4\ cm)^2}$$

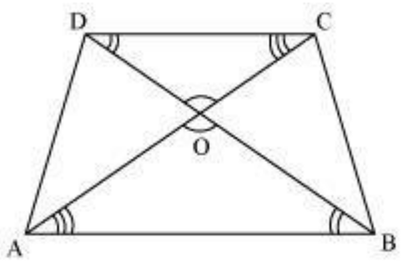
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right)cm$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right)cm = (8 \times 1.4)cm = 11.2\,cm$$

Question 2:

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB ~ ΔCOD (By AAA similarity criterion)

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Since AB = 2 CD,

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2\ CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

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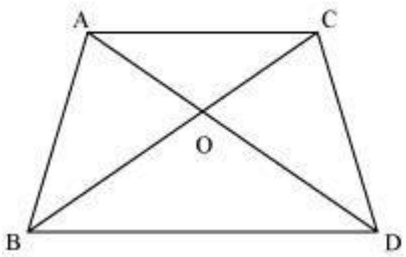
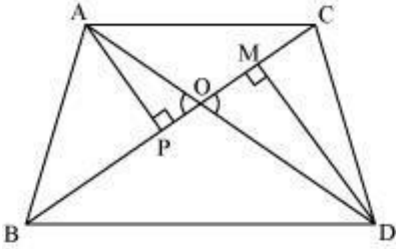
Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{AO}{DO}$



Answer:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$.

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.
Answer:

Let us assume two similar triangles as ΔABC ~ ΔPQR.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) =$ ar (ΔPQR)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

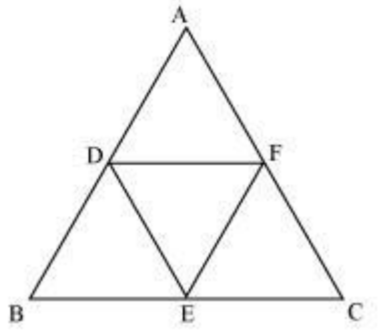
$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$\Rightarrow AB = PQ, BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.
Answer:



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2}AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4} ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4} ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4} ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - \left[ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)\right]$

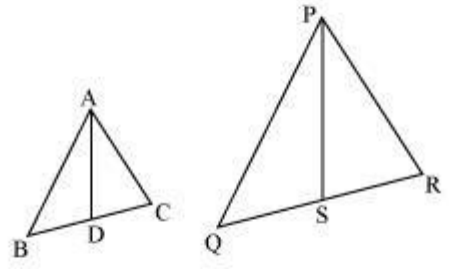
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4} ar(\Delta ABC) = \frac{1}{4} ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square

of the ratio of their corresponding medians.
Answer:



Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles.

∵ ΔABC ~ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ …(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PS are medians,

∴ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ … (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

∴ ΔABD ~ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ ... (4)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

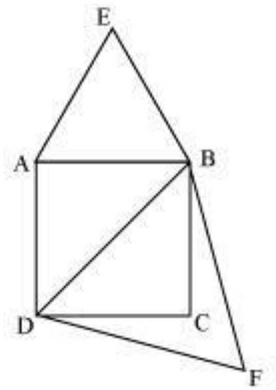
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer:



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \Delta \text{ABE}}{\text{Area of } \Delta \text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

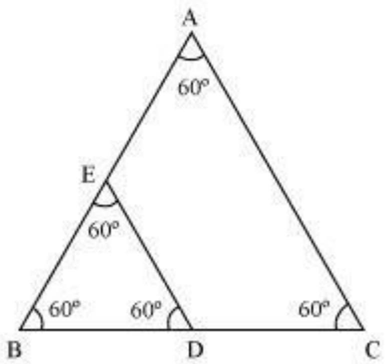
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore, side of $\Delta BDE = \frac{x}{2}$

$$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Question 1:

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer:

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

49 + 576 = 625

Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

However, 9 + 36 ≠ 64

Or, $3^2 + 6^2 \neq 8^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

However, 2500 + 6400 ≠ 10000

Or, $50^2 + 80^2 \neq 100^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv)Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, 144 +25 = 169

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

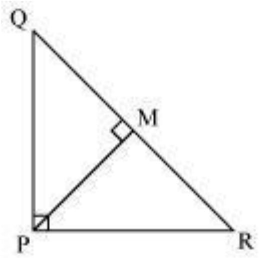
Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \times MR$.
Answer:



Let $\angle MPR = x$

In ΔMPR,

$\angle MRP = 180^\circ - 90^\circ - x$

$\angle MRP = 90^\circ - x$

Similarly, in ΔMPQ,

$\angle MPQ = 90^\circ - \angle MPR$

$= 90^\circ - x$

$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle MQP = x$

In Δ QMP and ΔPMR,

$\angle MPQ = \angle MRP$

$\angle PMQ = \angle RMP$

$\angle MQP = \angle MPR$

$\therefore \Delta QMP \sim \Delta PMR$ (By AAA similarity criterion)

$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$

$\Rightarrow PM^2 = QM \times MR$

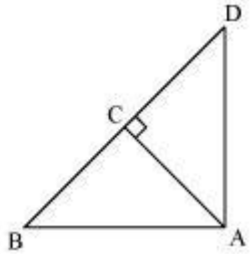
Question 3:

In the following figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Answer:

(i) In ΔADB and ΔCAB,

$\angle DAB = \angle ACB$ (Each 90°)

$\angle ABD = \angle CBA$ (Common angle)

$\therefore$ $\Delta ADB \sim \Delta CAB$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$$

$$\Rightarrow AB^2 = CB \times BD$$

(ii) Let $\angle CAB = x$

In ΔCBA,

$\angle CBA = 180^\circ - 90^\circ - x$

$\angle CBA = 90^\circ - x$

Similarly, in ΔCAD,

$\angle CAD = 90^\circ - \angle CAB$

$= 90^\circ - x$

$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle CDA = x$

In Δ CBA and Δ CAD,

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA$ (Each 90°)

$\therefore \Delta CBA \sim \Delta CAD$ (By AAA rule)

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA and ΔDAB,

∠DCA = ∠ DAB (Each 90º)

∠CDA = ∠ ADB (Common angle)

$\therefore \Delta DCA \sim \Delta DAB$ (AA similarity criterion)

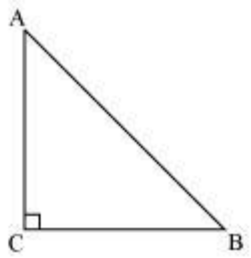
$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

Question 4:

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2\ AC^2$.

Answer:



Given that ΔABC is an isosceles triangle.

∴ AC = CB

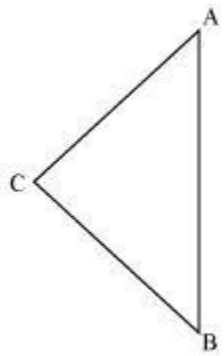
Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

$$AC^2 + CB^2 = AB^2$$
$$\Rightarrow AC^2 + AC^2 = AB^2 \qquad (AC = CB)$$
$$\Rightarrow 2AC^2 = AB^2$$

Question 5:

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Answer:



Given that,

$AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

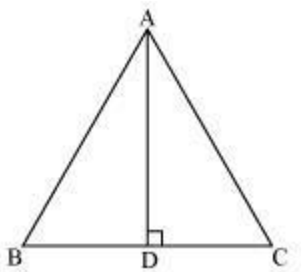
$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$

The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Question 6:

ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.
Answer:



Let AD be the altitude in the given equilateral triangle, ΔABC.

We know that altitude bisects the opposite side.

∴ BD = DC = *a*

In ΔADB,

$\angle ADB = 90°$

Applying pythagoras theorem, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 + a^2 = (2a)^2$

$\Rightarrow AD^2 + a^2 = 4a^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$

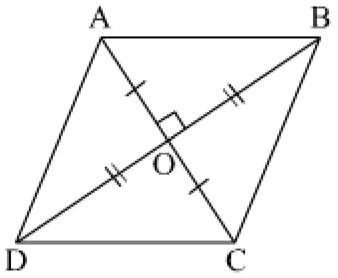
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer:



In Î”AOB, Î”BOC, Î”COD, Î”AOD,

Applying Pythagoras theorem, we obtain

$AB^2 = AO^2 + OB^2$ ... (1)

$BC^2 = BO^2 + OC^2$ ... (2)

$CD^2 = CO^2 + OD^2$ ... (3)

$AD^2 = AO^2 + OD^2$ ... (4)

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

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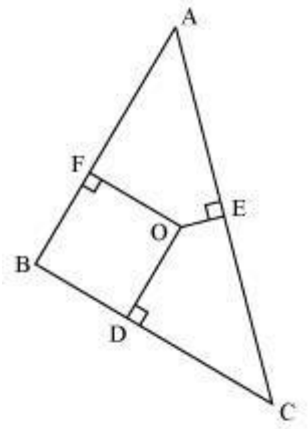
Question 8:

In the following figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

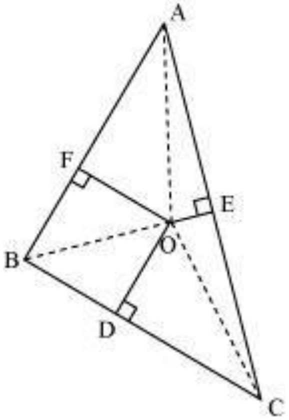


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer:

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in ΔAOF, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in ΔBOD,

$$OB^2 = OD^2 + BD^2$$

Similarly, in ΔCOE,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

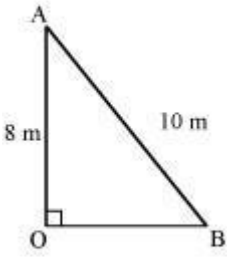
(ii) From the above result,

$$AF^2 + BD^2 + EC^2 = \left(OA^2 - OE^2\right) + \left(OC^2 - OD^2\right) + \left(OB^2 - OF^2\right)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.
Answer:



Let OA be the wall and AB be the ladder.

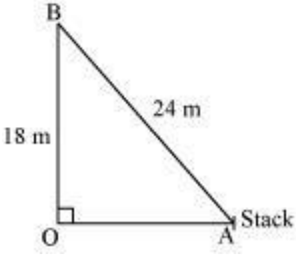
Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$
$$(10\text{ m})^2 = (8\text{ m})^2 + OB^2$$
$$100\text{ m}^2 = 64\text{ m}^2 + OB^2$$
$$OB^2 = 36\text{ m}^2$$
$$OB = 6\text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?
Answer:



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$$AB^2 = OB^2 + OA^2$$

$$(24\text{ m})^2 = (18\text{ m})^2 + OA^2$$

$$OA^2 = (576 - 324)\text{m}^2 = 252\text{ m}^2$$

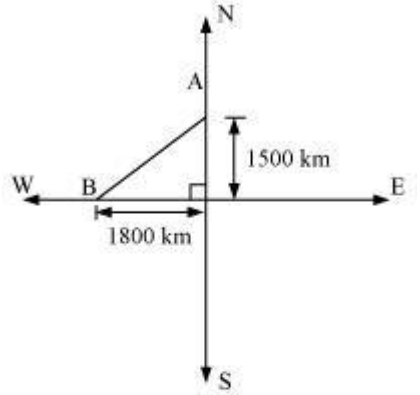
$$OA = \sqrt{252}\text{ m} = \sqrt{6\times6\times7}\text{ m} = 6\sqrt{7}\text{ m}$$

Therefore, the distance from the base is $6\sqrt{7}$ m.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer:



Distance travelled by the plane flying towards north in $1\frac{1}{2}\text{hrs} = 1,000\times 1\frac{1}{2} = 1,500\text{ km}$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\text{hrs} = 1,200\times 1\frac{1}{2} = 1,800\text{ km}$

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

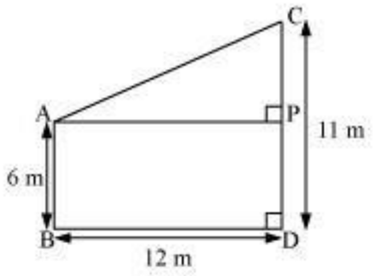
Distance between these planes after $1\frac{1}{2}\text{hrs}$, AB = $\sqrt{OA^2 + OB^2}$

$$= \left(\sqrt{(1,500)^2 + (1,800)^2}\right)\text{km} = \left(\sqrt{2250000+3240000}\right)\text{km}$$
$$= \left(\sqrt{5490000}\right)\text{km} = \left(\sqrt{9\times 610000}\right)\text{km} = 300\sqrt{61}\text{ km}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}\text{hrs}$.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.
Answer:



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, CP = 11 − 6 = 5 m

From the figure, it can be observed that AP = 12m

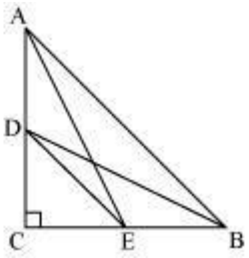
Applying Pythagoras theorem for ΔAPC, we obtain

$$AP^2 + PC^2 = AC^2$$
$$(12\text{ m})^2 + (5\text{ m})^2 = AC^2$$
$$AC^2 = (144 + 25)\text{m}^2 = 169\text{ m}^2$$
$$AC = 13\text{ m}$$

Therefore, the distance between their tops is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$
Answer:



Applying Pythagoras theorem in ΔACE, we obtain

$$AC^2 + CE^2 = AE^2 \quad \ldots (1)$$

Applying Pythagoras theorem in ΔBCD, we obtain

$$BC^2 + CD^2 = BD^2 \quad \ldots (2)$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \ldots (3)$$

Applying Pythagoras theorem in ΔCDE, we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in ΔABC, we obtain

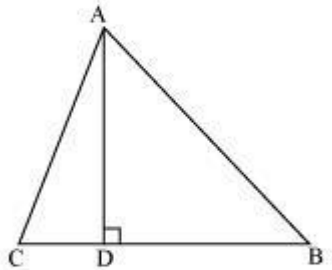
$$AB^2 = AC^2 + CB^2$$

Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$

Question 14:

The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3 CD. Prove that $2\,AB^2 = 2\,AC^2 + BC^2$



Answer:

Applying Pythagoras theorem for ΔACD, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \qquad \ldots (1)$$

Applying Pythagoras theorem in ΔABD, we obtain

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2 \quad \ldots (2)$

From equation (1) and equation (2), we obtain

$AC^2 - DC^2 = AB^2 - DB^2 \quad \ldots (3)$

It is given that 3DC = DB

$\therefore DC = \dfrac{BC}{4}$ and $DB = \dfrac{3BC}{4}$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

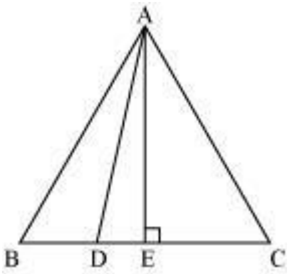
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that 9 AD^2 = 7 AB^2.

Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

And, AE = $\frac{a\sqrt{3}}{2}$

Given that, BD = $\frac{1}{3}$ BC

$\therefore$ BD = $\frac{a}{3}$

DE = BE − BD = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Applying Pythagoras theorem in ΔADE, we obtain

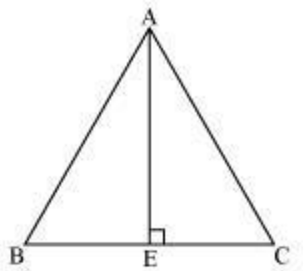
$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$
$$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right)$$
$$= \frac{28a^2}{36}$$
$$= \frac{7}{9}AB^2$$

$\Rightarrow 9\ AD^2 = 7\ AB^2$

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.
Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$4AE^2 = 3a^2$

⇒ 4 × (Square of altitude) = 3 × (Square of one side)

Question 17:

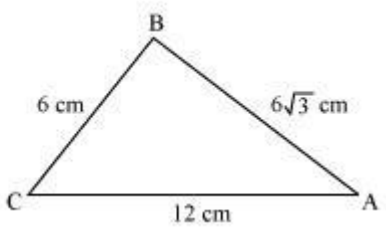
Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer:



Given that, AB = $6\sqrt{3}$ cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$AB^2 = 108$
$AC^2 = 144$
And, $BC^2 = 36$
$AB^2 + BC^2 = AC^2$

The given triangle, ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

∴ ∠B = 90°

Hence, the correct answer is (C).

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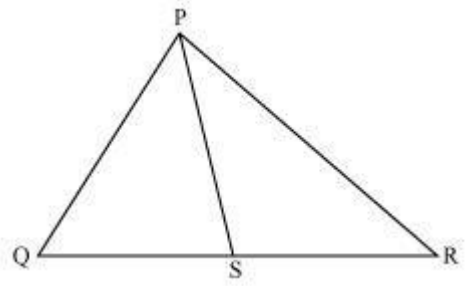
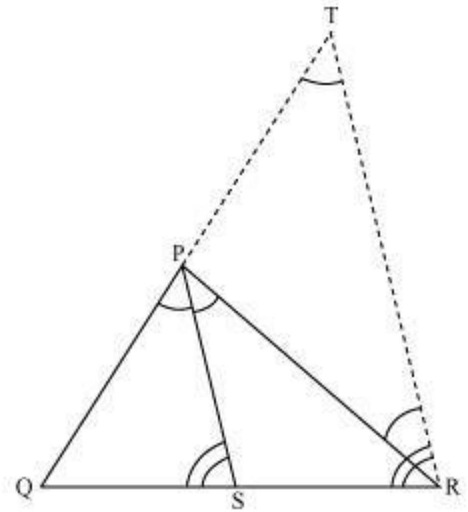
Question 1:

In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Answer:



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR … (1)

By construction,

∠SPR = ∠PRT (As PS || TR) … (2)

∠QPS = ∠QTR (As PS || TR) … (3)

Using these equations, we obtain

∠PRT = ∠QTR

∴ PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

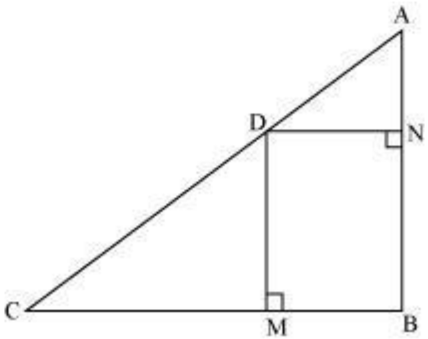
QSSR=QPPT

⇒QSSR=PQPR   ∵PT=PR

Question 2:

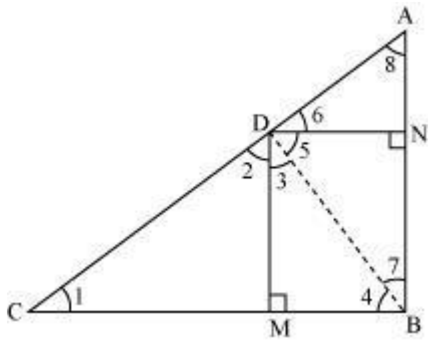
In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, Prove that:

(i) DM^2 = DN.MC
(ii) DN^2 = DM.AN



Answer:

(i)Let us join DB.



We have, DN || CB, DM || AB, and ∠B = 90°

∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° … (1)

In ΔCDM,

∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° … (2)

In ΔDMB,

∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° … (3)

From equation (1) and (2), we obtain

$\angle 1 = \angle 3$

From equation (1) and (3), we obtain

$\angle 2 = \angle 4$

In ΔDCM and ΔBDM,

$\angle 1 = \angle 3$ (Proved above)

$\angle 2 = \angle 4$ (Proved above)

∴ ΔDCM ~ ΔBDM (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \qquad (BM = DN)$$

$\Rightarrow DM^2 = DN \times MC$

(ii) In right triangle DBN,

$\angle 5 + \angle 7 = 90°$ … (4)

In right triangle DAN,

$\angle 6 + \angle 8 = 90°$ … (5)

D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle ADB = 90°$

$\Rightarrow \angle 5 + \angle 6 = 90°$ … (6)

From equation (4) and (6), we obtain

∠6 = ∠7

From equation (5) and (6), we obtain

∠8 = ∠5

In ΔDNA and ΔBND,

∠6 = ∠7 (Proved above)

∠8 = ∠5 (Proved above)

∴ ΔDNA ~ ΔBND (AA similarity criterion)

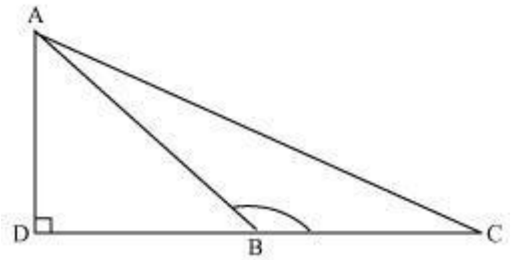
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

⇒ $DN^2 = AN \times NB$

⇒ $DN^2 = AN \times DM$ (As NB = DM)

Question 3:

In the given figure, ABC is a triangle in which ∠ABC> 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer:

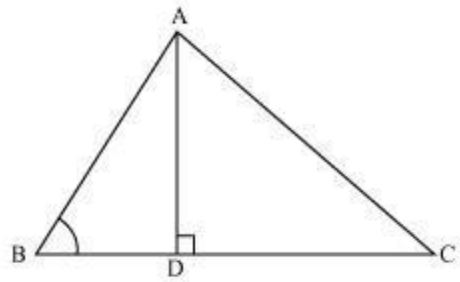
Applying Pythagoras theorem in ΔADB, we obtain

$AB^2 = AD^2 + DB^2$ … (1)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$
$AC^2 = AD^2 + (DB + BC)^2$
$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$
$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (1)]

Question 4:
In the given figure, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$
$\Rightarrow AD^2 = AB^2 - DB^2$ … (1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$= AB^2 + BC^2 - 2BC \times BD$

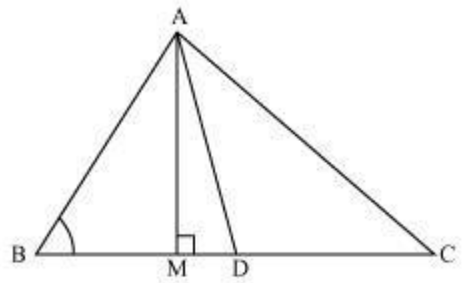
Question 5:

In the given figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Answer:

(i) Applying Pythagoras theorem in ΔAMD, we obtain

$AM^2 + MD^2 = AD^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$ [Using equation (1)]

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii)Applying Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$ … (2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$
$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$
$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$
$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

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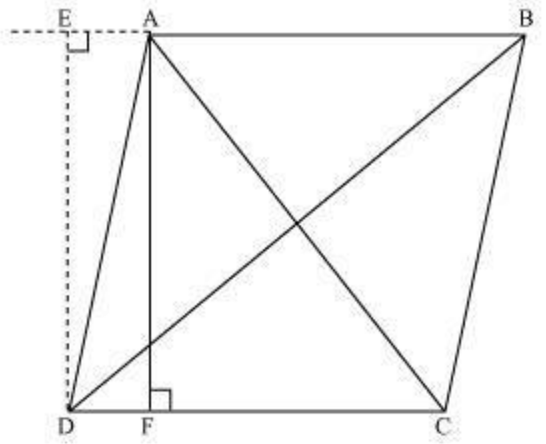
Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.
Answer:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in ΔDEA, we obtain

$DE^2 + EA^2 = DA^2$ … (*i*)

Applying Pythagoras theorem in ΔDEB, we obtain

$DE^2 + EB^2 = DB^2$
$DE^2 + (EA + AB)^2 = DB^2$
$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$
$DA^2 + AB^2 + 2EA \times AB = DB^2$ … (*ii*)

Applying Pythagoras theorem in ΔADF, we obtain

$AD^2 = AF^2 + FD^2$

Applying Pythagoras theorem in ΔAFC, we obtain

$AC^2 = AF^2 + FC^2$
$= AF^2 + (DC - FD)^2$
$= AF^2 + DC^2 + FD^2 - 2DC \times FD$
$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$
$AC^2 = AD^2 + DC^2 - 2DC \times FD$ … (*iii*)

Since ABCD is a parallelogram,

AB = CD … (*iv*)
And, BC = AD … (*v*)

In ΔDEA and ΔADF,

∠DEA = ∠AFD (Both 90°)

∠EAD = ∠ADF (EA || DF)

AD = AD (Common)

∴ ΔEAD ≅ ΔFDA (AAS congruence criterion)

⇒ EA = DF … (vi)

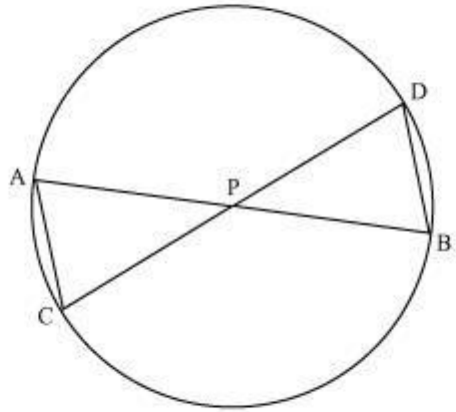
Adding equations (*i*) and (*iii*), we obtain
$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$
$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$
$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$
[Using equations (*iv*) and (*vi*)]
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In the given figure, two chords AB and CD intersect each other at the point P. prove that:

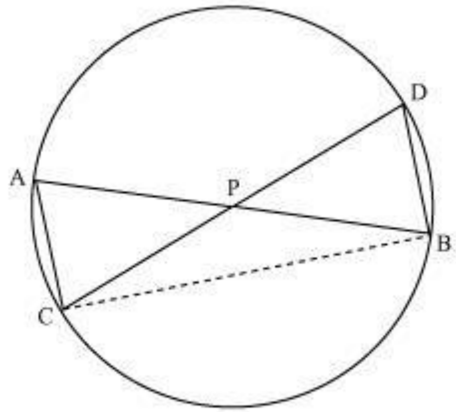
(i) ΔAPC ~ ΔDPB

(ii) AP.BP = CP.DP



Answer:

Let us join CB.



(i) In ΔAPC and ΔDPB,

∠APC = ∠DPB (Vertically opposite angles)

∠CAP = ∠BDP (Angles in the same segment for chord CB)

ΔAPC ~ ΔDPB (By AA similarity criterion)

(ii) We have already proved that

ΔAPC ~ ΔDPB

We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$$

$$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$$

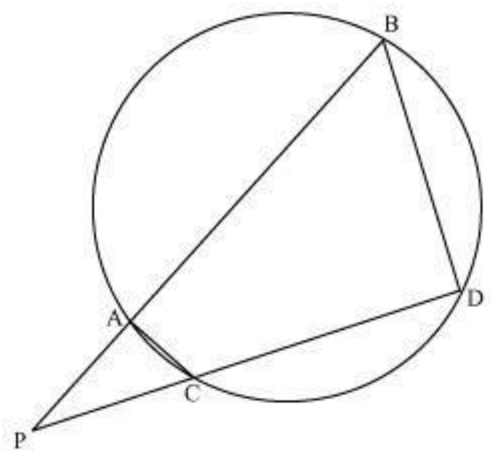
∴ AP. PB = PC. DP

Question 8:

In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) ΔPAC ~ ΔPDB

(ii) PA.PB = PC.PD



Answer:

(i) In ΔPAC and ΔPDB,

∠P = ∠P (Common)

∠PAC = ∠PDB (Exterior angle of a cyclic quadrilateral is ∠PCA = ∠PBD equal to the opposite interior angle)

∴ ΔPAC ~ ΔPDB

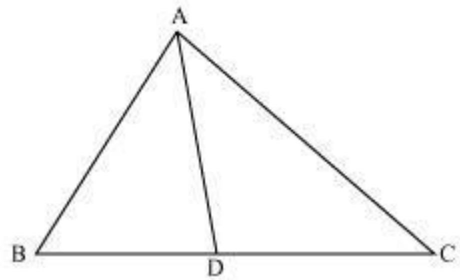
(ii)We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$$
$$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$$

∴ PA.PB = PC.PD

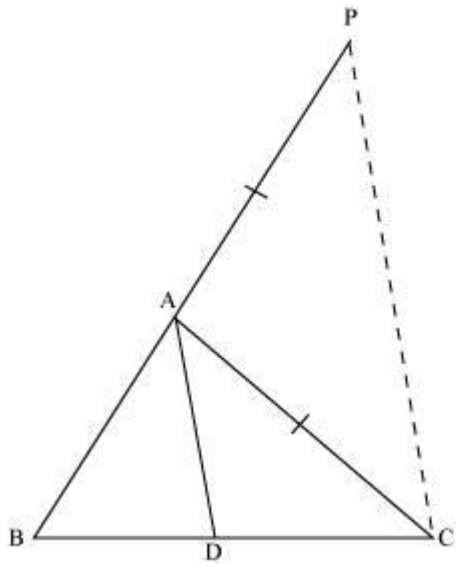
Question 9:

In the given figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠BAC.



Answer:

Let us extend BA to P such that AP = AC. Join PC.



It is given that,

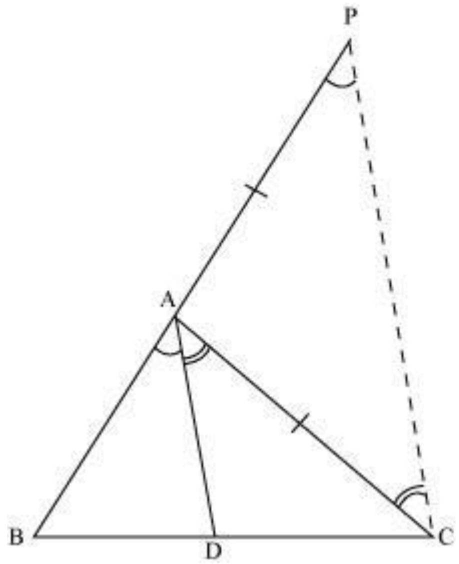
$$\frac{BD}{CD} = \frac{AB}{AC}$$
$$\Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

AD || PC

⇒ ∠BAD = ∠APC (Corresponding angles) ... (1)

And, ∠DAC = ∠ACP (Alternate interior angles) ... (2)



By construction, we have

AP = AC

⇒ ∠APC = ∠ACP … (3)

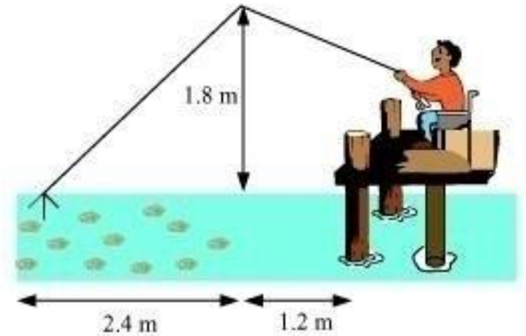
On comparing equations (1), (2), and (3), we obtain

∠BAD = ∠APC

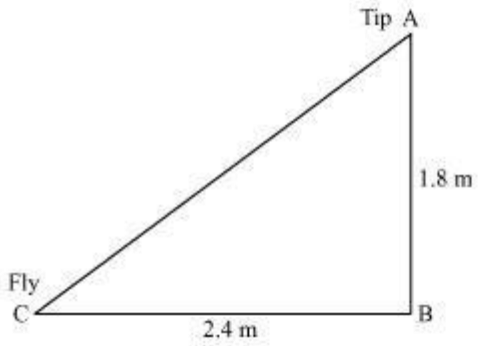
⇒ AD is the bisector of the angle BAC.

Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, ho much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in ΔABC.

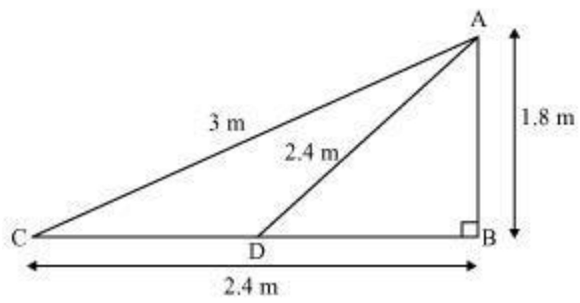
$AC^2 = AB^2 + BC^2$
$AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$
$AB^2 = (3.24 + 5.76)\text{ m}^2$
$AB^2 = 9.00\text{ m}^2$

$\Rightarrow AB = \sqrt{9}\text{ m} = 3\text{ m}$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

AD = AC − String pulled by Nazima in 12 seconds

= (3.00 − 0.6) m

= 2.4 m

In ΔADB,

$AB^2 + BD^2 = AD^2$
$(1.8\ m)^2 + BD^2 = (2.4\ m)^2$
$BD^2 = (5.76 - 3.24)\ m^2 = 2.52\ m^2$

BD = 1.587 m

Horizontal distance of fly = BD + 1.2 m

= (1.587 + 1.2) m

= 2.787 m

= 2.79 m

NCERT Solutions for Class 10 Math Chapter 7 – Coordinate Geometry

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Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (*a*, *b*), (− *a*, − *b*)
Answer:

(i) Distance between the two points is given by

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Therefore, distance between $(2, 3)$ and $(4, 1)$ is given by

$$l=\sqrt{(2-4)^2+(3-1)^2}=\sqrt{(-2)^2+(2)^2}$$

$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

(ii) Distance between $(-5,7)$ and $(-1,3)$ is given by

$$l=\sqrt{(-5-(-1))^2+(7-3)^2}=\sqrt{(-4)^2+(4)^2}$$

$$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

(iii) Distance between (a,b) and $(-a,-b)$ is given by

$$l=\sqrt{\left(a-(-a)\right)^2+\left(b-(-b)\right)^2}$$
$$=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$$

Question 2:

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer:

Distance between points $(0,0)$ and $(36,15)$

$$=\sqrt{(36-0)^2+(15-0)^2}=\sqrt{36^2+15^2}$$
$$=\sqrt{1296+225}=\sqrt{1521}=39$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A.

And hence, as calculated above, the distance between town A and B will be

39 km.

Question 3:

Determine if the points (1, 5), (2, 3) and (− 2, − 11) are collinear.
Answer:

Let the points (1, 5), (2, 3), and (−2, −11) be representing the vertices A, B, and C of the given triangle respectively.

Let $A=(1,5), B=(2,3), C=(-2,-11)$

$$\therefore AB=\sqrt{(1-2)^2+(5-3)^2}=\sqrt{5}$$

$$BC=\sqrt{(2-(-2))^2+(3-(-11))^2}=\sqrt{4^2+14^2}=\sqrt{16+196}=\sqrt{212}$$

$$CA=\sqrt{(1-(-2))^2+(5-(-11))^2}=\sqrt{3^2+16^2}=\sqrt{9+256}=\sqrt{265}$$

Since $AB+BC\neq CA$,

Therefore, the points (1, 5), (2, 3), and (−2, −11) are not collinear.

Question 4:

Check whether (5, − 2), (6, 4) and (7, − 2) are the vertices of an isosceles triangle.
Answer:

Let the points (5, −2), (6, 4), and (7, −2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB=\sqrt{(5-6)^2+(-2-4)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{1+36}=\sqrt{37}$$

$$BC=\sqrt{(6-7)^2+(4-(-2))^2}=\sqrt{(-1)^2+(6)^2}=\sqrt{1+36}=\sqrt{37}$$

$$CA=\sqrt{(5-7)^2+(-2-(-2))^2}=\sqrt{(-2)^2+0^2}=2$$

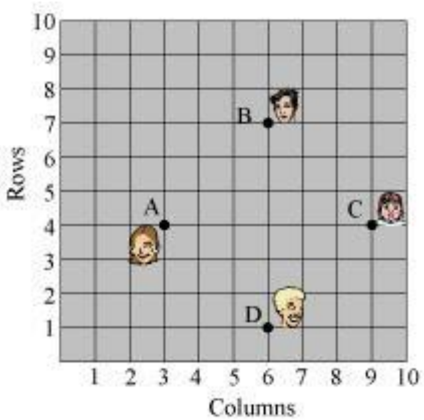
Therefore, $AB=BC$

As two sides are equal in length, therefore, ABCis an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABCD is a square?” Chameli disagrees.

Using distance formula, find which of them is correct.



Answer:

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

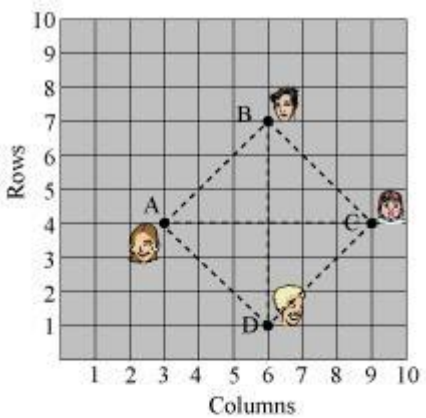
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

CD=9-62+4-12=32+32=9+9=18=32

$$\text{Diagonal } AC = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal } BD = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (− 1, − 2), (1, 0), (− 1, 2), (− 3, 0)

(ii) (− 3, 5), (3, 1), (0, 3), (− 1, − 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)
Answer:

(i) Let the points (−1, −2), (1, 0), (−1, 2), and (−3, 0) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal } AC = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal } BD = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii)Let the points (− 3, 5), (3, 1), (0, 3), and (−1, −4) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$
$$BC = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$
$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$
$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii)Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$
$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$
$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$
$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$
$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$
$$\text{Diagonal CD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Question 7:

Find the point on the *x*-axis which is equidistant from (2, − 5) and (− 2, 9).

Answer:

We have to find a point on *x*-axis. Therefore, its *y*-coordinate will be 0.

Let the point on *x*-axis be $(x,0)$.

$$\text{Distance between } (x,0) \text{ and } (2,-5)=\sqrt{(x-2)^2+(0-(-5))^2}=\sqrt{(x-2)^2+(5)^2}$$

$$\text{Distance between } (x,0) \text{ and } (-2,9)=\sqrt{(x-(-2))^2+(0-(-9))^2}=\sqrt{(x+2)^2+(9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2+(5)^2}=\sqrt{(x+2)^2+(9)^2}$$

$$(x-2)^2+25=(x+2)^2+81$$

$$x^2+4-4x+25=x^2+4+4x+81$$

$$8x=25-81$$

$$8x=-56$$

$$x=-7$$

Therefore, the point is (− 7, 0).

Question 8:

Find the values of *y* for which the distance between the points P (2, − 3) and Q (10, *y*) is 10 units.

Answer:

It is given that the distance between (2, −3) and (10, *y*) is 10.

Therefore, $\sqrt{(2-10)^2+(-3-y)^2}=10$

$\sqrt{(-8)^2+(3+y)^2}=10$

$64+(y+3)^2=100$

$(y+3)^2=36$

$y+3=\pm 6$

$y+3=6$ or $y+3=-6$

Therefore, $y=3$ or -9

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Question 9:

If Q (0, 1) is equidistant from P (5, − 3) and R (*x*, 6), find the values of *x*. Also find the distance QR and PR.

Answer:

$PQ = QR$

$\sqrt{(5-0)^2+(-3-1)^2}=\sqrt{(0-x)^2+(1-6)^2}$

$\sqrt{(5)^2+(-4)^2}=\sqrt{(-x)^2+(-5)^2}$

$\sqrt{25+16}=\sqrt{x^2+25}$

$41=x^2+25$

$16=x^2$

$x=\pm 4$

Therefore, point R is (4, 6) or (−4, 6).

When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$
$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (−4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$
$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (− 3, 4).

Answer:

Point (x, y) is equidistant from (3, 6) and (−3, 4).

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$
$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$
$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$
$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$
$$36 - 16 = 6x + 6x + 12y - 8y$$
$$20 = 12x + 4y$$
$$3x + y = 5$$
$$3x + y - 5 = 0$$

Question 1:

Find the coordinates of the point which divides the join of (− 1, 7) and (4, − 3) in the ratio 2:3.
Answer:

Let P(x, y) be the required point. Using the section formula, we obtain

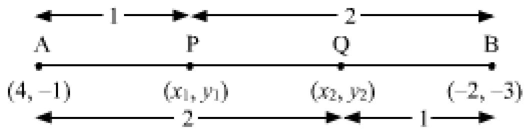
$$x = \frac{2 \times 4 + 3 \times (-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{2 \times (-3) + 3 \times 7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Therefore, the point is (1, 3).

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, − 1) and (− 2, − 3).
Answer:



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1\times(-2)+2\times 4}{1+2}, \quad y_1 = \frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

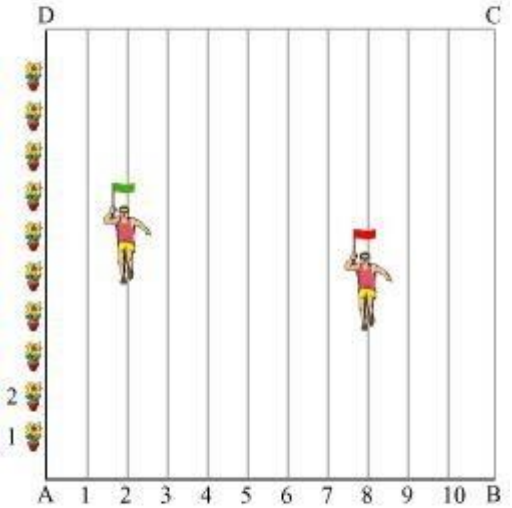
$$x_2 = \frac{2\times(-2)+1\times 4}{2+1}, \quad y_2 = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \qquad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20$ m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61}\text{ m}$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (*x*, *y*).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$
$$x = \frac{10}{2} = 5, \; y = \frac{45}{2} = 22.5$$
Hence, $A(x, y) = (5, 22.5)$

Therefore, Rashmi should post her blue flag at 22.5m on 5^{th} line.

Question 4:

Find the ratio in which the line segment joining the points (− 3, 10) and (6, − 8) is divided by (− 1, 6).
Answer:
Let the ratio in which the line segment joining (−3, 10) and (6, −8) is divided by point (−1, 6) be k : 1.

Therefore, $-1 = \frac{6k-3}{k+1}$
$$-k - 1 = 6k - 3$$
$$7k = 2$$
$$k = \frac{2}{7}$$
Therefore, the required ratio is $2:7$.

Question 5:
Find the ratio in which the line segment joining A (1, − 5) and B (− 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.
Answer:
Let the ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by x-axisbe $k:1$.

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$.

We know that *y*-coordinate of any point on *x*-axis is 0.

$$\therefore \frac{5k-5}{k+1} = 0$$
$$k = 1$$

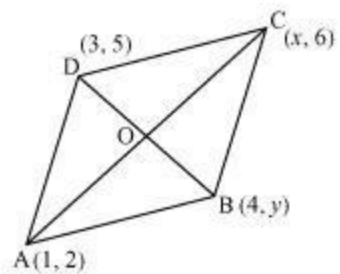
Therefore, *x*-axis divides it in the ratio 1:1.

Division point $= \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right) = \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$

Question 6:

If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Answer:



Let (1, 2), (4, *y*), (*x*, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2},\frac{2+6}{2}\right)\Rightarrow\left(\frac{x+1}{2},4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2},\frac{5+y}{2}\right)\Rightarrow\left(\frac{7}{2},\frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore\frac{x+1}{2}=\frac{7}{2}\text{ and }4=\frac{5+y}{2}$$
$$\Rightarrow x+1=7\text{ and }5+y=8$$
$$\Rightarrow x=6\text{ and }y=3$$

Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, − 3) and B is (1, 4)
Answer:

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, −3), which is the center of the circle.

$$\therefore (2,-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$$\Rightarrow x+1 = 4 \text{ and } y+4 = -6$$

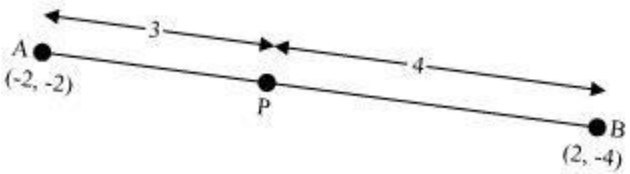
$$\Rightarrow x = 3 \text{ and } y = -10$$

Therefore, the coordinates of A are $(3, -10)$.

Question 8:

If A and B are (− 2, − 2) and (2, − 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Answer:



The coordinates of point A and B are (−2, −2) and (2, −4) respectively.

Since $AP = \frac{3}{7}AB$,

Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4.

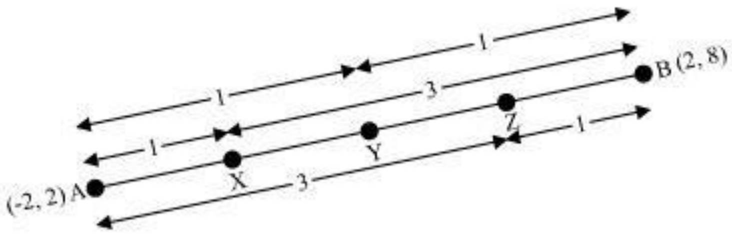
$$\text{Coordinates of P} = \left(\frac{3\times 2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Question 9:

Find the coordinates of the points which divide the line segment joining A (− 2, 2) and B (2, 8) into four equal parts.
Answer:



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of P} = \left(\frac{1\times 2+3\times(-2)}{1+3}, \frac{1\times 8+3\times 2}{1+3}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

$$\text{Coordinates of Q} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$

$$= (0, 5)$$

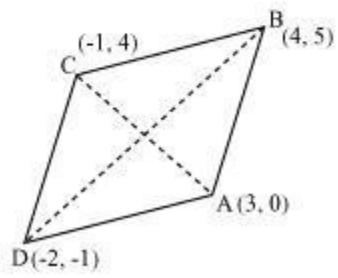
$$\text{Coordinates of R} = \left(\frac{3\times 2+1\times(-2)}{3+1}, \frac{3\times 8+1\times 2}{3+1}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

Question 10:

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (− 1, 4) and (− 2, −1) taken in order. [**Hint:** Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer:



Let (3, 0), (4, 5), (−1, 4) and (−2, −1) are the vertices A, B, C, D of a rhombus ABCD.

$$\text{Length of diagonal AC} = \sqrt{\left[3-(-1)\right]^2 + (0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$
$$\text{Length of diagonal BD} = \sqrt{\left[4-(-2)\right]^2 + \left[5-(-1)\right]^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$
$$\text{Therefore, area of rhombus ABCD} = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ square units}$$

Question 1:

Find the area of the triangle whose vertices are:

(i) (2, 3), (− 1, 0), (2, − 4) (ii) (− 5, − 1), (3, − 5), (5, 2)
Answer:

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\text{Area of the given triangle} = \frac{1}{2}\left[2\left\{0-(-4)\right\} + (-1)\left\{(-4)-(3)\right\} + 2(3-0)\right]$$

$$= \frac{1}{2}\left\{8+7+6\right\}$$

$$= \frac{21}{2} \text{ square units}$$

(ii)

$$\text{Area of the given triangle} = \frac{1}{2}\left[(-5)\left\{(-5)-(2)\right\} + 3\left(2-(-1)\right) + 5\left\{-1-(-5)\right\}\right]$$

$$= \frac{1}{2}\left\{35+9+20\right\}$$

$$= 32 \text{ square units}$$

Question 2:

In each of the following find the value of '*k*', for which the points are collinear.

(i) (7, − 2), (5, 1), (3, − *k*) (ii) (8, 1), (*k*, − 4), (2, − 5)

Answer:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, −2) (5, 1), and (3, *k*), area = 0

$$\frac{1}{2}\left[7\{1-k\}+5\{k-(-2)\}+3\{(-2)-1\}\right]=0$$
$$7-7k+5k+10-9=0$$
$$-2k+8=0$$
$$k=4$$

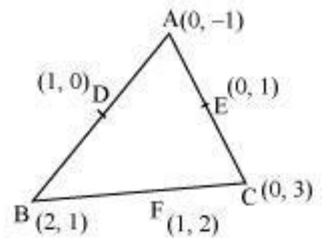
(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, −4), and (2, −5), area = 0

$$\frac{1}{2}\left[8\{-4-(-5)\}+k\{(-5)-(1)\}+2\{1-(-4)\}\right]=0$$
$$8-6k+10=0$$
$$6k=18$$
$$k=3$$

Question 3:

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, − 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
Answer:



Let the vertices of the triangle be A (0, −1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$E = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}$$

$$\text{Area of } \Delta DEF = \frac{1}{2}\left\{1(2-1) + 1(1-0) + 0(0-2)\right\}$$

$$= \frac{1}{2}(1+1) = 1 \text{ square units}$$

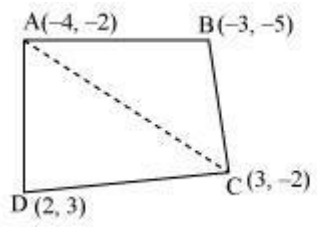
$$\text{Area of } \Delta ABC = \frac{1}{2}\left[0(1-3) + 2\left\{3-(-1)\right\} + 0(-1-1)\right]$$

$$= \frac{1}{2}\{8\} = 4 \text{ square units}$$

Therefore, required ratio $= 1:4$

Question 4:

Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3)
Answer:



Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\{(-5)-(-2)\} + (-3)\{(-2)-(-2)\} + 3\{(-2)-(-5)\}\right]$$

$$= \frac{1}{2}(12 + 0 + 9) = \frac{21}{2} \text{ square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\{(-2)-(3)\} + 3\{(3)-(-2)\} + 2\{(-2)-(-2)\}\right]$$

$$= \frac{1}{2}\{20 + 15 + 0\} = \frac{35}{2} \text{ square units}$$

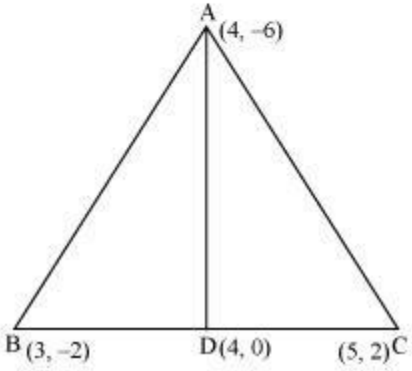
$$\text{Area of } \square ABCD = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2} + \frac{35}{2}\right) \text{square units} = 28 \text{ square units}$$

Question 5:

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, − 6), B (3, − 2) and C (5, 2)

Answer:



Let the vertices of the triangle be A (4, −6), B (3, −2), and C (5, 2).

Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4,0)$$

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right\}$$

$$\text{Area of } \Delta ABD = \frac{1}{2}\Big[(4)\{(-2)-(0)\}+(3)\{(0)-(-6)\}+(4)\{(-6)-(-2)\}\Big]$$

$$= \frac{1}{2}(-8+18-16) = -3 \text{ square units}$$

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

$$\text{Area of } \Delta ADC = \frac{1}{2}\Big[(4)\{0-(2)\}+(4)\{(2)-(-6)\}+(5)\{(-6)-(0)\}\Big]$$

$$= \frac{1}{2}(-8+32-30) = -3 \text{ square units}$$

However, area cannot be negative. Therefore, area of ΔADC is 3 square units.

Clearly, median AD has divided ΔABC in two triangles of equal areas.

Page No 171:

Question 1:

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, − 2) and B(3, 7)

Answer:

Let the given line divide the line segment joining the points A(2, −2) and B(3, 7) in a ratio $k : 1$.

Coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

$$\therefore 2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$
$$\Rightarrow \frac{6k+4+7k-2-4k-4}{k+1} = 0$$
$$\Rightarrow 9k - 2 = 0$$
$$\Rightarrow k = \frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, −2) and B(3, 7) is 2:9.

Question 2:

Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Answer:

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\text{Area} = \frac{1}{2}\left[x\left(2-0\right) + 1\left(0-y\right) + 7\left(y-2\right)\right]$$

$$0 = \frac{1}{2}\left[2x - y + 7y - 14\right]$$

$$0 = \frac{1}{2}\left[2x + 6y - 14\right]$$

$$2x + 6y - 14 = 0$$

$$x + 3y - 7 = 0$$

This is the required relation between x and y.

Question 3:

Find the centre of a circle passing through the points (6, − 6), (3, − 7) and (3, 3).
Answer:

Let O (x, y) be the centre of the circle. And let the points (6, −6), (3, −7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2}$$

$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$

$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

However, OA = OB (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$

$$\Rightarrow -6x - 2y + 14 = 0$$

$$\Rightarrow 3x + y = 7 \quad \ldots (1)$$

Similarly, OA = OC (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$

$$\Rightarrow -6x + 18y + 54 = 0$$

$$\Rightarrow -3x + 9y = -27 \quad \ldots (2)$$

On adding equation (1) and (2), we obtain

$10y = -20$
$y = -2$

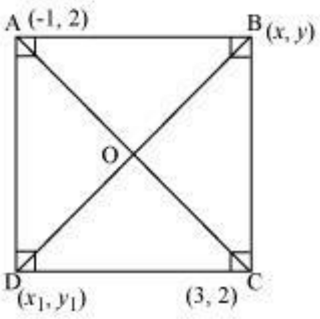
From equation (1), we obtain

$3x - 2 = 7$
$3x = 9$
$x = 3$

Therefore, the centre of the circle is (3, −2).

Question 4:

The two opposite vertices of a square are (− 1, 2) and (3, 2). Find the coordinates of the other two vertices.
Answer:



Let ABCD be a square having (−1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

∴ AB = BC

$$\Rightarrow \sqrt{(x+1)^2+(y-2)^2}=\sqrt{(x-3)^2+(y-2)^2}$$
$$\Rightarrow x^2+2x+1+y^2-4y+4=x^2+9-6x+y^2+4-4y$$
$$\Rightarrow 8x=8$$
$$\Rightarrow x=1$$

We know that in a square, all interior angles are of 90°.

In ΔABC,

$AB^2 + BC^2 = AC^2$

$$\Rightarrow \left(\sqrt{(1+1)^2+(y-2)^2}\right)^2+\left(\sqrt{(1-3)^2+(y-2)^2}\right)^2=\left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$\Rightarrow 2y^2 + 16 - 8y = 16$
$\Rightarrow 2y^2 - 8y = 0$
$\Rightarrow y(y - 4) = 0$
$\Rightarrow y = 0$ or 4

We know that in a square, the diagonals are of equal length and bisect each other at 90°. Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O} = \left(\frac{-1+3}{2}, \frac{2+2}{2}\right)$$

$$\left(\frac{1+x_1}{2}, \frac{y+y_1}{2}\right) = (1, 2)$$

$$\frac{1+x_1}{2} = 1$$

$$1 + x_1 = 2$$

$$x_1 = 1$$

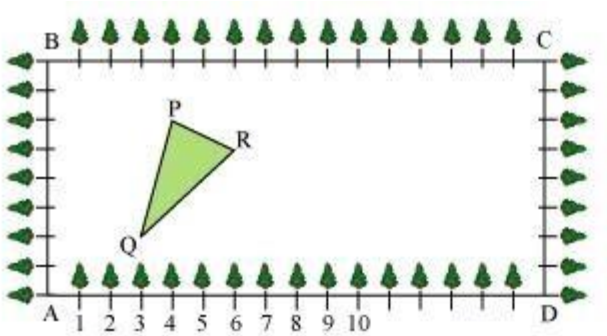
$$\text{and } \frac{y+y_1}{2} = 2$$

$\Rightarrow y + y_1 = 4$
If $y = 0$,
$y_1 = 4$
If $y = 4$,
$y_1 = 0$

Therefore, the required coordinates are (1, 0) and (1, 4).

Question 5:

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?
Answer:

(i) Taking A as origin, we will take AD as *x*-axis and AB as *y*-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$$
$$= \frac{1}{2}\left[4(2-5) + 3(5-6) + 6(6-2)\right]$$
$$= \frac{1}{2}\left[-12 - 3 + 24\right]$$
$$= \frac{9}{2}\text{ square units}$$

(ii) Taking C as origin, CB as *x*-axis, and CD as *y*-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

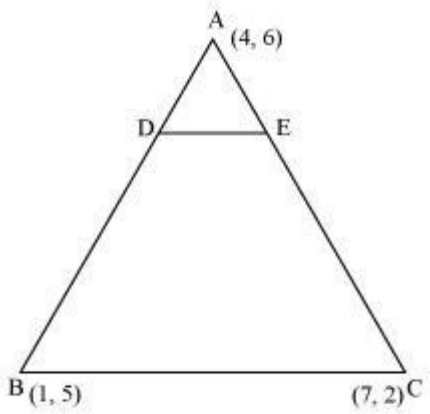
$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 + y_2\right)\right]$$
$$= \frac{1}{2}\left[12(6-3) + 13(3-2) + 10(2-6)\right]$$
$$= \frac{1}{2}\left[36 + 13 - 40\right]$$
$$= \frac{9}{2}\text{ square units}$$

It can be observed that the area of the triangle is same in both the cases.

Question 6:

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to

ratio of areas of two similar triangles)
Answer:



Given that, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\frac{AD}{AD + DB} = \frac{AE}{AE + EC} = \frac{1}{4}$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\text{Coordinates of Point D} = \left(\frac{1 \times 1 + 3 \times 4}{1 + 3}, \frac{1 \times 5 + 3 \times 6}{1 + 3}\right)$$

$$= \left(\frac{13}{4}, \frac{23}{4}\right)$$

$$\text{Area of a triangle} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$$

$$\text{Coordinates of point E} = \left(\frac{1\times 7 + 3\times 4}{1+3}, \frac{1\times 2 + 3\times 6}{1+3}\right) = \left(\frac{19}{4}, \frac{20}{4}\right)$$

$$\text{Area of } \Delta ADE = \frac{1}{2}\left[4\left(\frac{23}{4} - \frac{20}{4}\right) + \frac{13}{4}\left(\frac{20}{4} - 6\right) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right]$$

$$= \frac{1}{2}\left[3 - \frac{13}{4} + \frac{19}{16}\right] = \frac{1}{2}\left[\frac{48 - 52 + 19}{16}\right] = \frac{15}{32} \text{ square units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[4\left(5-2\right) + 1\left(2-6\right) + 7\left(6-5\right)\right]$$

$$= \frac{1}{2}\left[12 - 4 + 7\right] = \frac{15}{2} \text{ square units}$$

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

Therefore, ratio between the areas of ΔADE and ΔABC = $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Question 7:

Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

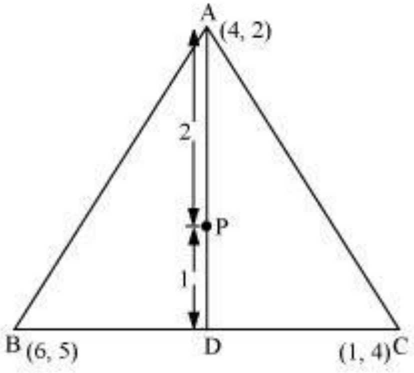
(i) The median from A meets BC at D. Find the coordinates of point D.

(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.
Answer:



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3).

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

$$\text{Coordinates of O} = \left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

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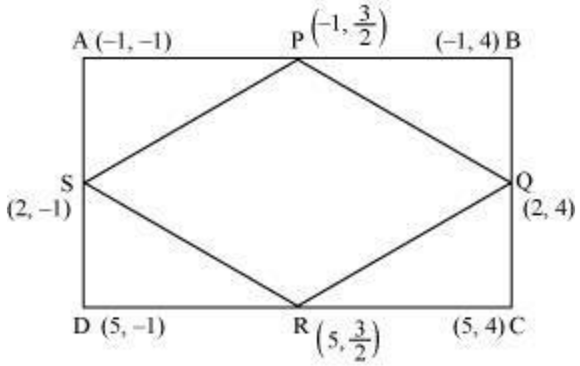
Question 8:

ABCD is a rectangle formed by the points A (− 1, − 1), B (− 1, 4), C (5, 4) and D (5, − 1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? a rectangle? or a rhombus? Justify your answer.
Answer:



P is the mid-point of side AB.

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Similarly, the coordinates of Q, R, and S are $(2, 4)$, $\left(5, \frac{3}{2}\right)$, and $(2, -1)$ respectively.

$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.