NCERT Solutions for Class 10 Math Chapter 4 – Quadratic Equations

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Question 1:

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$ (iii) (x-2)(x+1) = (x-1)(x+3)(iv) (x-3)(2x+1) = x(x+5)(v) (2x-1)(x-3) = (x+5)(x-1)(vi) $x^2 + 3x + 1 = (x-2)^2$ (vii) $(x+2)^3 = 2x(x^2-1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$ Answer:

(i)
$$(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii)
$$x^2 - 2x = (-2)(3 - x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii)
$$(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv)
$$(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v)
$$(2x-1)(x-3) = (x+5)(x-1) \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$$

It is of the form $ax^2 + bx + c = 0$

Hence, the given equation is a quadratic equation.

(vi)
$$x^{2} + 3x + 1 = (x - 2)^{2} \Rightarrow x^{2} + 3x + 1 = x^{2} + 4 - 4x \Rightarrow 7x - 3 = 0$$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(vii)
$$(x+2)^3 = 2x(x^2-1) \Rightarrow x^3+8+6x^2+12x = 2x^3-2x \Rightarrow x^3-14x-6x^2-8=0$$

It is not of the form $ax^2+bx+c=0$

Hence, the given equation is not a quadratic equation.

(viii)
$$x^3 - 4x^3 - x + 1 = (x - 2)^3 \Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x \Rightarrow 2x^2 - 13x + 9 = 0$$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

Question 2:

Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train. Answer:

(i) Let the breadth of the plot be x m.

Hence, the length of the plot is (2x + 1) m.

Area of a rectangle = Length × Breadth

 $\therefore 528 = x (2x + 1)$

$$\Rightarrow 2x^2 + x - 528 = 0$$

(ii) Let the consecutive integers be x and x + 1.

It is given that their product is 306.

 $\therefore x(x+1) = 306 \Longrightarrow x^2 + x - 306 = 0$

(iii) Let Rohan's age be *x*.

Hence, his mother's age = x + 26

3 years hence,

Rohan's age = x + 3

Mother's age = x + 26 + 3 = x + 29

It is given that the product of their ages after 3 years is 360.

 $\therefore (x+3)(x+29) = 360$ $\Rightarrow x^2 + 32x - 273 = 0$

(iv) Let the speed of train be x km/h.

Time taken to travel 480 km = $\frac{480}{x}$ hrs

In second condition, let the speed of train = (x-8) km/h

It is also given that the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}+3\right)$ hrs

Speed × Time = Distance

$$\left(x-8\right)\left(\frac{480}{x}+3\right) = 480$$

⇒480+3x-3840x-24=480

⇒3x-3840x=24

⇒3**x**2-24x-3840=0

⇒x2-8x-1280=0

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Question 1:

Find the roots of the following quadratic equations by factorisation:

(i)	$x^2 - 3x - 10 = 0$	(ii)	$2x^2 + x - 6 = 0$
(iii)	$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$	(iv)	$2x^2 - x + \frac{1}{8} = 0$
(v)	$100x^2 - 20x + 1 = 0$		
Answ	/er:		
(i)	$x^2 - 3x - 10$		

$$= x^{2} - 5x + 2x - 10$$

= x(x-5)+2(x-5)
= (x-5)(x+2)

Roots of this equation are the values for which (x-5)(x+2) = 0

:. x-5 = 0 or x+2 = 0

i.e., x = 5 or x = -2

(ii)
$$2x^{2} + x - 6$$

= $2x^{2} + 4x - 3x - 6$
= $2x(x+2) - 3(x+2)$
= $(x+2)(2x-3)$

Roots of this equation are the values for which (x+2)(2x-3) = 0 $\therefore x+2 = 0 \text{ or } 2x-3 = 0$

i.e.,
$$x = -2$$
 or $x = \frac{3}{2}$
(iii) $\sqrt{2x^2 + 7x + 5\sqrt{2}}$
 $= \sqrt{2x^2 + 5x + 2x + 5\sqrt{2}}$
 $= x(\sqrt{2x} + 5) + \sqrt{2}(\sqrt{2x} + 5)$
 $= (\sqrt{2x} + 5)(x + \sqrt{2})$

Roots of this equation are the values for which $(\sqrt{2}x+5)(x+\sqrt{2})=0$

:.
$$\sqrt{2x+5} = 0$$
 or $x+\sqrt{2} = 0$
i.e., $x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

(iv)
$$2x^{2} - x + \frac{1}{8}$$
$$= \frac{1}{8} (16x^{2} - 8x + 1)$$
$$= \frac{1}{8} (16x^{2} - 4x - 4x + 1)$$
$$= \frac{1}{8} (4x(4x - 1) - 1(4x - 1))$$
$$= \frac{1}{8} (4x - 1)^{2}$$

Roots of this equation are the values for which $(4x-1)^2 = 0$

Therefore, (4x-1) = 0 or(4x-1) = 0

i.e.,

$$\begin{aligned}
x &= \frac{1}{4} \text{ or } x = \frac{1}{4} \\
(v) & 100x^2 - 20x + 1 \\
&= 100x^2 - 10x - 10x + 1 \\
&= 10x(10x - 1) - 1(10x - 1) \\
&= (10x - 1)^2
\end{aligned}$$

Roots of this equation are the values for which $(10x-1)^2 = 0$

Therefore, (10x-1) = 0 or (10x-1) = 0

i.e.,
$$x = \frac{1}{10}$$
 or $x = \frac{1}{10}$

Question 2:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer:

(i) Let the number of John's marbles be *x*.

Therefore, number of Jivanti's marble = 45 - x

After losing 5 marbles,

Number of John's marbles = x - 5

Number of Jivanti's marbles = 45 - x - 5 = 40 - x

It is given that the product of their marbles is 124.

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\therefore (x-5)(40-x) = 124

\Rightarrow x^2 - 45x + 324 = 0

\Rightarrow x^2 - 36x - 9x + 324 = 0

\Rightarrow x(x-36) - 9(x-36) = 0

\Rightarrow (x-36)(x-9) = 0
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Either x - 36 = 0 or x - 9 = 0

i.e., x = 36 or x = 9

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 - 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 - 9 = 36

(ii) Let the number of toys produced be *x*.

 \therefore Cost of production of each toy = Rs (55 - x)

It is given that, total production of the toys = Rs 750

 $\therefore x(55-x) = 750$ $\Rightarrow x^2 - 55x + 750 = 0$ $\Rightarrow x^2 - 25x - 30x + 750 = 0$ $\Rightarrow x(x-25) - 30(x-25) = 0$ $\Rightarrow (x-25)(x-30) = 0$

Either x-25 = 0 or x - 30 = 0i.e., x = 25 or x = 30

Hence, the number of toys will be either 25 or 30.

Question 3:

Find two numbers whose sum is 27 and product is 182.

Answer:

Let the first number be x and the second number is 27 - x. Therefore, their product = x (27 - x)

It is given that the product of these numbers is 182.

Therefore, x(27-x) = 182 $\Rightarrow x^2 - 27x + 182 = 0$ $\Rightarrow x^2 - 13x - 14x + 182 = 0$ $\Rightarrow x(x-13) - 14(x-13) = 0$ $\Rightarrow (x-13)(x-14) = 0$

Either x-13 = 0 or x - 14 = 0i.e., x = 13 or x = 14If first number = 13, then Other number = 27 - 13 = 14 If first number = 14, then Other number = 27 - 14 = 13

Therefore, the numbers are 13 and 14.

Question 4:

Find two consecutive positive integers, sum of whose squares is 365. Answer:

Let the consecutive positive integers be x and x + 1.

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Given that x^{2} + (x+1)^{2} = 365

\Rightarrow x^{2} + x^{2} + 1 + 2x = 365

\Rightarrow 2x^{2} + 2x - 364 = 0

\Rightarrow x^{2} + x - 182 = 0

\Rightarrow x^{2} + 14x - 13x - 182 = 0

\Rightarrow x(x+14) - 13(x+14) = 0

\Rightarrow (x+14)(x-13) = 0
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Either x + 14 = 0 or x - 13 = 0, i.e., x = -14 or x = 13Since the integers are positive, x can only be 13. $\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Question 5:

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides. Answer:

Let the base of the right triangle be x cm.

Its altitude = (x - 7) cm

From pythagoras theorem, Base² + Altitude² = Hypotenuse² $\therefore x^2 + (x-7)^2 = 13^2$ $\Rightarrow x^2 + x^2 + 49 - 14x = 169$ $\Rightarrow 2x^2 - 14x - 120 = 0$ $\Rightarrow x^2 - 7x - 60 = 0$ $\Rightarrow x^2 - 12x + 5x - 60 = 0$ $\Rightarrow x(x-12) + 5(x-12) = 0$ $\Rightarrow (x-12)(x+5) = 0$

Either x - 12 = 0 or x + 5 = 0, i.e., x = 12 or x = -5Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 - 7) cm = 5 cm.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article. Answer:

Let the number of articles produced be *x*.

Therefore, cost of production of each article = Rs(2x + 3)

It is given that the total production is Rs 90.

 $\therefore x(2x+3) = 90$ $\Rightarrow 2x^{2} + 3x - 90 = 0$ $\Rightarrow 2x^{2} + 15x - 12x - 90 = 0$ $\Rightarrow x(2x+15) - 6(2x+15) = 0$ $\Rightarrow (2x+15)(x-6) = 0$

Either 2x + 15 = 0 or x - 6 = 0, i.e., $x = \frac{-15}{2}$ or x = 6As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3 = \text{Rs} \ 15$

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Question 1:

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$ (iii) $4x^2 + 4\sqrt{3x} + 3 = 0$ (iv) $2x^2 + x + 4 = 0$ Answer: (i) $2x^2 - 7x + 3 = 0$ $\Rightarrow 2x^2 - 7x = -3$ On dividing both sides of the equation by 2, we obtain $\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$ $\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$ On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation, we obtain $\Rightarrow \left(x\right)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$ $\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$ $\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$ $\Rightarrow \left(x - \frac{7}{4}\right) = \pm \frac{5}{4}$ $\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$ $\Rightarrow x = \frac{7}{4} + \frac{5}{4} \text{ or } x = \frac{7}{4} - \frac{5}{4}$ $\Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4}$ $\Rightarrow x = 3 \text{ or } \frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$ $\Rightarrow 2x^2 + x = 4$ On dividing both sides of the equation by 2, we obtain $\Rightarrow x^2 + \frac{1}{2}x = 2$ On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain $\Rightarrow (x)^{2} + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^{2} = 2 + \left(\frac{1}{4}\right)^{2}$ $\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$ $\Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4}$ $\Rightarrow x = \pm \frac{\sqrt{33}}{4} - \frac{1}{4}$ $\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$ $\Rightarrow x = \frac{\sqrt{33} - 1}{4} \text{ or } \frac{-\sqrt{33} - 1}{4}$

(iii)
$$4x^{2} + 4\sqrt{3}x + 3 = 0$$
$$\Rightarrow (2x)^{2} + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^{2} = 0$$
$$\Rightarrow (2x + \sqrt{3})^{2} = 0$$
$$\Rightarrow (2x + \sqrt{3}) = 0 \text{ and } (2x + \sqrt{3}) = 0$$
$$\Rightarrow x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

 $\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

 $\Rightarrow x^{2} + \frac{1}{2}x = -2$ $\Rightarrow x^{2} + 2 \times x \times \frac{1}{4} = -2$ On adding $\left(\frac{1}{4}\right)^{2}$ to both sides of the equation, we obtain $\Rightarrow (x)^{2} + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^{2} = \left(\frac{1}{4}\right)^{2} - 2$ $\Rightarrow \left(x + \frac{1}{4}\right)^{2} = \frac{1}{16} - 2$ $\Rightarrow \left(x + \frac{1}{4}\right)^{2} = -\frac{31}{16}$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

Question 2:

Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula. Answer:

(i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain a = 2, b = -7, c = 3

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4} \text{ or } \frac{7 - 5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain a = 2, b = 1, c = -4

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore x = \frac{-1 \pm \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 4, b = 4\sqrt{3}, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$
$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$
$$\therefore x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = 1, c = 4$$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$
$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

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Question 3:

Find the roots of the following equations:

(i)
$$x - \frac{1}{x} = 3, x \neq 0$$
 (ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$
Answer:

(i)
$$x - \frac{1}{x} = 3 \implies x^2 - 3x - 1 = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain a = 1, b = -3, c = -1

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore, $x = \frac{3 \pm \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$

(ii)
$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$
$$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$
$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$
$$\Rightarrow (x+4)(x-7) = -30$$
$$\Rightarrow x^2 - 3x - 28 = -30$$
$$\Rightarrow x^2 - 3x - 28 = -30$$
$$\Rightarrow x^2 - 3x + 2 = 0$$
$$\Rightarrow x^2 - 2x - x + 2 = 0$$
$$\Rightarrow x(x-2) - 1(x-2) = 0$$
$$\Rightarrow (x-2)(x-1) = 0$$
$$\Rightarrow x = 1 \text{ or } 2$$

Question 4:

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 3. Find his present age. Answer:

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Let the present age of Rehman be *x* years.

Three years ago, his age was (x - 3) years.

Five years hence, his age will be (x + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

 $\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$ $\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$ $\frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$ $\Rightarrow 3(2x+2) = (x-3)(x+5)$ $\Rightarrow 6x+6 = x^2 + 2x - 15$ $\Rightarrow x^2 - 4x - 21 = 0$ $\Rightarrow x^2 - 7x + 3x - 21 = 0$ $\Rightarrow x(x-7) + 3(x-7) = 0$ $\Rightarrow (x-7)(x+3) = 0$ $\Rightarrow x = 7, -3$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.