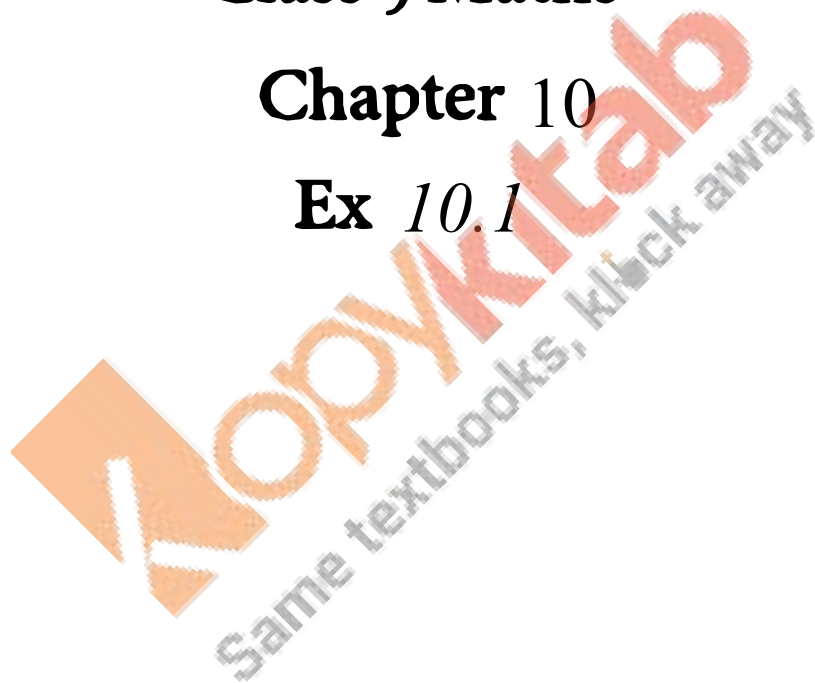# RD SHARMA

Solutions

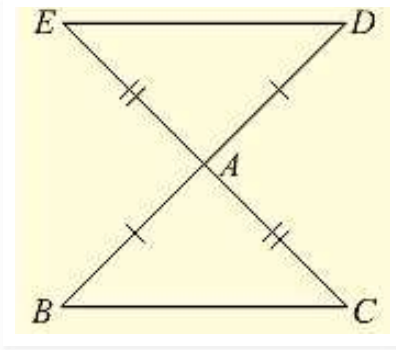# Class 9 Maths

Chapter 10

Ex 10.1

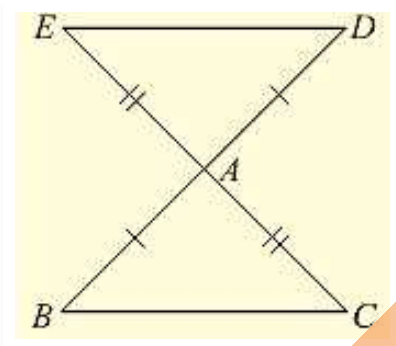(1) In Fig. (10).22, the sides BA and CA have been produced such that: BA = AD and CA = AE. Prove that segment DE ∥ BC.

Solution:

Given that, the sides BA and CA have been produced such that BA = AD and CA = AE and given to prove DE ∥ BC

Consider triangle BAC and DAE,

We have

BA = AD and CA= AE   [given in the data]

And also $\angle BAC = \angle DAE$ [vertically opposite angles]

So, by SAS congruence criterion, we have

$\angle BAC \simeq \angle DAE$

   BC = DE and $\angle DEA = \angle BCA, \angle EDA = \angle CBA$

[Corresponding parts of congruent triangles are equal]

Now, DE and BC are two lines intersected by a transversal DB such that $\angle DEA = \angle BCA$ i.e.. alternate angles are equal Therefore, DE BC ∥ BC.

(2) In a PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Solution: Given that,

In PQR, PQ = QR and L, M, N are midpoints of the sides PQ, QP and RP respectively and given to prove that LN = MN

Here we can observe that PQR is an isosceles triangle
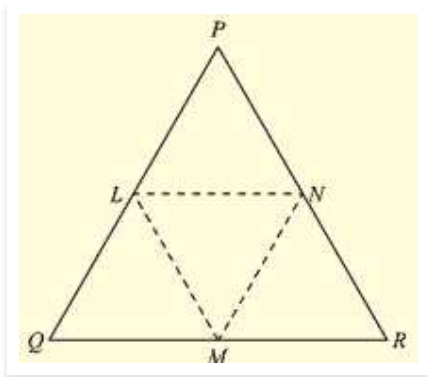
PQ = QR and ∠ QPR = ∠ QRP _____(i)

And also, L and M are midpoints of PQ and QR respectively

$$PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2}$$

And also, PQ = QR

Now, consider △ LPN and △ MRN, LP = MR   [From − (2)]

∠ LPN = ∠ MRN __ [From − (1)]

∠ QPR and ∠ LPN and ∠ QRP and ∠ MRN are same.

PN = NR   [N is midpoint of PR]
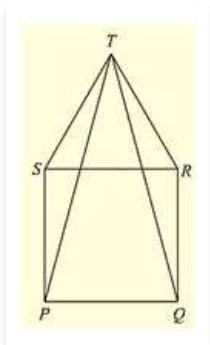
So, by SAS congruence criterion, we have △ LPN = △ MRN

LN = MN [Corresponding parts of congruent triangles are equal]

(3) In fig. (10).23, PQRS is a square and SRT is an equilateral triangle. Prove that (i) PT = QT (ii) ∠ TQR = $15°$
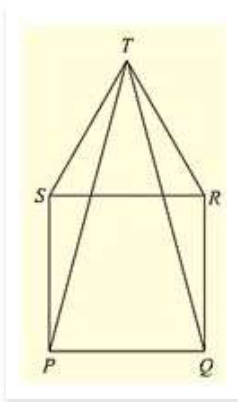
Solution: Given that PQRS is a square and SRT is an equilateral triangle. And given to prove that

(i) PT =QT and (ii) ∠ TQR =15°

Now, PQRS is a square

PQ = QR = RS = SP ____(i)
And ∠ SPQ = ∠ PQR = ∠ QRS = ∠ RSP = 90 ° = right angle

And also, SRT is an equilateral triangle.

SR = RT = TS ___(ii)

And ∠ TSR = ∠ SRT = ∠ RTS = 60°

From (i) and (ii)

PQ = QR = SP = SR = RT = TS ___(iii)

And also,

∠ TSP = ∠ TSR + ∠ RSP = 60° + 90° + 150°   ∠TRQ = ∠TRS + ∠ SRQ = 60° + 90° +
150°   => ∠ TSR = ∠ TRQ = 150°   ___ (iv)

SP = RQ   [From (iii)]

So, by SAS congruence criterion we have

△ TSP = △ TRQ

PT = QT   [Corresponding parts of congruent triangles are equal] Consider △ TQR.

QR = TR   [From (iii)]

△ TQR is a isosceles triangle.

∠ QTR = ∠ TQR [angles opposite to equal sides]

Now,

Sum of angles in a triangle is equal to 180°

=> ∠ QTR + ∠ TQR + ∠TRQ = 180°

=> 2 ∠ TQR + 150° = 180°   [From (iv)]

=> 2 ∠ TQR = 180° − 150°

=> 2 ∠ TQR = 30°   ∠ TQR = 15°]...

Hence proved

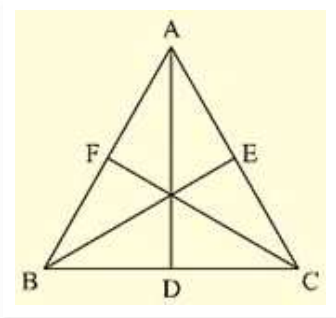(4) Prove that the medians of an equilateral triangle are equal.

Solution:

Given,

To prove the medians of an equilateral triangle are equal.

Median: The line Joining the vertex and midpoint of opposite side. Now, consider an equilateral triangle ABC.

Let D,E,F are midpoints of BC, CA and AB.

Then, AD, BE and CF are medians of ABC.

Now,

D Is midpoint of BC => BD = DC = $\frac{BC}{2}$

Similarly, CE = EA = $\frac{AC}{2}$

AF = FB = $\frac{AB}{2}$

Since $\triangle$ ABC is an equilateral triangle

=> AB = BC = CA   ____(i)

=> BD = DC = CE = EA = AF = FB = $\frac{BC}{2}$ = $\frac{AC}{2}$ = $\frac{AB}{2}$ ____(ii)

And also, $\angle$ ABC = $\angle$ BCA = $\angle$ CAB = 60° ____(iii)

Now, consider $\triangle$ ABD and $\triangle$ BCE AB = BC   [From (i)]

BD = CE   [From (ii)]

Now, in $\triangle$ TSR and $\triangle$ TRQ

TS = TR   [From (iii)]

$\angle$ ABD = $\angle$ BCE   [From (iii)] [$\angle$ ABD and $\angle$ ABC and $\angle$ BCE and $\angle$BCA are same]

So, from SAS congruence criterion, we have

$\triangle$ ABD = $\triangle$ BCE

AD = BE   ___(iv)

[Corresponding parts of congruent triangles are equal]

Now, consider $\triangle$ BCE and $\triangle$ CAF, BC = CA   [From (i)]

$\angle$ BCE = $\angle$ CAF   [From (ii)]

[$\angle$ BCE and $\angle$ BCA and $\angle$ CAF and $\angle$ CAB are same]

CE = AF   [From (ii)]

So, from SAS congruence criterion, we have

$\triangle$ BCE = $\triangle$ CAF

    BE = CF (v)

[Corresponding parts of congruent triangles are equal]

From (iv) and (v), we have

AD = BE = CF

Median AD = Median BE = Median CF
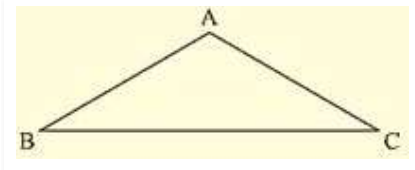
The medians of an equilateral triangle are equal.

Hence proved

(5) In a $\triangle$ ABC, if $\angle$ A = 120° and AB = AC. Find $\angle$ B and $\angle$ C.

Solution:

Consider a $\triangle$ ABC

Given Mat $\angle$ A = 120° and AB = AC and given to find $\angle$ B and $\angle$ C.

We can observe that $\triangle$ ABC is an isosceles triangle since AB = AC

$\qquad \angle$ B = $\angle$ C (i)

[Angles opposite to equal sides are equal]

We know that sum of angles in a triangle is equal to 180°

=> $\angle$ A + $\angle$ B + $\angle$ C= 180°

=> $\angle$ A + $\angle$ B + $\angle$ B= 180°   [From (i)]

=> 120° + 2$\angle$B = 180°

=> 2$\angle$B = 180° − 120°

=> $\angle$ B = $\angle$ C = 30°

(6) In a $\triangle$ ABC, if AB = AC and $\angle$ B = 70°. Find $\angle$ A.

Solution:

Consider a $\triangle$ ABC, if AB = AC and $\angle$ B = 70°

Since, AB = AC $\triangle$ ABC is an isosceles triangle

$\qquad \angle$ B = $\angle$ C [Angles opposite to equal sides are equal]
$\qquad \angle$ B = $\angle$ C = 70°

And also,

Sum of angles in a triangle = 180°

$\qquad \angle$ A + $\angle$ B +$\angle$ C = 180°
$\qquad \angle$ A + 70° + 70° = 180°
$\qquad \angle$ A = 180° − 140°
$\qquad \angle$ A = 40°

(7) The vertical angle of an isosceles triangle is (10)0°. Find its base angles.

Solution:

Consider an isosceles $\triangle$ABC such that AB = AC

Given that vertical angle A is (10)0°

To find the base angles

Since △ ABC is isosceles

∠ B = ∠ C [Angles opposite to equal sides are equal]

And also,

Sum of interior angles of a triangle = 180 °

∠ A + ∠ B +∠ C = 180 °
(10)0 ° + ∠ B ∠ B = 180 °
2∠ B = 180° − (10)0 °
∠ B = 40 °
∠ B = ∠ C = 40 °

(8) In Fig. (10).24, AB = AC and ∠ ACD = (10)5 °. Find ∠ BAC.

Solution:

Consider the given figure

We have,

AB = AC and ∠ ACD = (10)5 °

Since, ∠ BCD = 180° = Straight angle

∠ BCA + ∠ ACD =180°
∠ BCA + (10)5° = 180°
∠ BCA = I80° -(10)5°
∠BCA= 75°

And also,

△ABC is an isosceles triangle   [AB = AC]

∠ ABC = ∠ ACB [Angles opposite to equal sides are equal]

From (i), we have

∠ ACB = 75°

∠ ABC = ∠ ACB = 75°

And also,

Sum of Interior angles of a triangle = 180 °

∠ ABC = ∠ BCA + ∠ CAB =180°
75° + 75° + ∠ CAB =180°
150° + ∠ BAC = 180°
∠ BAC = 180° -150° = 30°
∠ BAC = 30°

(9) Find the measure of each exterior angle of an equilateral triangle.

Solution:

Given to find the measure of each exterior angle of an equilateral triangle consider an equilateral triangle ABC.

We know that for an equilateral triangle

AB = BC = CA and ∠ ABC = ∠ BCA= CAB = $\frac{180°}{3}$ = 60°   ___(i)

Now,

Extend side BC to D, CA to E and AB to F.

Here BCD is a straight line segment

    BCD = Straight angle =180°
    ∠ BCA + ∠ ACD = 180° [From (i)]
    60° + ∠ACD = 180°
    ∠ACD = 120°

Similarly, we can find ∠ FAB and ∠ FBC also as 120° because ABC is an equilateral triangle

∠ ACD = ∠ EAB − ∠ FBC = 120°

Hence, the median of each exterior angle of an equilateral triangle is 120°

(10) If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.
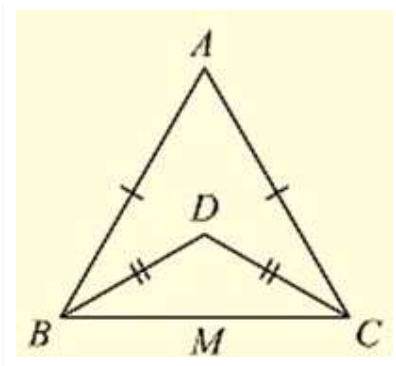
Solution:

ED is a straight line segment and B and C an points on it.

    ∠ EBC = ∠ BCD = straight angle = 180°
    ∠ EBA+ ∠ ABC = ∠ ACB + ∠ ACD
    ∠ EBA = ∠ ACD + ∠ ACB − ∠ ABC
    ∠ EBA = ∠ ACD [From (i) ABC = ACD]

∠ ABE = ∠ ACD
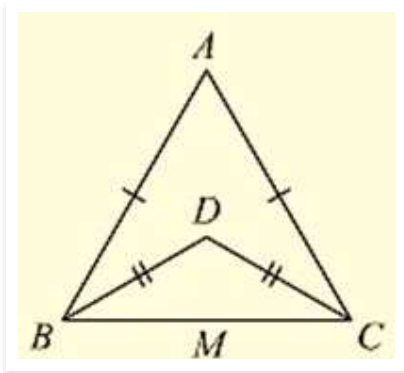
Hence proved

(11) In Fig. (10).2(5) AB = AC and DB = DC, find the ratio ∠ ABD : ∠ ACD.

Solution:

Consider the figure

Given,

AB = AC, DB = DC and given to find the ratio

∠ ABD = ∠ ACD

Now, Δ ABC and Δ DBC are isosceles triangles since AB = AC and DB = DC respectively

∠ ABC = ∠ ACB and ∠ DBC = ∠ DCB [Angles opposite to equal sides are equal]

Now consider,

∠ ABD : ∠ ACD

(∠ ABC − ∠ DBC) : (∠ ACB − ∠ DCB)
(∠ ABC − ∠ DBC) : (∠ ABC − ∠ DBC) [∠ ABC = ∠ ACB and ∠ DBC = ∠ DCB]
1:1

ABD: ACD = 1:1

(12) Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

 ABC is a right-angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C.

Solution:

ABC is a right angled triangle

Consider on a right − angled isosceles triangle ABC such that

∠ A = 90 ° and AB = AC Since,

AB = AC => ∠ C = ∠ B   ___(i)

[Angles opposite to equal sides are equal]

Now, Sum of angles in a triangle = 180 °

∠ A + ∠ B + ∠ C =180°

=> 90° + ∠ B+ ∠ B = 180°

=> 2∠ B = 90°
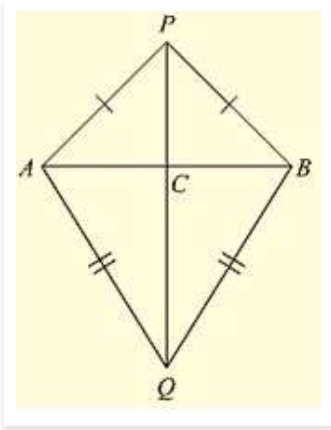
=> ∠ B = 45°

∠ B = 45°, ∠ C = 45°

Hence, the measure of each of the equal angles of a right-angled Isosceles triangle Is 45°

(13) AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See Fig. (10).26). Show that the line PQ is perpendicular bisector of AB.
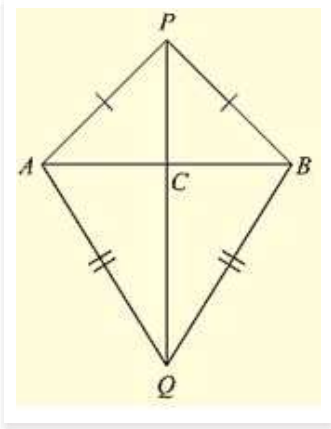
Solution:

Consider the figure.

We have

AB is a line segment and P, Q are points on opposite sides of AB such that

AP = BP   _____(i)

AQ = BQ   _____(ii)

We have to prove that PQ is perpendicular bisector of AB.

Now consider $\triangle$ PAQ and $\triangle$ PBQ,

We have

AP = BP   [From (i)]

AQ = BQ   [From (ii)]

And PQ − PQ   [Common site]

$\triangle$ PAQ $\simeq$ $\triangle$ PBQ   _____(iii) [From SAS congruence]

Now, we can observe that APB and ABQ are isosceles triangles. [From (i) and (ii)]

        $\angle$ PAB = $\angle$ ABQ and $\angle$ QAB = $\angle$ QBA

Now consider $\triangle$ PAC and $\triangle$ PBC

C is the point of intersection of AB and PQ

PA = PB   [From (i)]

∠ APC = ∠ BPC   [From (ii)]

PC = PC   [common side]

So, from SAS congruency of triangle $\Delta PAC \cong \Delta PBC$

AC = CB and ∠ PCA = ∠ PBC _____(iv)   [Corresponding parts of congruent triangles are equal]   And also, ACB is line segment
∠ ACP + ∠ BCP = 180 °
∠ ACP = ∠ PCB
∠ ACP = ∠ PCB = 90°<

We have AC = CB => C is the midpoint of AB

From (iv) and (v)

We can conclude that PC is the perpendicular bisector of AB

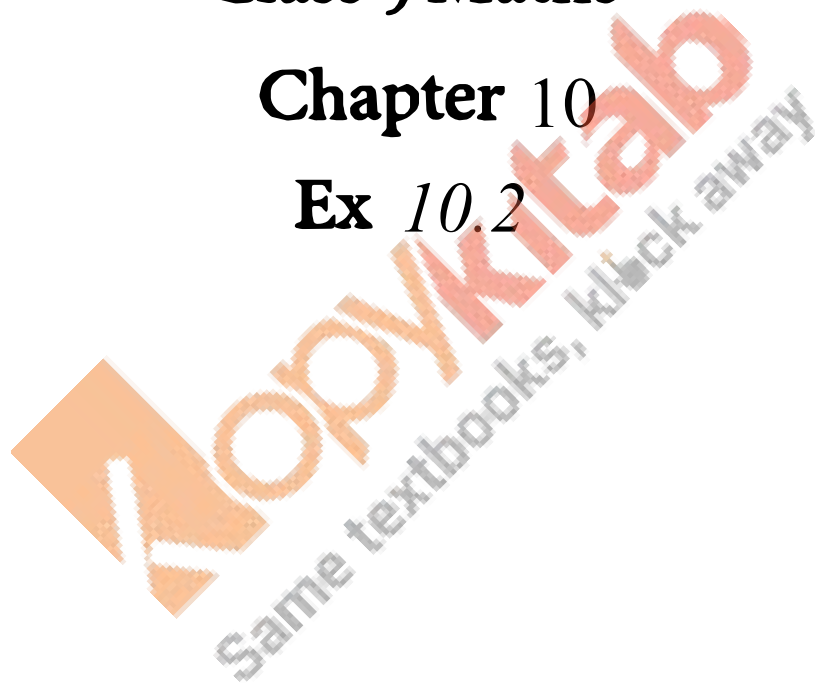Since C is a point on the line PQ, we can say that PQ is the perpendicular bisector of AB.

RD SHARMA

Solutions

Class 9 Maths

Chapter 10

Ex 10.2

(1) In fig. (10).40, it is given that RT = TS, ∠ 1 = 2 ∠ 2 and 4 = 2 ∠ (3) Prove that $\Delta \text{RBT} \cong \Delta \text{SAT}$.

Solution:

In the figure, given that

RT = TS   ......(i)

∠ 1 = 2 ∠ 2   ......(ii)

And ∠ 4 = 2 ∠ 3   ......(iii)

To prove that $\Delta \text{RBT} \cong \Delta \text{SAT}$.

Let the point of intersection RB and SA be denoted by O

Since RB and SA intersect at O

∠ AOR = ∠ BOS   [Vertically opposite angles]

    ∠ 1 = ∠ 4
    2 ∠ 2 = 2 ∠ 3 [From (ii) and (iii)]
    ∠ 2 = ∠ 3 ......(iv)

Now we have RT =TS in △ TRS

△ TRS is an isosceles triangle

∠ TRS = ∠ TSR   ......(v)

But we have

∠ TRS = ∠ TRB + ∠ 2   ......(vi)

∠ TSR = ∠ TSA + ∠ 3   ......(vii)

Putting (vi) and (vii) in (v) we get

∠ TRB + ∠ 2 = ∠ TSA + ∠ B

=> ∠ TRB = ∠ TSA   [From (iv)]

Now consider △ RBT and △ SAT

RT = ST [From (i)]

∠ TRB = ∠ TSA   [From (iv)] ∠ RTB = ∠ STA   [Common angle]

From ASA criterion of congruence, we have

△ RBT = △ SAT

(2) Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Solution: Given that lines AB and CD Intersect at O
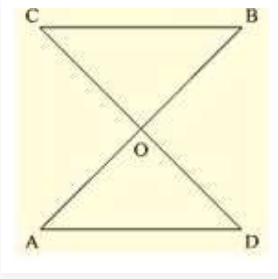
Such that BC ∥ AD and BC = AD ....(i)

We have to prove that AB and CD bisect at O.

To prove this first we have to prove that △ AOD ≅△ BOC

(3) BD and CE are bisectors of ∠ B and ∠ C of an isosceles △ ABC with AB = AC. Prove that BD = CE

Solution:

Given that △ ABC is isosceles with AB = AC and BD and CE are bisectors of ∠ B and ∠ C We have to prove BD = CE

Since AB = AC

=> △ ABC = △ ACB ......(i)
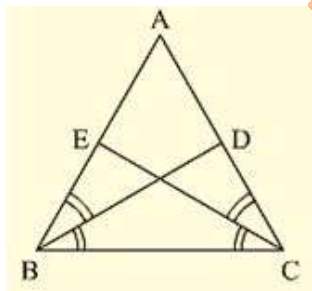
[Angles opposite to equal sides are equal]

Since BD and CE are bisectors of ∠ B and ∠ C

$$\angle ABD = \angle DBC = \angle BCE = ECA = \frac{\angle B}{2} = \frac{\angle C}{2}$$

Now,

Consider △ EBC = △ DCB

∠ EBC = ∠ DCB [∠ B = ∠ C] [From (i)]

BC = BC [Common side]

∠ BCE = ∠ CBD [From (ii)]
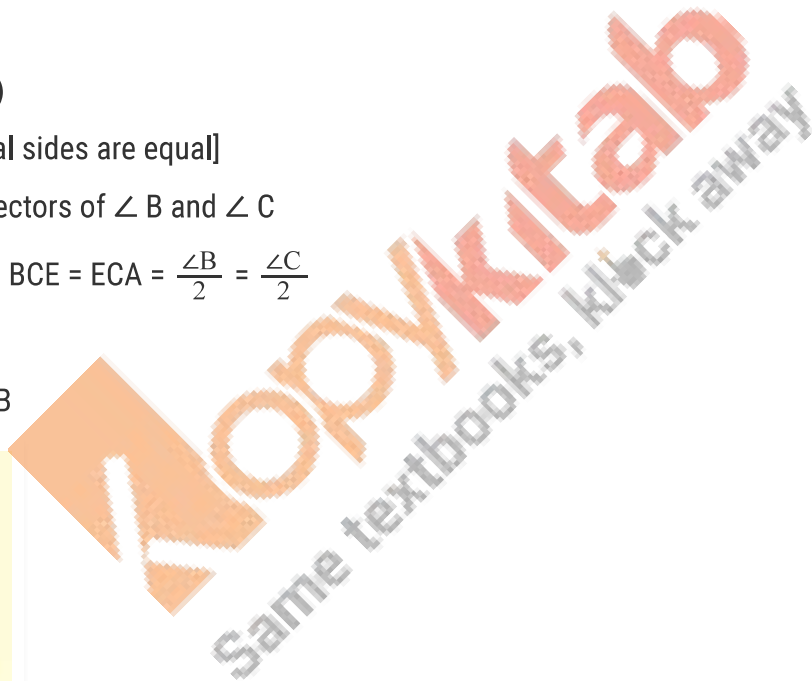
So, by ASA congruence criterion, we have △ EBC ≅△ DCB

Now,

CE = BD   [Corresponding parts of congruent triangles we equal]

or, BD = CE

Hence proved

Since AD || BC and transversal AB cuts at A and B respectively

∠ DAO = ∠ OBC …….(ii) [alternate angle]

And similarly AD || BC and transversal DC cuts at D and C respectively

∠ ADO = ∠ OBC   …..(iii) [alternate angle]

Since AB end CD intersect at O.

∠ AOD = ∠ BOC   [Vertically opposite angles]

Now consider △ AOD and △ BOD

∠ DAO = ∠ OBC [From (ii)]

AD = BC   [From (i)]
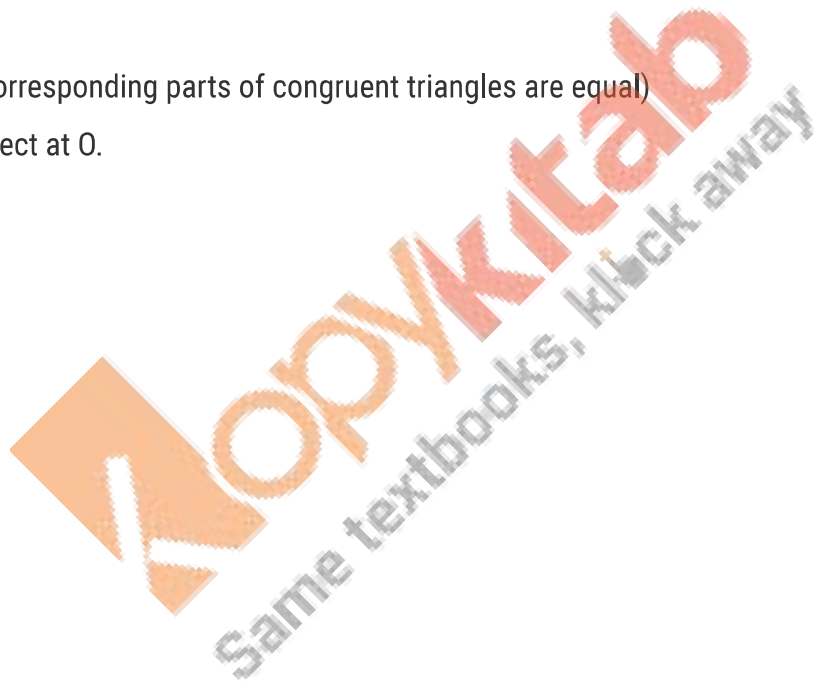
And ∠ ADO = ∠ OCB [From (iii)]

So, by ASA congruence criterion, we have

$\triangle AOD \cong \triangle BOC$

Now,

AO= OB and DO = OC [Corresponding parts of congruent triangles are equal)

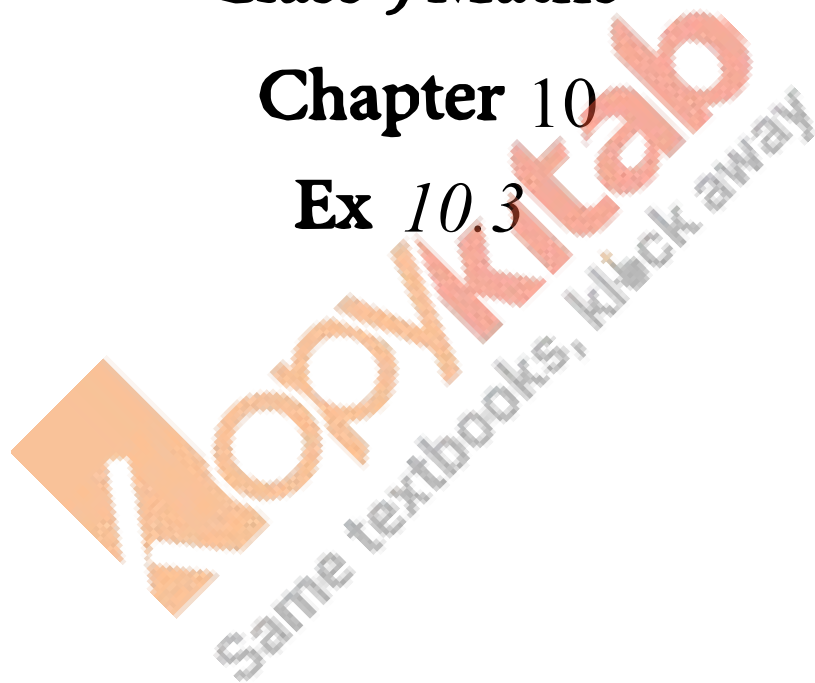   Lines AB and CD bisect at O.

Hence proved

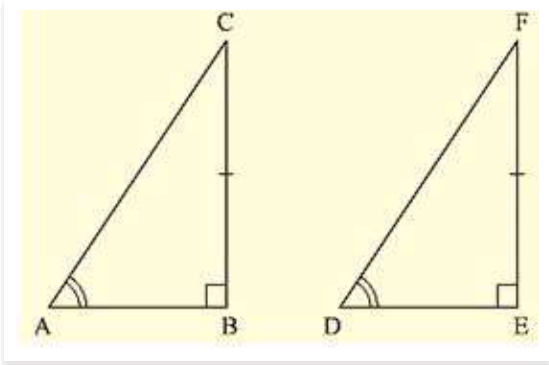# RD SHARMA

Solutions

Class 9 Maths

Chapter 10

Ex 10.3

(1) In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other.

We have to prove that the triangles are congruent.

Let us consider two right triangles such that

$\angle B = \angle C = 90°$ ....(i)

$AB = DE$ ....(ii)

$\angle C = \angle F$ From(iii)

Now observe the two triangles ABC and DEF

$\angle C = \angle F$ [From (iii)]

$\angle B = \angle E$ [From (i)]

and $AB = DE$ [From (ii)]

So, by AAS congruence criterion, we have $\triangle ABC \cong \triangle DEF$

Therefore, the two triangles are congruent

Hence proved

(2) If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Solution:

Given that the bisector of the exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles. Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and AD ∥ BC

Let $\angle EAD$ = (i), $\angle DAC$ =(ii), $\angle ABC$ =(iii) and $\angle ACB$ = (iv)
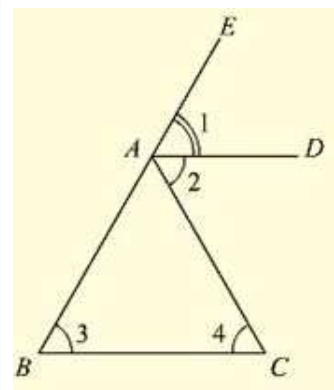
We have,

(i) = (ii)   [AD is a bisector of ∠ EAC]

(i) = (iii)   [Corresponding angles]
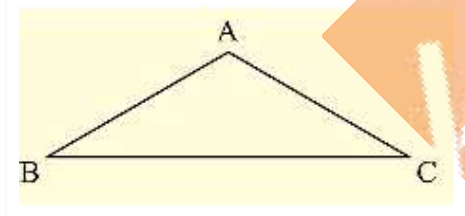
and (ii) = (iv)   [alternative angle]

(iii) = (iv)

    AB = AC

Since, in △ ABC, two sides AB and AC are equal we can say that △ ABC is isosceles triangle.

(3) In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Solution:

Let △ ABC be isosceles such that AB = AC.

    ∠ B = ∠ C

Given that vertex angle A is twice the sum of the base angles B and C. i.e., ∠ A= 2(∠ B + ∠ C)

∠ A = 2(∠ B + ∠ B)

∠ A = 2(2 ∠ B)

∠ A = 4(∠ B)

Now, We know that sum of angles in a triangle =180°

∠ A + ∠ B + ∠ C =180°

4 ∠ B + ∠ B + ∠ B = 180°
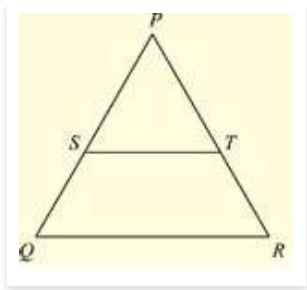
6 ∠ B =180°

∠ B = 30°

Since, ∠ B = ∠ C

∠ B = ∠ C = 30°

And ∠ A = 4 ∠ B

   ∠ A = 4 x 30° = 120°

Therefore, angles of the given triangle are 120°, 30° and 30°.

= 428 and LB = LC]

(4) PQR is a triangle in which PQ= PR and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Solution : Given that PQR is a triangle such that PQ = PR ant S is any point on the side PQ and ST ∥ QR.

To prove,

PS = PT

Since, PQ= PR

   PQR is an isosceles triangle.
   ∠ Q = ∠ R (or) ∠ PQR = ∠ PRQ

Now, ∠ PST = ∠ PQR and ∠ PTS = ∠ PRQ [Corresponding angles as ST parallel to QR]

Since, ∠ PQR = ∠ PRQ

   ∠ PST = ∠ PTS

Now, In △ PST, ∠ PST = ∠ PTS

△ PST is an isosceles triangle
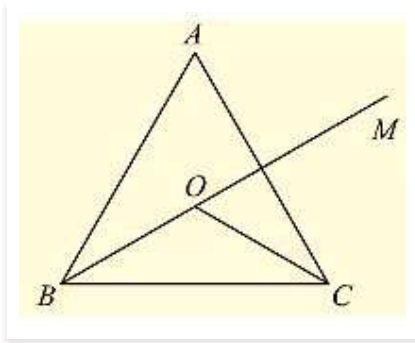
Therefore, PS = PT

(5) In a △ ABC, it is given that AB = AC and the bisectors of B and C intersect at O. If M is a point on BO produced, prove that ∠ MOC = ∠ ABC.

Solution:

Given that in △ ABC,

AB = AC and the bisector of ∠ B and ∠ C intersect at O. If M is a point on BO produced

We have to prove ∠ MOC = ∠ ABC

Since,

AB = AC

ABC is isosceles
∠ B = ∠ C (or)
∠ ABC = ∠ ACB

Now,

BO and CO are bisectors of ∠ ABC and ∠ ACB respectively

=> ABO = ∠ OBC =∠ ACO = ∠ OCB = $\frac{1}{2}$ ∠ ABC = $\frac{1}{2}$ ∠ ACB   ......(i)

We have, in $\triangle$ OBC

∠ OBC + ∠ OCB + ∠ BOC = 180°   ......(ii)

And also

∠ BOC + ∠ COM = 180° ......(iii) [Straight angle]

Equating (ii) and (iii)

∠ OBC + ∠ OCB + ∠ BOC = ∠ BOC + ∠ MOC
∠ OBC + ∠ OCB = ∠ MOC [From (i)]
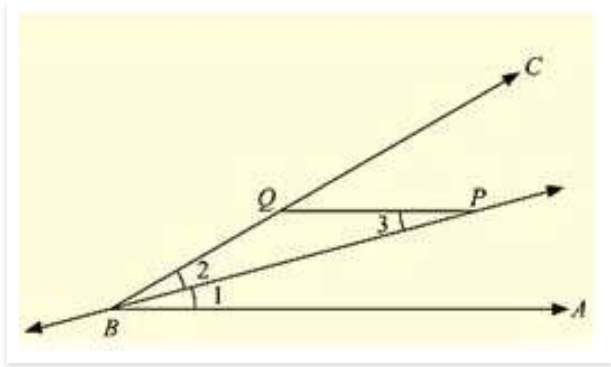2 ∠ OBC = ∠ MOC [From (i)]
2 ($\frac{1}{2}$ ∠ ABC) = ∠ MOC [From (i)]
∠ ABC = ∠ MOC

Therefore, ∠ MOC = ∠ ABC

(6) P is a point on the bisector of an angle ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

Sol:

Given that P is a point on the bisector of an angle ABC, and PQ ‖ AB.

We have to prove that △ BPQ is isosceles.

Since,

BP is bisector of ∠ ABC = ∠ ABP = ∠ PBC   ......(i)

Now,

PQ ∥ AB

   ∠ BPQ = ∠ ABP ......(ii) [alternative angles]

From (i) and (ii), we get

∠ BPQ = ∠ PBC (or) ∠ BPQ = ∠ PBQ

Now,

In BPQ,

∠ BPQ = ∠ PBQ

△ BPQ is an isosceles triangle.

Hence proved

(7) Prove that each angle of an equilateral triangle is 60°.

Sol:

Given to prove each angle of an equilateral triangle is 60°.

Let us consider an equilateral triangle ABC.
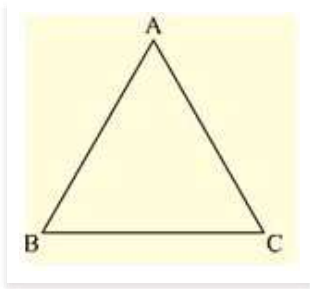
Such that AB = BC = CA

Now, AB = BC

   ∠ A = ∠ C ......(i) [Opposite angles to equal sides are equal]

And BC = AC

   ∠ B = ∠ A ......(ii)

From (i) and (ii), we get

∠ A = ∠ B = ∠ C   ......(iii)

We know that

Sum of angles in a triangle = 180

$\angle A + \angle B + \angle C = 180$
$\angle A + \angle A + \angle A = 180$
$3 \angle A = 180$
$\angle A = 60$

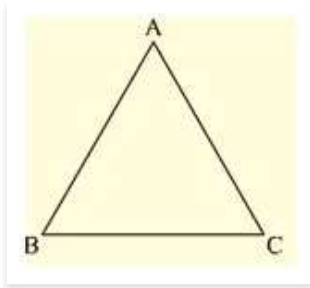$\angle A = \angle B = \angle C = 60$

Hence, each angle of an equilateral triangle is 60°.

(8) Angles $\triangle$ A, B, C of a triangle ABC are equal to each other. Prove that ABC is equilateral.

Sol:

Given that angles A, B, C of a triangle ABC equal to each other.

We have to prove that $\triangle$ ABC is equilateral

We have, $\angle A = \angle B = \angle C$

Now,

$\angle A = \angle B$

   BC=AC [opposite sides to equal angles are equal]

And $\angle B = \angle C$

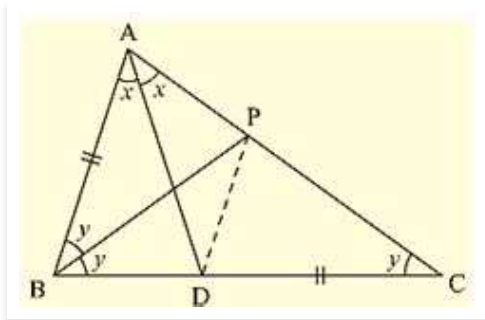   AC=AB

From the above we get

AB = BC = AC

   $\triangle$ ABC is equilateral.

(9) ABC is a triangle in which $\angle$ B = 2 $\angle$ C. D is a point on BC such that AD bisects $\angle$ BAC and AB = CD. Prove that [$\angle$ BAC = 72 .

Solution: Given that in ABC, $\angle$ B= 2 $\angle$ C and D is a point on BC such that AD bisectors $\angle$ BAC and

AB = CD.

We have to prove that ∠ BAC = 72°

Now, draw the angular bisector of ∠ ABC, which meets AC in P.

Join PD

Let C = ∠ ACB = y

∠ B = ∠ ABC = 2 ∠ C = 2y and also
Let ∠ BAD = ∠ DAC
∠ BAC = 2x [AD is the bisector of ∠ BAC]

Now, in △ BPC,

∠ CBP = y [BP is the bisector of ∠ ABC]
∠ PCB = y
∠ CBP = ∠ PCB = y [PC = BP]

Consider, △ ABP and △ DCP, we have

△ ABP = △ DCP = y

AB = DC   [Given]

And PC = BP   [From above]

So, by SAS congruence criterion, we have $\triangle ABP \cong \triangle DCP$

Now,

∠ BAP = ∠ CDF and AP = DP [Corresponding parts of congruent triangles are equal]

∠ BAP = ∠ CDP = 2

Consider, △ APD,

We have AP = DP

= ∠ ADP = ∠ DAP

But ∠ DAP = x

∠ ADP = ∠ DAP = x

Now

In △ ABD.

∠ ABD + ∠ BAD + ∠ ADB = 180

And also ∠ ADB + ∠ ADC = 180° [Straight angle]

From the above two equations, we get

∠ ABD + ∠ BAD + ∠ ADB = ∠ ADB + ∠ ADC

2y + x = ∠ ADP + ∠ PDC
2y + x = x + 2x
2y=2x
y = x (or) x = y

We know,

Sum of angles in a triangle = 180°

So, In △ ABC,

∠ A + ∠ B + ∠ C =180°
2x + 2y + y = 180° [∠ A = 2x, ∠ B = 2y, ∠ C = y]
2(y)+3y =180° [x = y]
5y = 180°
y = 36°

Now, ∠ A = ∠ BAC = 2 x 36° = 72°

(10) ABC is a right angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C.

Solution: Given that ABC is a right angled triangle such that ∠ A = 90° and AB = AC

Since, AB= AC

△ ABC is also isosceles.

Therefore, we can say that △ ABC is right angled isosceles triangle.

∠ C = ∠ B and ∠ A = 90° ......(i)

Now, we have sum of angled in a triangle = 180°

∠ A + ∠ B + ∠ C = 180°
90° + ∠ B + ∠ B = 180° [From(i)]
2∠ B = 180° − 90°
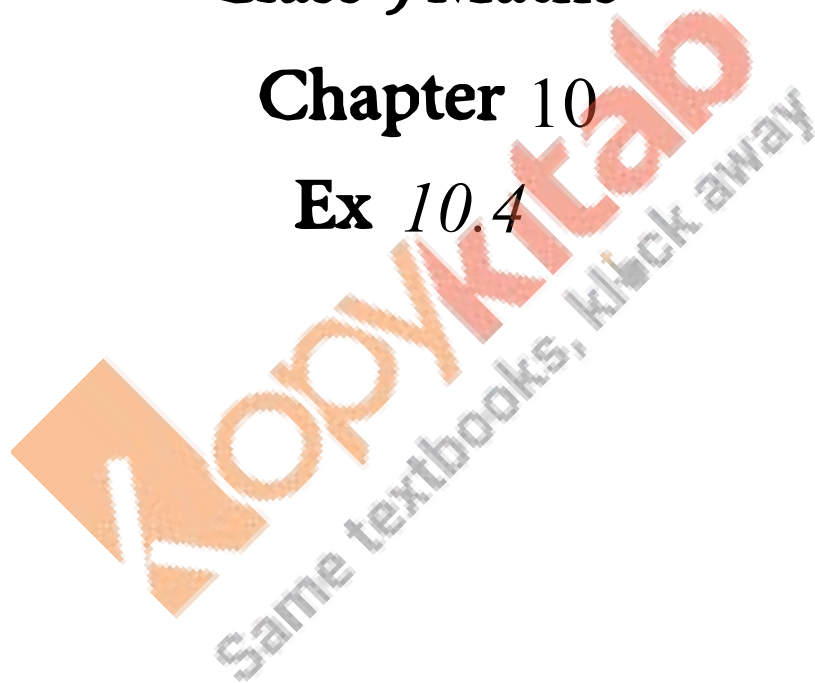∠ B = 45°

Therefore, ∠ B = ∠ C = 45°

RD SHARMA

Solutions

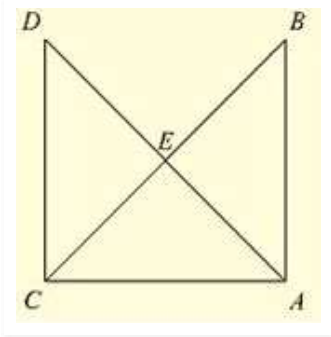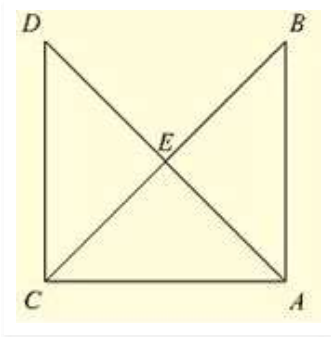## Class 9 Maths

Chapter 10

Ex 10.4

(1) In fig (10).9(2) It is given that AB = CD and AD = BC. Prove that $\triangle ADC \cong \triangle CBA$.

Solution:

Given that in the figure AB = CD and AD = BC.

We have to prove $\triangle ADC \cong \triangle CBA$

Now,

Consider $\triangle$ ADC and $\triangle$ CBA.

We have

AB = CD	[Given]
BC = AD	[Given]
And AC = AC	[Common side]

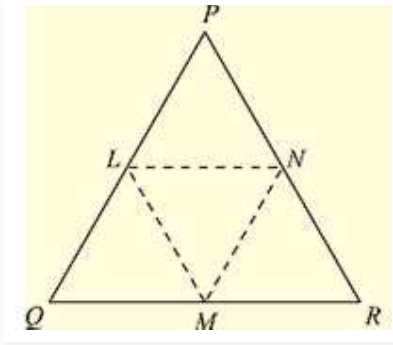So, by SSS congruence criterion, we have

$\triangle ADC \cong \triangle CBA$

Hence proved

(2) In a $\triangle$ PQR. IF PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Sol: Given that in $\triangle$ PQR, PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively

We have to prove LN = MN.

Join L and M, M and N, N and L

We have PL = LQ, QM = MR and RN = NP

[Since, L, M and N are mid-points of Pp. QR and RP respectively]

And also PQ = QR

$\quad$ PL = LQ = QM = MR = $\dfrac{PQ}{2}$ = $\dfrac{QR}{2}$ ......(i) Using mid-point theorem,

We have

MN ∥ PQ and MN = $\dfrac{PQ}{2}$

$\quad$ MN = PL = LQ ......(ii)

Similarly, we have

LN ∥ QR and LN = (1/2)QR

$\quad$ LN = QM = MR ......(iii)

From equation (i), (ii) and (iii), we have

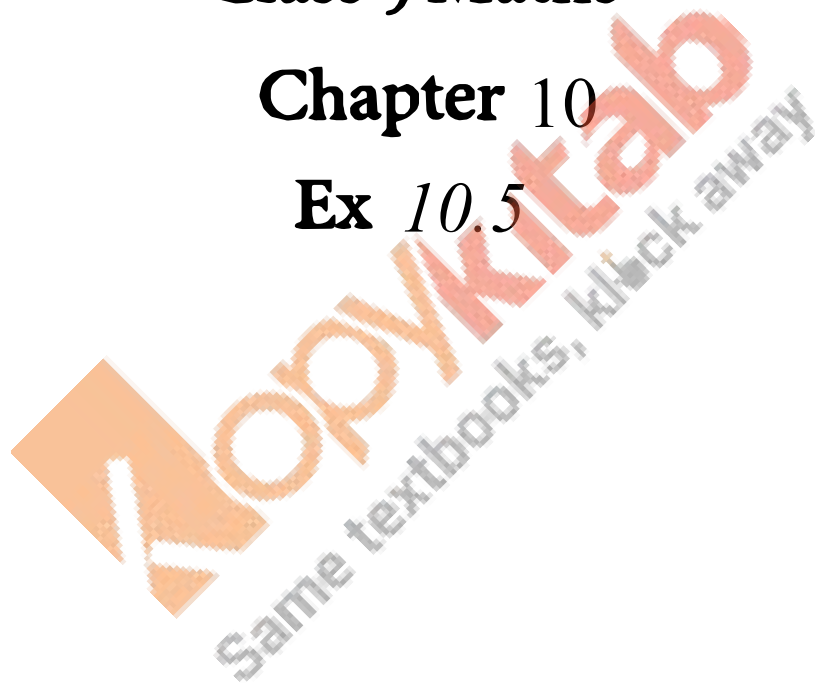PL = LQ = QM = MR = MN = LN

LN = MN

RD SHARMA

Solutions

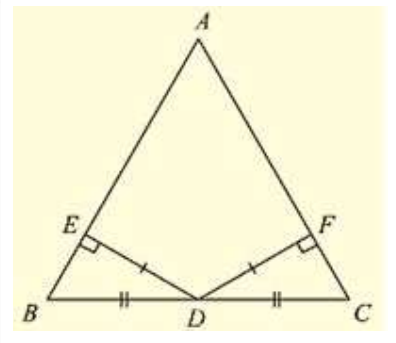## Class 9 Maths

Chapter 10

Ex 10.5

(1) ABC is a triangle and D is the mid-point of BC. The perpendiculars from B to AB and AC are equal. Prove that the triangle is isosceles.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angle of the other

We have to prove that the triangles are congruent

Let us consider two right triangles such that

$\angle B = \angle E = 90$ ......(i)

$AB = DE$...... (ii)

$\angle C = \angle F$......(iii)

Now observe the two triangles ABC and DEF

$\angle C = \angle F$[From (iii)]

$\angle B = \angle E$ [From (i)]

And $AB = DE$[From (ii)]

So, by AAS congruence criterion, we have

$\triangle ABC \cong \triangle DEF$

Therefore, the two triangles are congruent
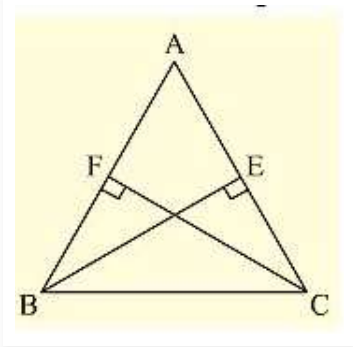
Hence proved

(2) ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that $\triangle$ ABC is isosceles

Sol: Given that ABC is a triangle in which BE and CF are perpendicular to the sides AC and AS respectively such that BE = CF.

To prove, $\triangle$ ABC is isosceles

Now, consider $\triangle$ BCF and $\triangle$ CBE,

We have

∠ BFC = CEB = 90 [Given]

BC = CB[Common side]

And CF = BE[Given]

So, by RHS congruence criterion, we have

$\Delta BFC \cong \Delta CEB$

Now,

∠ FBC = ∠ EBC   [Incongruent triangles corresponding parts are equal]

∠ ABC = ∠ ACB
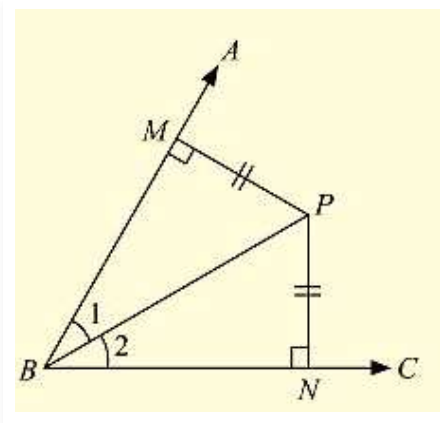
AC = AB[Opposite sides to equal angles are equal in a triangle]

△ ABC is isosceles

(3) If perpendiculars from any point within an angle on its arms are congruent. Prove that it lies on the bisector of that angle.

Solution:

Given that, if perpendicular from any point within, an angle on its arms is congruent, prove that it lies on the bisector of that angle.

Now,

Let us consider an angle ABC and let BP be one of the arm within the angle

Draw perpendicular PN and PM on the arms BC and BA such that they meet BC and BA in N and M respectively.

Now, in △ BPM and △ BPN

We have ∠ BMP = ∠ BNP = 90° [given]

BP = BP[Common side]

And MP = NP[given]

So, by RHS congruence criterion, we have

$$\Delta BPM \cong \Delta BPN$$
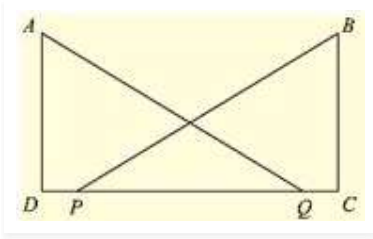
Now, ∠ MBP = ∠ NBP [Corresponding parts of congruent triangles we equal]

   BP is the angular bisector of ∠ABC.

Hence proved

(4) In fig. (10).99, AD ⊥ CD and CB ⊥ CD. If AQ = BP and DP = CQ, prove that ∠ DAQ = ∠ CBP.

Sol:

Given in the fig. (10).99, AD ⊥ CD and CB ⊥ CD.

And AQ = BP and DP = CQ,

To prove that ∠ DAQ = ∠ CBP

Given that DP = QC

Add PQ on both sides

   DP + PQ = PQ + QC
   DQ = PC ......(i)

Now, consider triangle DAQ and CBP,

We have

∠ ADQ = ∠ BCP = 90°   [given] AQ = BP [given]

And DQ = PC[From (i)]

So, by RHS congruence criterion, we have

$$\Delta DAQ \cong \Delta CBP$$

Now,

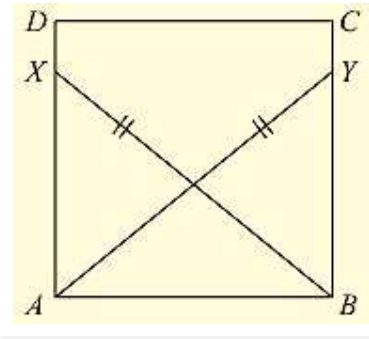∠ DAQ = ∠ CBP   [Corresponding parts of congruent triangles are equal]

Hence proved

(5) ABCD is a square, X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠ BAY = ∠ ABX.

Solution:

Given that ABCD is a square, X and Y are points on sides AD and BC respectively such that AY = BX.

To prove BY = AX and ∠ BAY = ∠ ABX

Join B and X, A and Y.

Since, ABCD is a square

∠ DAB = ∠ CBA = 90°
∠ XAB = ∠ YAB = 90° ......(i)

Now, consider triangle XAB and YBA

We have

∠ XAB = ∠ YBA = 90°. [From (i)] BX = AY[given]

And AB = BA[Common side]

So, by RHS congruence criterion, we have $\Delta XAB \cong \Delta YBA$

Now, we know that corresponding parts of congruent triangles are equal.

BY = AX and ∠ BAY = ∠ ABX

Hence proved

(6) Which of the following statements are true (T) and which are false (F):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal

(iii) The measure of each angle of an equilateral triangle is 60

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.

(v) The bisectors of two equal angles of a triangle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

 (viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.

(ix) Two right-angled triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Solution:

(i) False (F)

Reason: Sides opposite to equal angles of a triangle are equal

(ii) True (F)

Reason: Since the sides are equal, the corresponding opposite angles must be equal

(iii) True (T)

Reason: Since all the three angles of equilateral triangles are equal and sum of the three angles is 180 , each angle will be equal to $\frac{180°}{3} = 60°$

(iv) False (F)

Reason: Here the altitude from the vertex is also the perpendicular bisector of the opposite side.

The triangle must be isosceles and may be an equilateral triangle.

(v) True (T)

Reason: Since it an isosceles triangle, the lengths of bisectors of the two equal angles are equal

(vi) False (F)

Reason: The angular bisector of the vertex angle is also a median

=> The triangle must be an isosceles and also may be an equilateral triangle.

(vii) False (F)

Reason: Since two sides are equal, the triangle is an isosceles triangle. The two altitudes corresponding to two equal sides must be equal.

(viii) False (F)

Reason: The two right triangles may or may not be congruent

(ix) True (T)

Reason: According to RHS congruence criterion the given statement is true.

(7) Fill the blanks In the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are ___

(ii) Angle opposite to equal sides of a triangle are ___

(iii) In an equilateral triangle all angles are ___

(iv) In △ ABC, if ∠ A = ∠ C, then AB =

(v) If altitudes CE and BF of a triangle ABC are equal, then AB ___

(vi) In an isosceles triangle ABC with AB = AC, if BD and CE are its altitudes, then BD is ___ CE.

(vii) In right triangles ABC and DEF, if hypotenuse AB = EF and side AC = DE, then $\triangle ABC \cong \triangle$___

Solution:

(i) Sides opposite to equal angles of a triangle are equal

(ii) Angles opposite to equal sides of a triangle are equal

(iii) In an equilateral triangle all angles are equal Reason: Since all sides are equal in a equilateral triangle. the angles opposite to equal sides will be equal .

(iv) In a △ ABC, if ∠ A = ∠ C, then AB = BC

(v) If altitudes CE and BF of a triangle ABC are equal, then AB = AC

(vi) In an isosceles triangle $\triangle$ ABC with AB = AC, if BD and CE are its altitudes, then BD is equal to CE

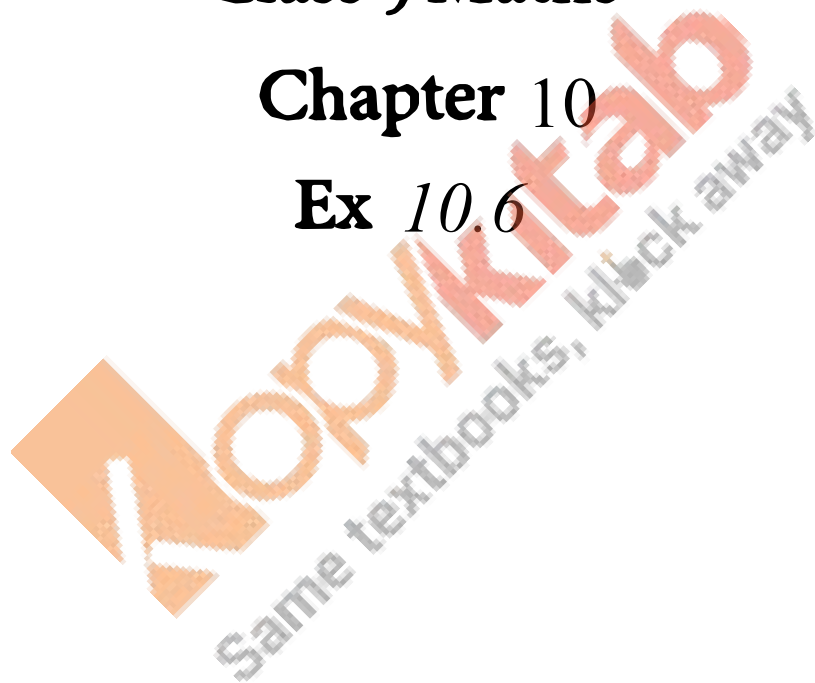(vii) In right triangles ABC and DEF, if hypotenuse AB = EF and side AC = DE, then, $\triangle$ ABC = $\triangle$EFD.

RD SHARMA

Solutions

Class 9 Maths

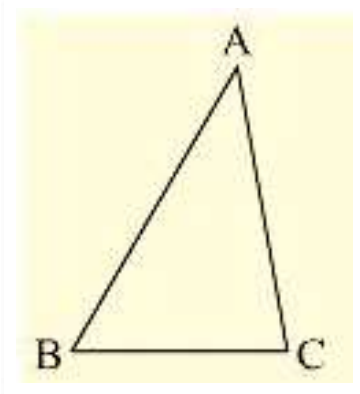## Chapter 10

Ex 10.6

(1) In △ ABC, if ∠ A = 40° and ∠ B = 60°. Determine the longest and shortest sides of the triangle.

Solution:

Given that in △ ABC, ∠ A = 40° and ∠ B = 60°

We have to find longest and shortest side

We know that,

Sum of angles in a triangle 180°

∠ A + ∠ B + ∠ C = 180°

40° + 60° + ∠ C = 180°

∠ C = 180° − ((10)0°) = 80°

∠ C = 80°

Now,

=> 40° < 60° < 80° = ∠ A < ∠ B < ∠ C

=> ∠ C is greater angle and ∠ A is smaller angle.

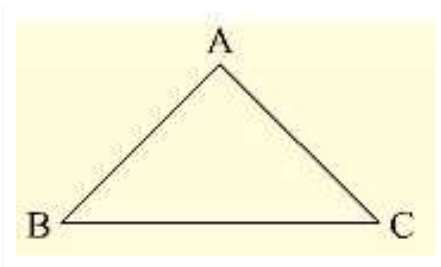Now, ∠ A < ∠ B < ∠ C

=> BC < AAC < AB [Side opposite to greater angle is larger and side opposite to smaller angle is smaller]

AB is longest and BC is smallest or shortest side.

(2) In a △ ABC, if ∠ B = ∠ C = 45°, which is the longest side?

Solution: Given that in △ ABC,

∠ B = ∠ C = 45°

We have to find longest side

We know that.

Sum of angles in a triangle =180°

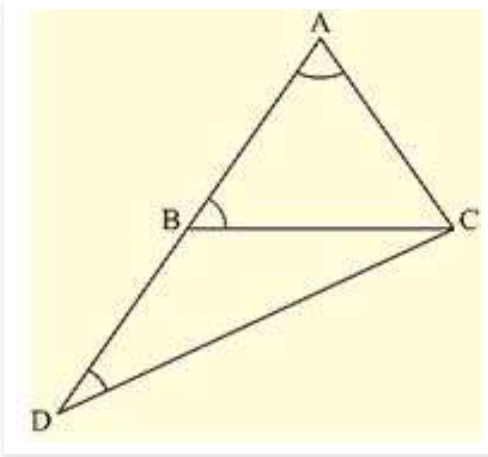$\angle A + \angle B + \angle C = 180°$

$\angle A + 45° + 45° = 180°$

$\angle A = 180° - (45° + 45°) = 180° - 90° = 90°$

$\angle A = 90°$

(3) In $\triangle$ ABC, side AB is produced to D so that BD = BC. If $\angle$ B = 60° and $\angle$ A = 70°. prove that: (i) AD > CD (ii) AD > AC

 Sol: Given that, in $\triangle$ ABC, side AB is produced to D so that BD = BC.

$\angle$ B = 60°, and $\angle$ A = 70°

To prove,

(i) AD > CD (ii) AD > AC

First join C and D

We know that,

Sum of angles in a triangle =180°

$\angle A + \angle B + \angle C = 180°$

$70° + 60° + \angle C = 180°$

$\angle C = 180° - (130°) = 50°$

$\angle C = 50°$

$\angle ACB = 50°$......(i)

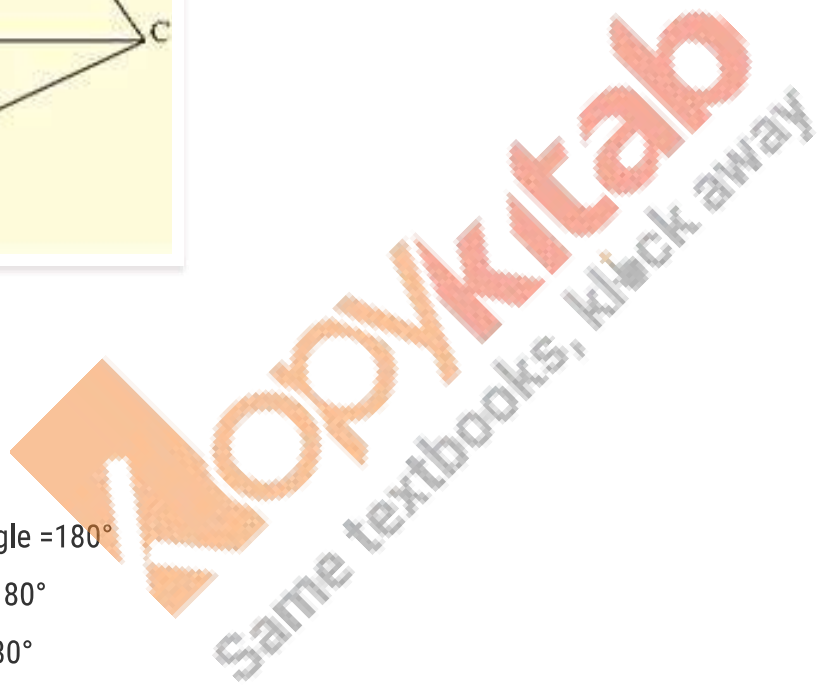And also in $\triangle$ BDC

$\angle$ DBC =180 $- \angle$ ABC [ABD is a straight angle]

$180 - 60° = 120°$

and also BD = BC[given]

$\angle$ BCD = $\angle$ BDC [Angles opposite to equal sides are equal]

Now,

$\angle$ DBC + $\angle$ BCD + $\angle$ BDC = 180° [Sum of angles in a triangle =180°]

=> 120° + ∠ BCD + ∠ BCD = 180°

=> 120° + 2∠ BCD = 180°

=> 2∠ BCD = 180° − 120° = 60°

=> ∠ BCD = 30°

=> ∠ BCD = ∠ BDC = 30° ….(ii)

Now, consider △ ADC.

∠ BAC => ∠ DAC = 70° [given]

∠ BDC => ∠ ADC = 30° [From (ii)]

∠ ACD = ∠ ACB+ ∠ BCD

= 50° + 30°[From (i) and (ii)] =80°

Now, ∠ ADC < ∠ DAC < ∠ ACD

AC < DC < AD [Side opposite to greater angle is longer and smaller angle is smaller]

AD > CD and AD > AC

Hence proved

Or,

We have,

∠ ACD > ∠ DAC and ∠ ACD > ∠ ADC

AD > DC and AD> AC   [Side opposite to greater angle is longer and smaller angle is smaller]

(4) Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Sol:

Given lengths of sides are 2cm, 3cm and 7cm.

To check whether it is possible to draw a triangle with the given lengths of sides

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.

2 + 3 ≯ 7 or 2 + 3 < 7

2 + 7 > 3

and 3 + 7 > 2

Here 2 + 3 ≯ 7

So, the triangle does not exit.

(5) O is any point in the interior of △ ABC. Prove that

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > (1/2)(AB + BC +CA)

Solution:
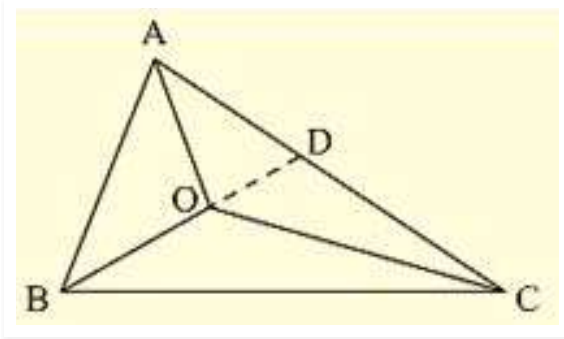
Given that O is any point in the interior of $\triangle$ ABC

To prove

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > (1/2)(AB + BC +CA)

We know that in a triangle the sum of any two sides is greater than the third side.

So, we have

In $\triangle$ ABC

AB + BC > AC

BC + AC > AB

AC + AB > BC

In $\triangle$ OBC

OB + OC > BC.......(i)

In $\triangle$ OAC

OA + OC > AC......(ii)

In $\triangle$ OAB

OA + OB > AB......(iii)

Now, extend (or) produce BO to meet AC in D.

Now, in $\triangle$ ABD, we have

AB + AD > BD

AB + AD > BO + OD ....(iv) [BD = BO + OD]

Similarly in $\triangle$ ODC, we have

OD + DC > OC......(v)

(i) Adding (iv) and (v), we get

AB + AD + OD + DC > BO + OD + OC