



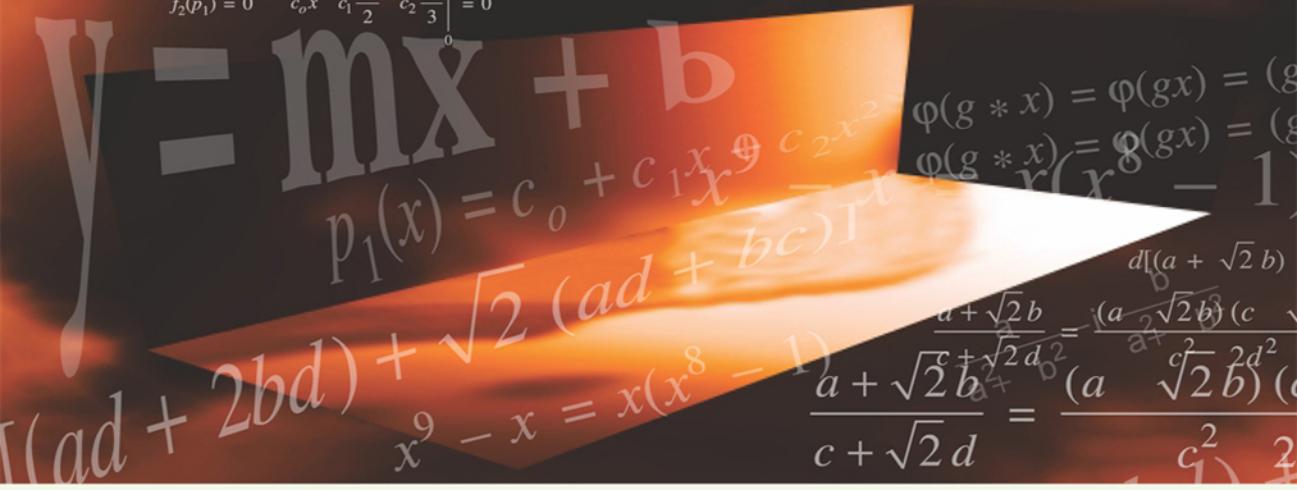
VIKAS®

A COURSE IN ABSTRACT ALGEBRA

FOURTH EDITION

$$f_2(p_1) = 0 \quad c_0x^0 - c_1\frac{x^2}{2} + c_2\frac{x^3}{3}^2 = 0$$

$$d[(a + \sqrt{2}b)(c + \sqrt{2}d)] = d[(ad + 2bd) + \sqrt{2}(ad + bc)]$$



Vijay K Khanna • S K Bhambri

A C O U R S E I N
ABSTRACT ALGEBRA

Fourth Edition

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Preface to the Fourth Edition

Today, when we look back, it is more than twenty years since the first edition of the book was published. Over the years, it has been subjected to a lot of changes (improvements, if we dare say) keeping in mind the requirements of the readers for whom it was intended. The present volume goes on to do the same and we have tried to further improve upon the contents. Almost all the chapters have undergone revision and a fairly large number of new examples and worked out problems have been added to make the theory easier to grasp. The portion on Fields may seem to be taking the back seat, but then it has had its share of revision in the previous editions. At places, new/alternate proofs have been added and some reshuffling has also been done to move certain results to better-suited places.

The subject matter has been planned so as not to remain constrained by the syllabus of a particular course. It is meant for anyone and everyone pursuing a serious course in Abstract Algebra. The book also comes to you in a bigger and better looking format, designed to make reading more focused.

While thanking all those readers who have been advising us for improvements, a word of thanks to that vast majority of silent admirers cannot be overlooked and we do feel indebted to them for making the book a success that it is.

Utmost care has been taken to keep the misprints out and we hope we have been successful in our effort. Since there is always room for improvement, it goes without saying that any suggestions, howsoever trivial, would be highly welcome.

Vijay K Khanna
S K Bhambri

Preface to the First Edition

The present volume has grown out of our association with the teaching of Algebra to the honours and postgraduate classes for the last several years and our exposure to the problems faced by the students in grasping the abstract nature of the subject. This experience is the foundation and, we hope, the strength of the text. Earnest efforts have been exerted to present the subject matter in a well-knit manner so as not only to stimulate the interest of the student but also to provide with an insight into the complexities of a subject of great intrinsic beauty.

The book is intended to serve as a text for undergraduate students especially those opting for an honours course in Mathematics. However, postgraduate students will find it equally useful.

The first chapter on Preliminaries is a curtain-raiser to the main contents of the book. It gives a rather terse summary of the results from Set Theory (some of which we presume the reader would already be familiar with). A few results from Number Theory are incorporated in the later half of this chapter. We debated between ourselves whether or not to give a ‘full chapter status’ to Number Theory results, and after a careful thought decided to keep these as only a part of the first chapter since we feared that a full chapter on these might impair the balance of the book.

The main text can be divided into four sections on Groups, Rings, Vector spaces (Linear Algebra) and Fields. Fairly sufficient ground has been covered in the first three sections. It is only in the last section on Fields that we can possibly be accused of being stingy. But then there are constraints and it was paucity of space and time (and not of ideas) that finally made us keep Galois Theory out. Maybe in a subsequent edition it would find its way in.

Different concepts have been explained with the help of examples. A large number of problems with solutions have been provided to assist one get a firm grip on the ideas developed. There is plenty of scope (in the form of exercises) for the reader to try and solve problems on his own. In all, a substantial variety of challenges (and rewards) is assured.

We are deeply indebted to all those authors whose books (research papers) on Algebra influenced our learning of the subject and take this opportunity to express our sincere gratitude to them. We are also thankful to those friends and colleagues with whom we had fruitful discussions from time to time.

It is our earnest belief that no ‘work’ is ever complete till it has had its share of criticism and hence we’ll be only too glad to receive comments and suggestions for the betterment of the book.

Vijay K Khanna
S K Bhambri

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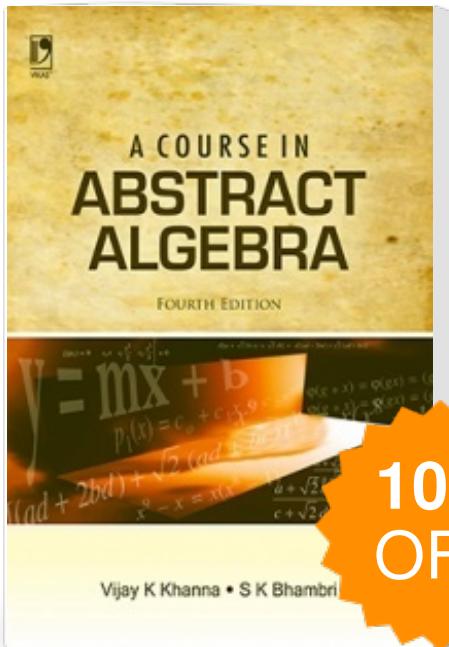
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A Course in Abstract Algebra



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