Eastern Economy Edition

Partial Differential Equations

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PHI

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My Grandsons

Mr. Hrusikesh Biswal

and

Mr. Byamakesh Dalai

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Preface

This textbook has evolved over a period of years and meets the needs of a one-term course in partial differential equation. No specific prerequisite except basic calculus is required to understand this textbook. Most of the books on partial differential equations available in the market are voluminous. In this book, author tried his best to present all related formula with few standard worked out examples according to the derived formula to make the book precise. The notations used in this textbook are commonly used by mathematicians. Considerable use has been made of illustrations to stimulate the students' visual understanding of partial differential equation.

A careful and judicious selection of examples has made it simple and lucid for class room instruction, at the same time often conveying an interesting logical fact. And some standard problems with sufficient hints have been included at the end of each section to gauge the students' understanding and grasp of the theory. It is unusual to find so many examples and problems in one textbook. Therefore, the author fervently hopes that this book will truly fulfil the requirement for an accessible textbook suitable for courses all over the universities in India.

I am thankful to Mrs. Sunanda Samal for her continuous and unfailing encouragement throughout the period of writing the manuscript. I am indeed indebted to Prof. A. Ramachandra Rao, Department of Mathematics, Indian Institute of Science, Bangalore, who taught me much about this subject during my Ph.D. work under his guidance.

Finally, I wish to extend my sincere thanks to all those who contributed to this text book morally and materially till the end.

Almost every book contains errors and this one will hardly be an exception. Please let me know by the email at purnabiswal@rediffmail.com about any errors that you may notice while reading this book.

Purna Chandra Biswal

Chapter 1

Partial Differential Equation in Engineering

Many natural phenomena can be described in terms of partial differential equations. The aim of this chapter is to provide a feeling for how these equations originate, the assumptions that are made in deriving them. We will consider the main parts of partial differential equations that are relevant for engineering applications. But, first we need to recall some useful results from vector analysis.

1.1 Divergence Theorem

Consider the body illustrated in figure 1.1. Gauss's divergence theorem states that if V is a volume with surface S and if

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} f_1(x,\, y,\, z) \\ f_2(x,\, y,\, z) \\ f_3(x,\, y,\, z) \end{pmatrix}$$

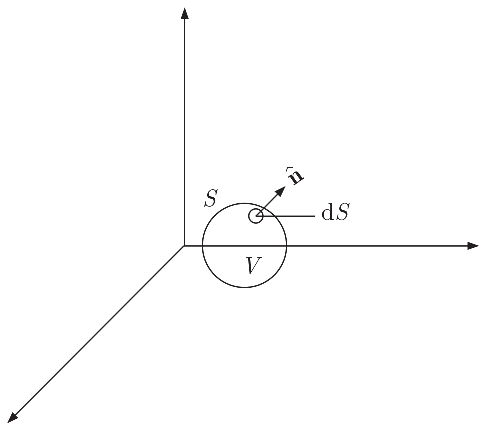

Figure 1.1 Geometry for the divergence theorem.

is a differentiable vector field, then

$$\int_V \nabla \,.\, \mathbf{f}\, \mathrm{d}V = \int_S \mathbf{f} \,.\, \hat{\mathbf{n}}\, \mathrm{d}S$$

where $\hat{\mathbf{n}}$ is the outward normal to the surface of the volume at a given point on the surface and $\nabla \,.\, \mathbf{f}$ is the divergence of $\mathbf{f}$ defined in Cartesian co-ordinates by

$$\nabla \,.\, \mathbf{f} = \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{pmatrix} . \begin{pmatrix} f_1(x,\, y,\, z) \\ f_2(x,\, y,\, z) \\ f_3(x,\, y,\, z) \end{pmatrix}$$

$$= \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

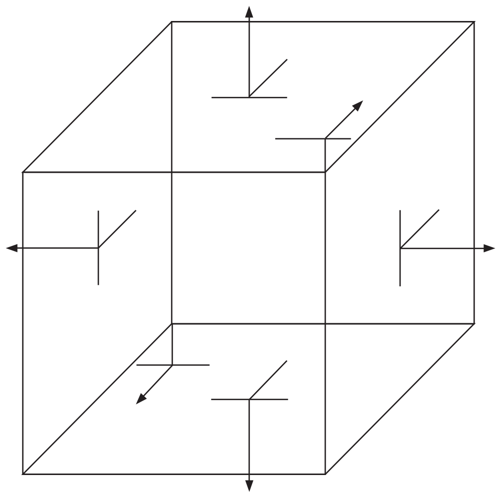

Figure 1.2 The box used in the discussion of the proof of the divergence theorem.

We can motivate the physical meaning of the divergence theorem by sketching a proof for a box

$$x_0 \le x \le x_1, \;\; y_0 \le y \le y_1, \;\; z_0 \le z \le z_1$$

This box has six faces, and the unit normals on each faces are as shown in figure 1.2. We have

$$\int_V \nabla \,.\, \mathbf{f}\, \mathrm{d}V = \int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} \left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} \right) \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$$