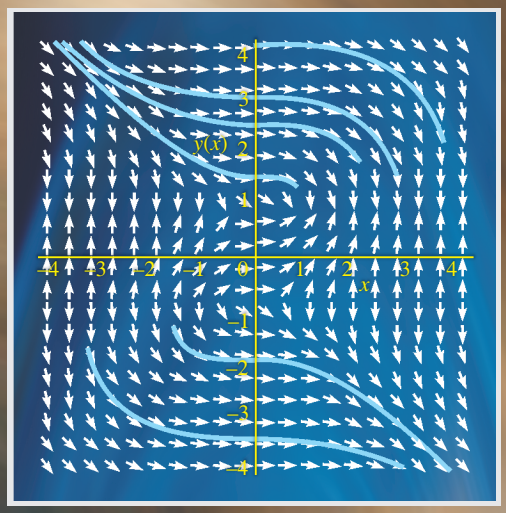
Second Edition

Eastern Economy Edition

Ordinary and Partial Differential Equations



Theory and Applications

Nita H. Shah

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ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Theory and Applications

Second Edition

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ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS—Theory and Applications
Second Edition
Nita H. Shah

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To the memory of
my sweet mother Damyanti H. Shah
who was the source of light, energy and inspiration to me

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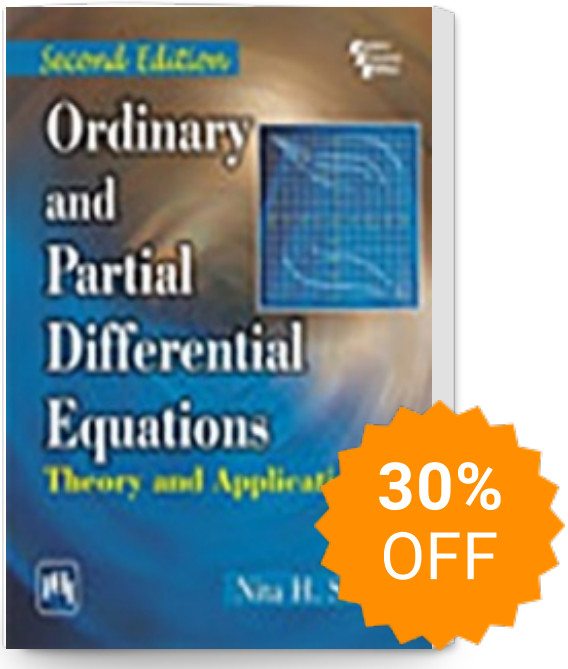
Ordinary And Partial Differential Equations: Theory And Applications



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