APPLIED MATHEMATICS

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FOREWORD

Date....24.12.'09........

The application of mathematics in day to day activities becomes part and parcel of our routine. Gone were the days, when mathematics was considered to be one of the basic and fundamental courses to the students of Engineering, Economics, Computer Science, Physics etc. Now the application of mathematics in the biological streams like Agriculture, Medicine, Veterinary Science has been given impetus and many of the Universities in India have made mathematics, a compulsory subject even at under graduate level, mainly to make the life sciences dynamic and application oriented.

Though a lot of computer aided programmes and packages are available to analyse the data, the interpretation and discussion of the output requires thorough knowledge on the critical aspects of mathematics to draw meaningful conclusions and policy prescriptions from the available computer output.

In this context, teaching the students with easy to understand resource materials is very vital. The available resource materials should be not only supportive to the students but also provide an atmosphere for self learning. The contents should be very simple, interesting, self explanatory and easy to follow to comprehend the subject thoroughly.

Under this context, the book "Applied Mathematics" written by Dr. C. Kailasam, Professor of Mathematics, Ms. R.Pangayar Selvi, Assistant Professor of Mathematics and Mrs. R. Vasanthi, Assistant Professor of Mathematics Department of Physical Sciences & Information Technology are highly appropriate, timely and relevant.

Dr.C.Kailasam is not only a good teacher but also an expert in Mathematical Modeling having more than 35 years of experience in the Area of Applied Mathematical Research. Under his able guidance and direction many of the Post Graduates and Doctoral Research Scholars have conducted innovative research and come out with workable policy options, particularly in the areas of Biological and Social Sciences. During his tenure as Head of Physical Sciences and Information Technology, he was instrumental in introducing number of courses involving applied mathematics in the curricula. By blending his vast teaching and research expertise he has prepared this book very nicely with large number of illustrations; so that it could be a piece of reference to the academics, scholars and researchers of the universities particularly state agricultural universities and ICAR institutions. The book can also prescribed as a text book in universities which offer courses as "Applied Mathematics".

On the whole the book is well written and provides many new insights. The authors deserve appreciation for bringing out a valuable book encompassing all the essential issues and subjects of applied mathematics. The authors certainly has to be complemented for their effort.

24/12/09

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PREFACE

Applied Mathematics is basically a multidisciplinary subject which covers a wide spectrum of mathematical issues and it generally draws upon concepts and methods of mathematics from the fields of application as well as brings innovative ideas, techniques, and scientific knowledge to expand the horizons of the application of mathematics.

The book is probably one of the few dealing with structural analysis of the relationship between mathematics and other biological sciences such as agriculture. The book contains ten chapters that highlight and discuss the various dimensions of applied mathematics. The book is written in lucid and simple language and it explains the application of mathematics for problem solving that too with easy to understand workout examples and cases.

It is hoped that the book will provide the basic and fundamental knowledge of understanding the concepts of applied mathematics explicitly. This book is prepared, taking into consideration the changing needs of the undergraduate and post graduate curricula of various universities involved in offering courses on applied mathematics. The book will also be a guide for the researchers involved in the application of mathematical models. The book will be of immense use to the students aspiring for civil service examinations and other state level officers examinations. The book would also provide the required mathematical knowledge to the students of State Agricultural Universities and other ICAR Institutions to sharpen their understanding of applied mathematics.

Dr. C. Kailasam
Ms. R. Pangayar Selvi
Mrs. R. Vasanthi

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We take this opportunity to thank our Vice Chancellor, Dr. P. Murugesa Boopathi, Tamil Nadu Agricultural University, Coimbatore for having provided the necessary working atmosphere, conductive environment and permission to bring out this valuable edition.

We would like to express our heartiest thanks to Dr. S. Santhanabosu, Dean (Engg) for his encouragement and well wishes.

We wish to express our regards to Dr. R. Palanisamy, Professor & Head, Department of Physical Sciences & Information Technology for his and continued cooperation to bring this publication in time.

Our sincere thanks are due to Dr. C. R. Ranganathan, Professor of Mathematics, Mr. M. Suresh, Assistant Professor of Statistics, Mrs. G. Mangayarkarasi, Assistant Professor of Mathematics and other faculties in the Department of Physical Sciences & Information Technology, AEC&RI, TNAU, for their motivation and well wishes.

We would like to thank all those individuals who have directly or indirectly contributed their time and energy to bring this edition successfully.

We thank Agrobios (India), Jodhpur, for their efforts in bringing out this book in record time.

We invite suggestions and critical comments from the reader for further improvement and refinement of this book.

C. Kailasam
R. Pangayar Selvi
R. Vasanthi

CONTENTS

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PERMUTATION AND COMBINATION

INTRODUCTION

Permutations and combinations are an integral part of modern life and we are using them all the time without even realizing it. They are widely applied in solving problems of probability, genetic engineering and life sciences. Suppose if we want to plant different seedlings in the boundary of the field or in the border of the roads, permutation plays a very significant role in decision making. For breeder's selection of different plant characters or genes breeding for efficient and more yielding crops, combinations play a very important role. We will discuss permutations and combinations, first defining these concepts, then showing examples, and relating them to practical applications.

FACTORIAL

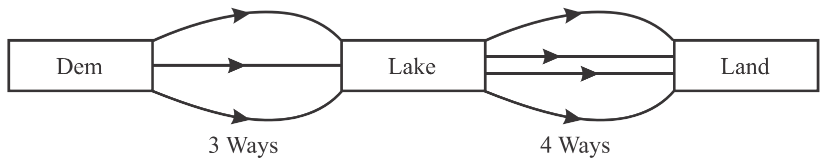
Permutation and Combinations involve operations with factorial notation. The factorial is the product of the integer's 'n' through '1' as n factorial and use the symbol n! or n to denote this; that is,

$$2! = 2 \times 1$$
$$3! = 3 \times 2 \times 1$$
$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$n! = n \times (n-1) \times \ldots \times 3 \times 2 \times 1$$

FUNDAMENTAL COUNTING PRINCIPLE

If one thing can be done in ***m*** ways and another thing can be performed in ***n*** ways then the total number of ways in which both the things can be done in succession is $m \times n$.

Let us see it with an example. Water is an important determinant factor of production of crops in agriculture sector. Intensive and extensive cultivation of land depend mainly on the availability of water. Consider the water sources like dam and lake and the water has to irrigate to land. As the system, lakes get supply from a permanent storage like dams etc. Assume that there are 3 ways to supply water from dam to lake and 4 ways from lake to land. Then the total number of ways we can supply water from dam to land via lake is $3 \times 4 = 12$ ways. This can be explained as follows.



For every channel supply of water from dam to lake and then to field there are 4 ways. Since there are 3 ways of channel from dam to lake, therefore the total number of ways to supply water is $3 \times 4 = 12$**.** From the twelve sources one can choose the most economic and efficient way.

PERMUTATION

Permutation means *arrangement* of things. The word *arrangement* is used, if the order of things *is* considered. Let us assume that there are 3 ornamental plants P_1, P_2, P_3 to be planted in front of the house. These 3 plants can be planted in three locations following 6 ways namely

Arrangement 1	P_1	P_2	P_3
Arrangement 2	P_1	P_3	P_2
Arrangement 3	P_2	P_1	P_3
Arrangement 4	P_2	P_3	P_1
Arrangement 5	P_3	P_1	P_2
Arrangement 6	P_3	P_2	P_1

Each arrangement is called a permutation. Thus there are 6 possible arrangements (permutations) of 3 plants taking all the 3 plants at a time. This we write as $3P_3$. Therefore $3P_3 = 3! = 3\times 2\times 1 = 6$.

Suppose out of the 3 seedlings we choose only 2 seedlings at a time to plant them in the two pots. How many arrangements are possible? For this consider 2 pots as shown in figure.

I Pot	II Pot

Since we want to plant only two seedlings and we have totally 3 seedlings, the first pot can be planted by any one of the 3 seedlings (i.e.) the first pot can be planted in 3 ways. After planting the first pot we are left with only 2 seedlings and the second pot can be planted by any one of these two seedlings. Therefore from Fundamental Counting Principle the total number of ways in which both the pots can be filled is $3 \times 2 = 6$. This we write as $3P_2 = 6$.

In general the number of permutations of n objects taking r objects at a time is denoted by nPr. Its value is given by:

$$
\begin{aligned}
n\mathrm{Pr} &= n(n-1)(n-2)...(n-r+1) \\
&= \frac{n(n-1)(n-2)....(n-r+1)\times(n-r)(n-r-1)...2.1}{(n-r)(n-r-1)....2.1}
\end{aligned}
$$

i.e.

$$n\mathrm{Pr} = \frac{n!}{(n-r)!}$$

Note:

(a) nPn = n !

(b) $nP_1 = n$.

(c) $nP_0 = 1$.

EX. 1. Evaluate $8P_3$

SOLUTION

$$8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!}$$

$$= \frac{8\times7\times6\times5\,!}{5\,!} = 336$$

EX. 2. Evaluate $11P_2$

SOLUTION

$$11P_2 = \frac{11!}{(11-2)!} = \frac{11!}{9!} = \frac{11\times10\times9!}{9\,!} = 110$$

EX. 3. There are 6 varieties on brinjal, in how many ways these can be arranged in 6 plots, which are in a line?

SOLUTION

Six varieties of brinjal can be arranged in 6 plots in $6P_6$ ways.

$$6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! \qquad [0! = 1]$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$$

Therefore 6 varieties of brinjal can be arranged in 720 ways.

EX. 4. In how many ways can 8 different flowers be strung into a garland so that 2 specified flowers are together?

SOLUTION

When garlanding out of 8 different flowers 2 specified flowers should be together. Consider the 2 specified flowers as one single unit. Now totally we have 7 units. The flowers can be arranged in $7P_7$ ways. i.e 7 ! ways. In every 7! Permutation 2 varieties of specified flowers can be rearranged among themselves in 2! ways.

Therefore the total number of ways:

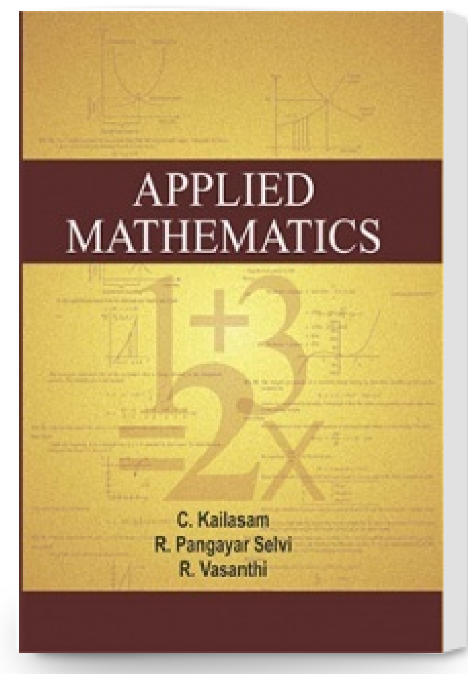
$$= 7!\ X2! = 5040\ x2 = 10080 \text{ ways}$$

EX. 5. There are 5 varieties of roses and 2 varieties of jasmine to be arranged in a row, for a photograph. In how many ways can they be arranged, if:

(i) all varieties of jasmine together

(ii) all varieties of jasmine are not together.

Applied Mathematics



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