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Analog Communication

Simplified Approach

SECOND EDITION

K. N. Hari Bhat

D Ganesh Rao

Analog Communications

A Simplified Approach

2nd Edition

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Dedicated to

Shobha, Vaishnavi, Aparna, Gururaj, Bhawana, and Vikram
Dr. K.N. Hari Bhat

Lord Shirdi Sai Baba, Parents, Teachers and
founder chairman M.S. Ramaiah, MSR group of institutions
Dr. D. Ganesh Rao

Preface

This book presents the basic principles and analysis involved in Analog Communication Systems. The text material provided may be broadly divided in to two viz: Analog communication techniques and their behavior under the influence of noise. The mathematical techniques involved in this book is at a moderate level. This is kept in mind in view of the readers who have a moderate background of differential and integral calculus. And still, the book provides exposure to some of the sophisticated analytical techniques such as Fourier transform, Spectral analysis and the performance of systems in the presence of noise. The material has been developed in an organized manner by including the necessary basic topics in separate chapters. For example, Chapter 1 gives an overview on Fourier transforms and its properties keeping in mind that knowledge of the subject is necessary to understand the various techniques discussed on Analog communication.

Chapter 2 develops various forms of Amplitude modulation techniques that include conventional AM, Double Side Band (DSB), Single Side Band (SSB), and Vestigial Side Band (VSB). Various forms of generation and detection of these schemes have been discussed. A detailed study in terms of bandwidth, spectral forms and other factors has been provided for each of the modulation techniques. A brief introduction on Television has been provided at the end of the chapter.

The major forms of Angle modulation techniques have been developed in Chapter 3. This includes Phase modulation and Frequency modulation. Starting with the basic terms such as time and frequency domain representations, it develops estimation of transmission bandwidth and discusses in detail the different types of generating wide-band and narrow band modulation techniques. Various forms of demodulation processes have been considered. An article on FM stereo multiplexing has been provided.

A discussion on Probability, Random Variables and Random Processes have been introduced in Chapter 4, in view of the essentiality of the subject to understand the behavior of noise present in different communication systems. This chapter is developed by discussing the basic elements and properties of probability and random process and focus on Gaussian distribution, Stationarity, white noise process, power spectral density and autocorrelation function.

Two chapters have been devoted to noise in view of the importance of the subject. Chapter 5 discusses the various forms of noise and their power spectral densities and bandwidths in a general perspective. The concept of noise figure and effective noise temperature are introduced. Analysis of cascade noisy system is performed. Chapter 6 deals with the noise in continuous-wave modulation systems. Noise in various types of AM and FM receivers and terms like signal-to-noise ratios, carrier-to-noise ratios, figure of merit have been developed.

Appendix A provides an insight to the basics of communication system discussing various points such as Electromagnetic wave propagation, their frequency ranges and typical applications, bandwidth spectrum and application of decibel – gain, loss, measurement, reference standards etc.

Appendix B provides a brief introduction to radio receivers and related terms like sensitivity and selectivity. Importance is given to Superheterodyne principle, different stages used and its advantages over other receivers.

The basic objective of preparing this book has been to make the readers to understand the difficult concepts by following a simplified approach and make studying the subject a pleasant experience by explaining it in a lucid and organized manner. A lot of illustrations, spectral and block diagrams and detailed derivations have been provided in the development of material. The authors sincerely believe that the goal is achieved and they will be happy to inform that this book is really useful to the students, academicians and working professionals as well.

The first author profusely thanks the Management, and the Principal Dr. K.S. Deshikachar, Nagarjuna College of Engineering and Technology, Bangalore for their support during the preparation of the book.

The second author acknowledges the encouragement given by the Management, and the Principal Dr. K. Rajanikant, M.S.Ramaiah Institute of Technology, Bangalore during the preparation of this book. Authors express their gratitude to their family members for their co-operation while penning the manuscript. The authors also acknowledge the contribution of Mr. R. Subramanian in different stages of development of the book.

The authors thank the typesetting team members of Cally Imaging Pvt. Ltd., for their wholehearted effort in making the book in a good format and the Publisher Mr. Lal. M. Prasad, M/s. Sanguine Technical Publishers, Bangalore for bringing out the book in an excellent form.

Any design is not complete and liable for improvement. Therefore we request the readers to mail their valuable comments, suggestions and criticism. We are available at sanguine_publishers@vsnl.net.

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Dr. D. Ganesh Rao

Contents

Chapter 1

THE FOURIER TRANSFORM

The Fourier transform (FT) provides a frequency-domain description of aperiodic signals. In brief, FT may be regarded as an extension of Fourier series (FS) as applied to aperiodic signals.

The Fourier transform of an aperiodic signal $x(t)$ is defined as follows:

$$X(f) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j2\pi ft}\, dt \tag{1.1}$$

The inverse Fourier transform, which allows us to obtain $x(t)$ from $X(f)$ is defined as follows:

$$x(t) = \int_{-\infty}^{\infty} X(f)\mathrm{e}^{j2\pi ft}\, df \tag{1.2}$$

The signal $x(t)$ and its Fourier transform $X(f)$ form a unique transform pair, and their relationship is shown symbolically using a double arrow:

$$x(t) \xleftrightarrow{\text{FT}} X(f) \tag{1.3}$$

The FT $X(f)$ is, in general complex and may be represented in the following exponential form:

$$X(f) = |X(f)|\mathrm{e}^{j\theta(f)} \tag{1.4}$$

For real signals, $X(f)$ is conjugate symmetric with $X(-f) = X^*(f)$. This implies that the magnitude $X(f)$ displays even symmetry and the phase $\theta(f)$ displays odd symmetry.

1.1. Effect of Signal Symmetry on the FT of Real-Valued Signals

- If $x(t)$ is real and even symmetric, then the FT $X(f)$ is real and even symmetric.
- If $x(t)$ is real and odd symmetric, then the FT $X(f)$ is imaginary and odd symmetric.
- If $x(t)$ is real and has no symmetry, then $\mathrm{Re}\{X(f)\}$ is even symmetric, and $\mathrm{Im}\{X(f)\}$ is odd symmetric.

1.2. Fourier Transform Pairs and Properties

Table 1.1 gives the Fourier transforms of some useful signals. It is to be noted that the Fourier transform of signals that grow exponentially or faster, does not exist. The reason for this lies in the nature of convergence of Fourier transform, which we reserve for discussion towards the end of this chapter. The Fourier transform is a linear operation and obeys the principle of superposition. Most commonly used properties of FT are summarised in Table 1.2 as a ready reckoner.

Table 1.1 ■ Some useful transform pairs

Sl. no.	$x(t)$	$X(f)$
1	$\delta(t)$	1
2	$\mathrm{rect}(t)$	$\mathrm{sinc}(f)$
3	$\mathrm{tri}(t)$	$\mathrm{sinc}^2(f)$
4	$\mathrm{sinc}(t)$	$\mathrm{rect}(f)$
5	$\cos(2\pi f_0 t)$	$\frac{1}{2}[\delta(f+f_0)+\delta(f-f_0)]$
6	$\sin(2\pi f_0 t)$	$\frac{1}{2j}[\delta(f+f_0)-\delta(f-f_0)]$
7	$\mathrm{e}^{-at}u(t)$	$\frac{1}{a+j2\pi f}$
8	$t\mathrm{e}^{-at}u(t)$	$\frac{1}{(a+j2\pi f)^2}$
9	$\mathrm{e}^{-a\|t\|}$	$\frac{2a}{a^2+4\pi^2 f^2}$
10	$\mathrm{e}^{-\pi t^2}$	$\mathrm{e}^{-\pi f^2}$
11	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
12	$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
13	$\mathrm{e}^{-at}\cos(2\pi f_0 t)u(t)$	$\frac{a+j2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$
14	$\mathrm{e}^{-at}\sin(2\pi f_0 t)u(t)$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
15	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

Table 1.2 ■ Properties of FT

Property	$x(t)$	$X(f)$
Duality	$X(t)$	$x(-f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Folding	$x(-t)$	$X(-f)$
Time shift	$x(t-t_0)$	$e^{-j2\pi f t_0}X(f)$
Frequency shift	$e^{j2\pi\beta t}x(t)$	$X(f-\beta)$
Convolution	$x(t) * h(t)$	$X(f)H(f)$
Multiplication	$x(t)h(t)$	$X(f) * H(f)$
Derivative	$\frac{dx(t)}{dt}$	$j2\pi f\ X(f)$
Modulation	$x(t)\cos 2\pi f_0 t$	$\frac{1}{2}[X(f+f_0)+X(f-f_0)]$
Multiplication by t	$-j2\pi t\ x(t)$	$\frac{dX(f)}{df}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{2}X(0)\delta(f)+\frac{1}{j2\pi f}X(f)$
Conjugation	$x^*(t)$	$X^*(-f)$
Correlation	$x(t)**y(t)$	$X(f)Y^*(f)$
Autocorrelation	$x(t)**x(t)$	$X(f)X^*(f)=\lvert X(f)\rvert^2$

1.3. Fourier Transform of Important Functions

(a) *The unit impulse:* Let $x(t)=\delta(t)$. Then

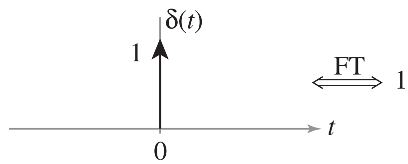
$$X(f) \triangleq \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\ dt$$

$$\Longrightarrow X(f) = \int_{-\infty}^{\infty} \delta(t)e^{-j2\pi f t}\ dt$$

$$= e^{-j2\pi f t}|_{t=0} \text{ (sifting property)}$$

$$= 1$$

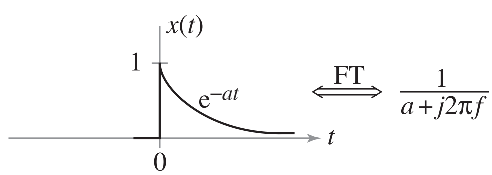
Thus, the spectrum of an impulse function is constant for all frequencies.



(b) *The decaying exponential:* Let $x(t) = \mathrm{e}^{-at}u(t)$. Then

$$\begin{aligned}X(f) &= \int_0^\infty \mathrm{e}^{-at}\mathrm{e}^{-j2\pi ft}\,dt \\ &= \int_0^\infty \mathrm{e}^{-(a+j2\pi f)t}\,dt \\ &= \frac{1}{a+j2\pi f} \\ \Longrightarrow |X(f)| &= \frac{1}{\sqrt{a^2+(2\pi f)^2}}\end{aligned}$$

The above expression means that the magnitude spectrum decays monotonically with frequency f.



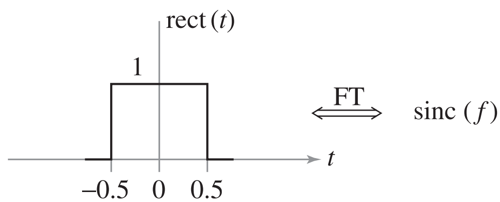
(c) *The rect function:* The signal

$$\begin{aligned}x(t) &= \mathrm{rect}(t) \\ &\triangleq \begin{cases}1, & |t| < 0.5 \\ 0, & |t| > 0.5\end{cases}\end{aligned}$$

Hence,

$$\begin{aligned}X(f) &= \int_{-0.5}^{0.5} 1 \times \mathrm{e}^{-j2\pi ft}\,dt \\ &= \left.\frac{\mathrm{e}^{-j2\pi ft}}{-j2\pi f}\right|_{t=-0.5}^{0.5} = \frac{\sin \pi f}{\pi f} \triangleq \mathrm{sinc}(f)\end{aligned}$$

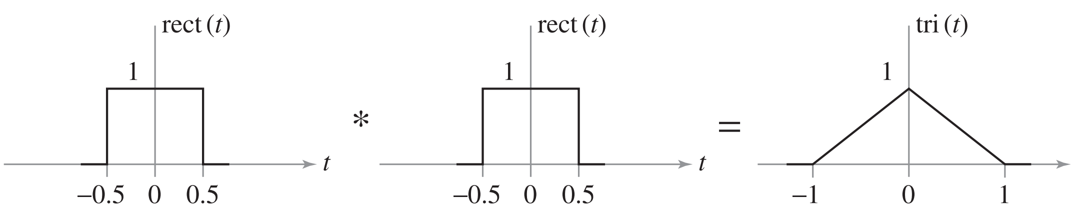
Thus, the magnitude spectrum of a rectangular pulse has a sinc form.



(d) *The tri function:* The signal

$$x(t) = \text{tri}(t)$$
$$\triangleq \begin{cases} 1 - |t|, & |t| \leqslant 1 \\ 0, & \text{elsewhere (width} = 2) \end{cases}$$

We can realise a triangular function as the convolution of two rectangular pulses as shown in the figure below.



That is, $\text{tri}(t) = \text{rect}(t) * \text{rect}(t)$. Hence

$$\text{tri}(t) \overset{\text{FT}}{\longleftrightarrow} \text{sinc}(f)\text{sinc}(f)$$
$$= \text{sinc}^2(f)$$

1.4. Fourier Transform of Periodic Signals

Consider a periodic signal $x(t)$. Then, the signal $x(t)$ has an exponential Fourier series representation given by

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{j2\pi k f_0 t} \tag{1.5}$$

Recall the transform pair: $e^{j2\pi kf_0 t} \overset{\text{FT}}{\longleftrightarrow} \delta(f - kf_0)$. Taking FT on both sides of equation (1.5), we get

$$X(f) = \sum_{k=-\infty}^{\infty} X(k)\delta(f - kf_0) \tag{1.6}$$

Thus, the FT of a periodic signal is an impulse train. The impulses are located at integer multiples of f_0, and the impulse strengths are equal to the Fourier coefficients $X(k)$. The impulse train is not, in general periodic, because the strengths $X(k)$ are different.

1.5. Parseval's Theorem

The Fourier transform is an energy-conserving relation, and the energy may be found from the time signal $x(t)$ or its spectrum $|X(f)|$:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df \tag{1.7}$$

This can be proved as follows. Let $x(t)$ be an energy signal. Then, the total energy of the signal $x(t)$ is

$$E = \int_{t=-\infty}^{\infty} |x(t)|^2 \, dt = \int_{t=-\infty}^{\infty} x(t)x^*(t) \, dt \tag{1.8}$$

Recall the inverse transform relationship:

$$x(t) = \int_{f=-\infty}^{\infty} X(f)e^{j2\pi ft} \, df$$

Taking conjugates on both sides, we get

$$x^*(t) = \int_{f=-\infty}^{\infty} X^*(f)e^{-j2\pi ft} \, df \tag{1.9}$$

Substituting equation (1.9) in equation (1.8), we get

$$E = \int_{t=-\infty}^{\infty} x(t) \int_{f=-\infty}^{\infty} X^*(f)e^{-j2\pi ft} \, df \, dt \tag{1.10}$$

Interchanging the order of integrals, we get

$$
\begin{aligned}
E &= \int_{f=-\infty}^{\infty} X^*(f) \int_{t=-\infty}^{\infty} x(t)e^{-j2\pi ft}\, dt\, df \\
&= \int_{f=-\infty}^{\infty} X^*(f)X(f)\, df \\
&= \int_{f=-\infty}^{\infty} |X(f)|^2\, df \qquad (1.11)
\end{aligned}
$$

It may be noted that in the above expression $|X(f)|$ is the magnitude spectrum. The Parseral's theorem for periodic signals is stated below without proof.

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |X(k)|^2$$

1.6. Existence of Fourier Transforms

For the existence of FT, Dirichlet conditions are sufficient conditions that are given below.

- $x(t)$ is absolutely integrable. That is,

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty.$$

- $x(t)$ has a finite number of maxima and minima and a finite number of discontinuities.

The Dirichlet conditions guarantee a Fourier transform if they are satisfied. However, they are only sufficient conditions. That is, Fourier transforms may still exist for certain functions even if the Dirichlet conditions are violated. For example, the signals $u(t)$ and $r(t)$ are not absolutely integrable but still posses Fourier transforms (that invariably contains impulses). In a nutshell, we can state that the FT of absolutely integrable signals always exist, the FT of signals that are not absolutely integrable but do not grow exponentially almost always include impulses, and the FT of signals that grow exponentially, for example, $e^{\alpha t^2}$, does not exist.

1.7. System Analysis Using the Fourier Transform

The FT is a tool for computing the system response of an initially relaxed LTI system because of the fact that FT maps time-domain convolution into frequency-domain multiplication. That is,

$$y(t) = x(t) * h(t) \xleftrightarrow{\text{FT}} Y(f) = X(f)H(f) \qquad (1.12)$$

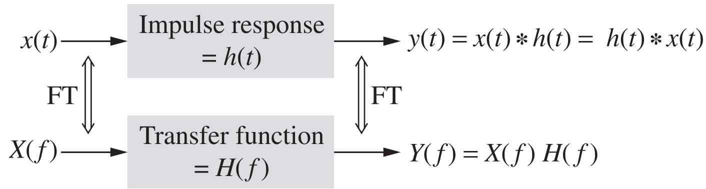


Figure 1.1 ■ Input–output relations in the time-domain and frequency-domain.

In the above expression, the quantity $H(f)$ is defined as the transfer function or the system function. It may be noted that $H(f)$ is also the FT of the impulse response $h(t)$ of the system. The input–output relations of an LTI system in the time-domain and frequency-domain are depicted in Figure 1.1.

1.8. Energy and Power Spectral Density

The Parseval's theorem for energy signals, $E = \int_{-\infty}^{\infty} |X(f)|^2\, df$ says that the total signal energy equals the area of the squared magnitude spectrum $|X(f)|^2$. We define, the quantity, $E_x(f) = |X(f)|^2$ as the energy spectral density (ESD). It has the unit Joules per hertz (JHz^{-1}). It should be noted that $E_x(f)$ is always a real, nonnegative, and even function of f. The inverse Fourier transform of ESD $E_x(f)$ is an autocorrelation function. That is,

$$\underset{\substack{\text{(autocorrelation}\\ \text{function)}}}{r_{xx}(t)} \quad \overset{\text{FT}}{\longleftrightarrow} \quad \underset{\text{(ESD)}}{E_x(f)} \tag{1.13}$$

The above equation is known as Wiener–Khintchine theorem. Involving the definition of inverse Fourier transform, we have

$$\begin{aligned} r_{xx}(t) &= \int_{f=-\infty}^{\infty} E_x(f) e^{j2\pi ft}\, df \\ \Longrightarrow r_{xx}(0) &= \int_{-\infty}^{\infty} E_x(f)\, df \\ &= \int_{-\infty}^{\infty} |X(f)|^2\, df = E \end{aligned} \tag{1.14}$$

Thus, $r_{xx}(0)$ equals the energy in $x(t)$.

For power signals, we use averaged measures consistent with power. This leads to the concept of power spectral density (PSD) denoted by $S(f)$. The PSD of aperiodic signal is a continuous function whose area is equal to the total signal power. The unit of PSD is watts per hertz (WHz^{-1}). The spectrum of a periodic signal with period T is a train of impulses located at $f = kf_0$ (where $f_0 = 1/T$) with strengths $X(k)$. The PSD of a periodic signal is a train of impulses at $f = kf_0$ with strengths $|X(k)|^2$ whose total area (the sum of impulse strengths) is equal to the total signal power. That is, PSD of periodic signal $x(t)$

$$S_x(f) = \sum_{k=-\infty}^{\infty} |X(k)|^2 \delta(f - kf_0)$$

and the total power

$$\int_{-\infty}^{\infty} S_x(f)\, df = \sum_{-\infty}^{\infty} |X(k)|^2 \tag{1.15}$$

It is important to realise that the PSD is not a unique indicator of the underlying time signal since many time signals can give the same PSD. Also note that for power signals, $r_{xx}(0)$ is equal to the average power in $x(t)$.

SOLVED PROBLEMS

Problem 1.1 ■ Find the Fourier transform $X(f)$ for each of the signals given below.

(a) $x(t) = 2t\ \mathrm{rect}(t)$
(b) $x(t) = \mathrm{e}^{-2|t|}$

Solution (a) Refer the sketch given in Figure P1.1(a).

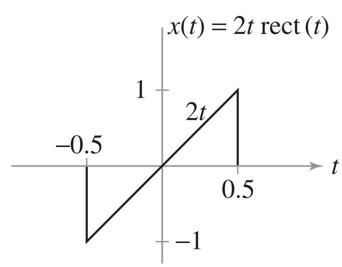


Figure P1.1(a) ■

$$X(f) \triangleq \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j2\pi ft}\ dt$$

$$\Longrightarrow X(f) = \int_{-1/2}^{1/2} 2t\mathrm{e}^{-j2\pi ft}\ dt$$

$$= \int_{-1/2}^{1/2} 2t[\cos 2\pi ft - j\sin 2\pi ft]\ dt$$

$$= \int_{-1/2}^{1/2} 2t\cos 2\pi ft\ dt - j\int_{-1/2}^{1/2} 2t\sin 2\pi ft\ dt$$