

# OPERATIONS RESEARCH METHODS

RELATED PRODUCTION, DISTRIBUTION, AND  
INVENTORY MANAGEMENT APPLICATIONS



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# PREFACE

Business managements make daily decisions on many issues, such as how much and where to produce, for which market, what prices to set, and how much stock to keep. Mathematical models can help to make the best decisions, among the possible alternatives.

The objective of this book is to present a survey of selected Operations Research methods, and some of their applications to managerial decision making, concerning production, inventory, distribution, and integrated supply chain modelling. Also included are listing of some computer programs to compute operations research models.<sup>1</sup>

*Operations Research* (O.R.) has been termed *The Science of Better*. A problem in the real world is modelled, usually in mathematical terms, then mathematical techniques, together with data analysis and computational algorithms, are applied, in order to find ways to do the job better. The word *Operations* derives from the many successful applications of O.R. to military operations in the 1940s. But, since then, most O.R. applications have been to peaceful activities, especially to business management, of which planning industrial production, and scheduling airlines, and other transportation, have been prominent. The name *Management Science* denotes the same discipline, with some emphasis on business management. Practitioners of *Operations Management* will find many of these techniques relevant. The areas of Logistics, Supply Chain Management, Decision Sciences, and Manufacturing Management deal with similar applications.<sup>1</sup>

This book is concerned with O.R. methods. The commencing chapter (numbered chapter 0 – computers count up from zero) gives a general discussion of model building, various examples of their applications, and some discussion of the limitations of some models. It is suggested that an O.R. practitioner should understand the models and the techniques, and, while using computers and computer packages extensively, should not depend on them to decide a model (possibly inappropriate) to use.

Chapter 1 discusses *linear programming*. As well as the mathematics, section 1.16 on *cost data* discusses when such models are appropriate, and what sort of data must be sought.

Chapter 2 discusses *dynamic programming*, and several of its many applications, to capital budgetting and to a rental problem. The effect of random elements is also considered.

Chapter 3 discusses the *critical path method*.

Chapter 4 discusses *planning over time*. This includes discussion of interest rates and present value, effect of inflation, risk-averse utility, decision trees and planning over time, and forecasting. A concise introduction to the use of spreadsheets for such planning and calculations is given in chapter 9.

Chapter 5 discusses *inventory*, presenting various versions of

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<sup>1</sup> See Ragsdale (2001), and Gass and Harris (2001), for various such applications.

Some programs written in FORTRAN AND GAMS are included.

*economic order quantity*, with applications including seasonal demand, discounting, and the issues that arise when inventory and production must be managed together, as especially in *supply chain* management.

Chapter 6 outlines the topic of *networks*, which often arise in planning models.

Chapter 7 discusses various methods for *nonlinear optimization*, and lists some of their applications. Although many models are described by linear equations, nonlinear models are also often needed.

Chapter 8 discusses *simulation* techniques, with applications to queue, inventory and storage models, and also to networks models. Often simulation can obtain results, when no exact formulas are possible. The discussion of simulation methods, both those based on events and those planned on a time scale, should enable a user to understand what a computer package for simulation should be doing, and perhaps write his own simulation program.

In chapter 9, a number of related computer programs are listed.

For these various methods and applications, the basic methods and primary applications are presented, together with some reviews of recent developments.

In comparison with some other books on the methods of Operations Research, the present book gives a concise account of the methods and applications. A moderate mathematical background is assumed (some calculus and matrix algebra). A reader is assumed to want to understand the methods, and the modelling assumptions made, rather than leave it all to a computer package. Some aspects that may be innovative include the following:

- A detailed discussion (in chapter 0) of O.R. modelling, including the difficulties involved, and various areas of application to real problems.
- A discussion (section 1.16) of cost data, replacing the common (inadequate) assumption that everything is linear.
  - Various topics (in chapter 4) concerning planning over time, including discussion of risk-averse utility, and stability analysis.
  - Some critical discussion (sections 5.11 and 5.12) of supply chain models.
  - Some critical discussion (sections 7.19 and 7.20) of multiobjective optimization, and of optimization when the usual convexity assumptions fail.

This book presents phases and aspects of applied operations research studies with the emphasis on business, logistics, and operation management applications. This book includes many examples of applications of operations research methods in those areas. It can be used as a text or a reference for a course on operations management and logistics/supply chain management at the Masters or Doctoral level and a text for undergraduate students in operations research. It will also be of importance to corporate executives, academics, practitioners, and business consultants.

# TABLE OF CONTENTS

Chapter 0 Introduction to Operations Research and Applications	
0.1 Mathematical models for management	1
0.2 Some reference books	2
0.3 Initial example	3
0.4 Check-list, when setting up an Operations Research model	3
0.5 Some other examples of Operations Research models in business	6
0.6 Importance of model building, and limitations of computers	11
0.7 Why the name?	12
0.8 The usefulness of spreadsheets	12
0.9 Presentation of conclusions and recommendations	12
Chapter 1 Linear Programming	
1.1 Introduction – management models	14
1.2 Standard form for a linear program	16
1.3 Idea of simplex method	16
1.4 Simplex tableaus	17
1.5 Information from tableau	18
1.6 Two-phase method	19
1.7 Unsigned variables	20
1.8 Revised simplex method	20
1.9 Dual linear program	22
1.10 Dual properties	23
1.11 Sensitivity	25
1.12 Dual simplex method	25
1.13 Jumps in shadow costs	26
1.14 Integer linear programming	27
1.15 Transportation problem	29
1.16 Cost data (piecewise linear cost function)	33
1.17 Some exercises on linear programming	36
1.18 Computer input/output for linear programming	40
1.19 A note on interior-point methods	43
Appendix: Some theory of linear programming	44
Chapter 2 Dynamic Programming	
2.1 Introduction	46
2.2 A routing problem	46
2.3 Why does this work?	47
2.4 Forward analysis	48
2.5 An investment (or capital budgetting) problem	48
2.6 A model for inventory and production	49
2.7 A rental problem	51
2.8 Stochastic inventory	52
2.9 Some exercises and examples	53
Chapter 3 Critical Path Method	
3.1 Introduction	55

3.2	Example of CPM	55
3.3	Method of calculation	56
3.4	Example of computer output	57
3.5	Alternative presentation	58
3.6	Random variation in time durations	59
3.7	Exercises	59
Chapter 4 Planning over Time, Uncertainty and Forecasting		
4.1	Present value	61
4.2	Annual cost and rate of return	62
4.3	Allowing for inflation?	62
4.4	Different interest rates	63
4.5	Decision trees	64
4.6	Risk-averse utility	68
4.7	Decision tree examples and exercises	69
4.8	Stability over time	70
4.9	Indifference curves	71
4.10	Forecasting and exponential smoothing	72
Chapter 5 Inventory Management		
5.1	What is inventory?	74
5.2	Economic order quantity	74
5.3	Inventory model for constant sales rates	75
5.4	Inventory model for variable demand	76
5.5	Sequencing a number of products	77
5.6	What stock to carry of a critical component?	78
5.7	Managing inventory and production together	78
5.8	Seasonal demand	80
5.9	Discounted and floor prices	80
5.10	The newsboy problem	81
5.11	Supply chain management	81
5.12	Examples of supply chain models	83
5.13	Some exercises	85
Chapter 6 Network Planning		
6.1	Why networks?	86
6.2	Idea of out-of-kilter algorithm	87
6.3	Idea of a shortest-path algorithm	88
6.4	Idea of Ford-Fulkerson transportation method	88
6.5	Branch and bound	89
6.6	Remarks on mixed-integer linear programming	92
6.7	Job-shop scheduling	92
6.8	Examples	94
Chapter 7 Nonlinear Optimization Methods		
7.1	Introduction	95
7.2	Separable programming	95
7.3	Convergence and convergence rate	96
7.4	Lagrange multipliers	97

7.5	Sensitivity	99
7.6	Computing an iterative algorithm	99
7.7	Descent methods	100
7.8	Fletcher–Reeves algorithm	101
7.9	Davidon–Fletcher–Powell (DFP) algorithm	101
7.10	Example	102
7.11	Linesearch	103
7.12	Descent methods and stability	105
7.13	Constrained minimization	106
7.14	Sequential unconstrained minimization technique (SUMT)	107
7.15	Projected gradient	107
7.16	Quadratic programming by Wolfe’s method	108
7.17	Further comments on constrained minimization	109
7.18	Multiobjective optimization	112
7.19	Invexity	113
7.20	Some applications of nonlinear programming	114
Chapter 8 Simulation		
8.1	Idea of simulation	115
8.2	Queue and storage models	115
8.3	Examples of simulation	118
8.4	Getting pseudorandom numbers	120
8.5	Some simulation models	122
8.6	Some other methods and models	122
Chapter 9 Some Computer Programs		
9.1	Remarks on computer programs for O. R. problems	125
9.2	Using spreadsheets	126
9.3	Using Excel Solver	129
9.4	An example of a GAMS package for capital budgetting	131
9.5	Introduction to the FORTRAN programming language	133
9.6	A linear programming package lpZSq	138
9.7	Documentation for the linear programming package lpZSq	144
9.8	A FORTRAN program for critical path	147
9.9	A program for one application of dynamic programming	151
9.10	FORTRAN programs to illustrate methods for unconstrained optimization	153

Bibliography

160

## Chapter 0 INTRODUCTION TO OPERATIONS RESEARCH AND APPLICATIONS

### 0.1 Mathematical models for management

A business firm must make decisions on many issues relating to production, distribution, consumers, shareholders, information processing, employees, society, pricing etc.. These decisions typically concern production and inventory planning, sales forecasting, capital budgeting, investment planning, materials requirement planning, locational decisions, personnel management and planning, pricing, distribution, management and planning, integrated supply chain or logistics management and planning, as they affect the several parts of the firm. These questions are sometimes considered under the heading of Operations Management.

Business decision making requires the choice of the best decision among alternatives, or at any rate a decision that gives a substantial improvement. The objectives may be revenue maximisation, cost minimisation, satisfactory performance regarding social responsibility, shareholder value maximisation, etc., or sometimes the survival of the firm in adverse circumstances. The decisions are constrained by requirements such as budget and resource constraints. Methods of Operations Research (O.R.) are well adapted to such decision making in business. Applications of OR to business management have been discussed in different disciplines such as Management Science, Operations Management, Logistics Management, Supply Chain Management, and Decision Sciences.

In Operations Research, we set up and use mathematical models, usually related to questions of planning in business, industry, or management. Any model of a real-life situation must simplify it greatly, by picking out those factors we think important for our purpose, and neglecting the rest. Otherwise, we can't calculate, or predict. (Newton approximated the sun and earth by masses concentrated at points.

This approximation succeeded). We must consider whether we have made a right selection. What check or validation is possible?

To put our model in *mathematical terms* makes us formulate our ideas precisely – what exactly are we assuming? – and so we are less likely to make hidden assumptions without knowing it. Moreover, once we have a model in mathematical terms, we can manipulate it effectively using concise mathematical language. Also calculations, using a computer, are more readily set up. The model results provide useful information for making business decisions.

The factors we recognize, as affecting the situation, consist of (i) external factors ("exogenous factors"), which we regard as inputs to the system, and whose effects we shall study, (ii) factors the model tries to explain ("output", "dependent", or "endogenous" factors), and

(iii) factors we shall neglect. For example, if we are hired by a firm to determine the level of production that will maximize profits, then profit becomes an "output" for the system, and market factors become the "inputs". But, for an economist who wants to explain production levels, profit could be an internal variable (internal to the system being studied, neither an input nor an output). We should be clear what is the aim of our investigation. It often falls to the O.R. investigator to clarify the objective. This involves much discussion with the propounder of the problem, to find out "where the shoe is really pinching". For example, it could be irrelevant to optimize a production schedule, if the real problem is labor relations.

We can fall on our face by neglecting some essential factors. A century ago, some mathematicians proved that aeroplanes were impossible; they had neglected the essential "boundary layer", where viscosity of the air plays an essential role. Likewise, in a business model, it is possible to omit an essential factor – for example there may be an upper bound on certain resources, which must not be forgotten. This course has to present examples from books; however, in a real-world situation, always go and see for yourself (the factory floor, or the airport, or whatever) if you possibly can – it is usually different from what was supposed.

## 0.2 Some reference books (a few out of the many published.)

The codes <DG> etc. are here for cross-referencing.

Note that a book such as <DG>, which includes discussion of building suitable models, is of more value to an O.R. practitioner than a book that mainly presents the computational algorithms. In practice, the calculations are done on computers; however, one needs some understanding of the algorithms, otherwise one never knows whether the computer is producing sense or nonsense.

<DG> H.C. Daellenbach, J.A. George, D.C.. McNickle, 1984 (1st. ed.) or 1978 (2nd. ed.). Introduction to Operations Research (Allyn & Bacon).

<a good balance between model building and the mathematics>

<HW> R. Hesse and E. Woolsey, 1980. Applied Management Science (a quick and dirty approach), (Science Research Associates).

<good for practical " real-world" details of application>

<B+> K.H. Bradshaw and others, 1982. An Operations Research Casebook (Longman Cheshire).

<the source of many miniproject topics for an O. R. course>

J.C. Ecker and M. Kupferschmid, 1988. Introduction to Operations Research (Wiley).

<MD> M.J.C. Martin and R.A. Denison, 1971. Case Exercises in Operations Research ,(Wiley-Interscience)

<a good source of many project topics>

E.A. Bender, 1970. An introduction to mathematical

modelling (Wiley/Interscience).

H. A. Taha, 1982. Operations Research (Macmillan).

<better on calculation examples than model building>

B. Srinivasan and C.L. Sandblom, 1989, Quantitative Analysis for Business Decisions (McGraw-Hill).

<includes some useful examples of decision trees>

<SDK> M. Syslo, N. Deo, J. Kowalik, 1983. Discrete Optimization Algorithms with Pascal Programs, (Prentice-Hall).

<See especially chapter 3: Optimization on Networks>

<WS> R.E.D. Woolsey and H. Swanson, 1969, 1975. Operations Research for Immediate Application A Quick and Dirty Manual, (Harper & Row). <see especially chapters 6 and 7>

<MW> S. Makridakis and S. C. Wheelwright, 1989. Forecasting Methods for Management (Wiley).

This list is not meant to exclude other, and more recent, textbooks. Reference may be made to Thompson and Thore (1992), Ragsdale (2001), and Vollman et al. (2005), listed in the bibliography, for more examples of O. R. models in management. But it is not recommended to use a text that is mainly a manual for a particular computer package (see section 0.6).

### 0.3 Initial example

Think of a factory, whose running cost (which is to be minimized) is a function of various production variables (how much do we make of which product on which machine for which market?), subject to constraints (limits on material resources, labor, time available, market requirements). This may (but need not always) reduce to a linear programming model). Note that we must, later, be specific as to what we mean by cost. The following check-list (in 0.4) applies specifically to linear programming models, but also gives general guidance in setting up other kinds of O.R. models.

### 0.4 Check-list, when setting up an Operations Research model

The stages of an applied operations research study have been listed as follows (following Taha, 1992):

- a. defining the research or the policy problem
- b. developing the operations research model for the relevant system
- c. implementing/solving the model numerically by some suitable algorithms and computer programs
- d. undertaking validation tests of the model and its results
- e. analysing the implications of the model results for actions or decisions related to the issues under the study.

A more detailed check-list is as follows:

- (1) . *Find out what the problem really is!* It won't come to you packages

in mathematical terms. Ask simple, relevant questions. Go and see for yourself. "Obvious" assumptions can be wrong.

(2) *What are the variables of the problem?* What, in fact, can you vary? If you seek to optimize, what is the objective function? Or are there several? Check that the variables relate to the decisions that must be made. Would they give us enough information to act on? Many variables cannot be negative. What alternatives are possible?

(3) *List the constraints.* They often include: input constraints (restrictions or requirements on raw materials), capacity constraints (e.g. available machine time, pipeline capacity, storage available, rate at which a product can be made), materials balance ("what goes in must come out", e.g. old inventory + production – part used to make new product = new inventory + deliveries to customers), output constraints (requirements, or limits, on sales, production, or rate of return). Check that units of measurement are consistent, or make them so. (A factory once measured its storage tank capacities in cubic feet, but its flow rates – inconsistently – in gallons per minute. For mathematical modelling, the flow rates were converted to cubic feet per hour – to avoid hopeless confusion. Today, SI units (metres, kilograms, etc.) would be preferred).

(4) *What data are needed?* You may need all your tact to get it! Exact figures may be unobtainable, or make little sense; maybe get upper and lower estimates instead. Are the cost figures you get relevant? If you are planning production schedules, then likely costs of raw materials and energy enter; wages may or may not, depending on the time scale of planning.

(5) *Check out a simplified example!* So, find any blunders in your model, or your computer program, before you risk your reputation. And get quickly a "ballpark estimate" to please the boss! (Don't be frightened of a rough "back of envelope" calculation for this limited purpose – if you can do it quickly!) Do the results make practical sense? Are the computations, perhaps, only telling you what everyone in the industry has known for twenty years?

terms, not mathematical symbols)? Note the importance of writing a report, intelligible to management. *Don't* expect managers to cope with a schedule of how much to make of which product, if it is expressed as:  $X_i = 17.5 \leq x \leq 29.7$  and so on!

Instead, tell them, in their language! Something like:

*Daily production schedule* Dinguses 17.5 tonnes Whatsits 29.7 tonnes and so on.

(7) Model Validation and Verification.

In Gass and Harris (2001, p. 865), model validation is defined as *the process of determining how well the outputs of a mathematical model of real-world problem conform to reality*. Model validation is often synonymously used with other terms such as model verification and model testing. Model validation is one of the important steps of an O.R. study. In an applied O.R. study, the reliability of the O.R. model and its results need to be tested. For this purpose, the usual model validation criteria can be adopted. According to Hazell and Norton (1986, p. 269) model validation and its purposes are as follows "Validation of model is a process that leads to (1) a numerical report of the models fidelity to the historical data, (2) improvements of the model as a consequence of imperfect validation, (3) a qualitative judgment on how reliable the model is for its stated purposes, and (4) a conclusion (preferably explicit) for the kinds of uses it should not be used for"

Several criteria can be used to test the validation of a model (see Gass and Harris, 2001; Labys, 1982; Taha, 1992; Hazell and Norton, 1986) at three levels of validation tests: descriptive, analytical and experimental. Three different types of validation criteria are applied to these three levels of validation tests (although 2 types of validations are suggested in Gass and Harris (2001) which are face validity and predictive validity) (see Kresge 1980 for further details):

(i) Descriptive validation criteria:

The attainment of the objectives of the model.

The appropriateness of the model structure and the plausibility of results.

(ii) Analytical validation criteria:

The plausibility and characteristics of models and their results.

The robustness of the results.

(iii) Experimental validation criteria:

Methodological tests of model documentation.

Cost and efficiency in model, storage, transfer and extension.

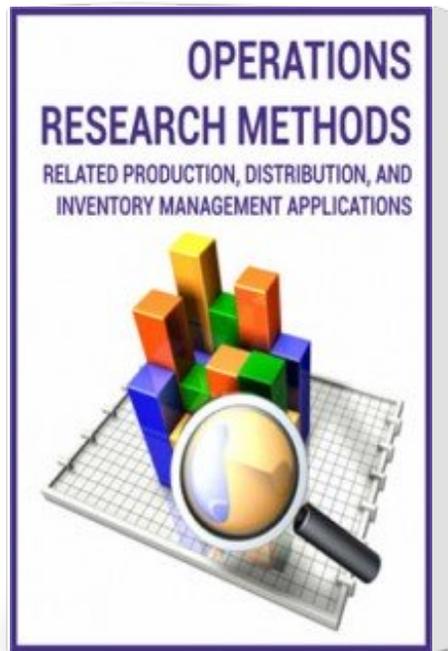
Tests for accuracy and efficiency of implementation as well as cost of and efficiency in the software transfer, storage and extension.

The above is a detailed set of model validation criteria for O.R. models. In a real life O.R. study often some simple validation tests are used (Hazell and Norton, 1986): plausibility of results, and comparisons of model results with actual or historical results, and other model results. Also, it is important to compare the model predictions with an *independent* set of observations. Those may be observations from a different time period, from the period whose data were used to estimate the parameters of the model.

(8) *What happens if something changes?*

Murphy's law makes sure it does ! How sensitive are your conclusions to small changes in the data ? (The jargon phrases are "post-optimality analysis" for linear programming, or (more generally)

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